8. Relaciones binarias y conjuntos ordenados

Ejemplos con Mathematica

1. Relaciones binarias

```
A = {a, b, c};
R = {{a, a}, {b, b}, {a, b}, {b, c}};
```

■ Ejemplo 8.1.

Sean A = {a, b, c, d, e, f, g, h} y R = {(a, a), (b, b), (c, c), (d, d), (e, e), (f, f), (g, g), (h, h), (a, b), (b, a)}. Determinar si R es reflexiva, simétrica, transitiva o antisimétrica, y comprobar si R es relación de orden o de equivalencia.

```
A = {a, b, c, d, e, f, g, h};
R = {{a, a}, {b, b}, {c, c}, {d, d},
   {e, e}, {f, f}, {g, g}, {h, h}, {a, b}, {b, a}};
Reflexiva = True; For[n = 1, n ≤ Length[A], n++,
 If[Intersection[{{A[[n]], A[[n]]}}, R] == {{A[[n]], A[[n]]}},
  Null, Reflexiva = False]]; Simetrica = True;
For[m = 1, m ≤ Length[R], m++, If[Intersection[{{R[[m, 2]], R[[m, 1]]}}, R] ==
   {{R[[m, 2]], R[[m, 1]]}}, Null, Simetrica = False]];
Transitiva = True; For[p = 1, p ≤ Length[R], p++,
 For[q = 1, q ≤ Length[R], q++,
   If[R[[p, 1]] == R[[q, 2]], If[Intersection[{{R[[q, 1]], R[[p, 2]]}}, R] ==
       {{R[[q, 1]], R[[p, 2]]}}, Null, Transitiva = False]];];];
Antisimetrica = True; For[r = 1, r ≤ Length[R], r++,
 If[Intersection[{{R[[r, 2]], R[[r, 1]]}}, R] == {{R[[r, 2]], R[[r, 1]]}} &&
   ! (ToString[R[[r, 1]]] == ToString[R[[r, 2]]]), Antisimetrica = False]];
If[Reflexiva, Print["R es reflexiva"], Print["R no es reflexiva"]]
 If[Simetrica, Print["R es simétrica"], Print["R no es simétrica"]]
 If[Transitiva, Print["R es transitiva"], Print["R no es transitiva"]]
 If[Antisimetrica, Print["R es antisimétrica"],
  Print["R no es antisimétrica"]];
If[Reflexiva && Simetrica && Transitiva,
 Print["R es una relación de equivalencia"],
 Print["R no es relación de equivalencia"]];
If[Reflexiva && Antisimetrica && Transitiva,
 Print["R es una relación de orden"], Print["R no es relación de orden"]]
```

```
R es reflexiva
```

```
R es simétrica
```

```
R es transitiva
```

```
R no es antisimétrica
```

```
R es una relación de equivalencia
```

```
R no es relación de orden
```

2. Máximos y mínimos

■ Ejemplo 8.2.

Sea A = {a, b, c, d, e, f, g, h, i} un conjunto ordenado con la relación binaria: R = {(a, a), (a, b), (a, c), (a, d), (a, e), (a, f), (a, h), (a, i), (b, b), (b, e), (b, f), (b, h), (b, i), (c, c), (d, d), (d, e), (d, f), (d, h), (d, i), (e, e), (e, f), (e, h), (e, i), (f, f), (f, i), (g, g), (g, i), (h, h), (h, i), (i, i)}. Calcular, si existen, el máximo y el mínimo de A.

```
A = {a, b, c, d, e, f, g, h, i};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
maximo = {}; Do[maxi = True; Do[If[Intersection[{{A[[m]], A[[n]]}}, R] == {},
   maxi = False], {m, 1, Length[A]}];
 If[maxi, AppendTo[maximo, A[[n]]]];, {n, 1, Length[A]}];
If[maximo == {}, Print["No tiene máximo"], Print["Máximo: ", maximo[[1]]]]
```

```
No tiene máximo
```

```
A = {a, b, c, d, e, f, g, h, i};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
minimo = {}; Do[mini = True; Do[If[Intersection[{{A[[n]], A[[m]]}}, R] == {},
   mini = False], {m, 1, Length[A]}];
 If[mini, AppendTo[minimo, A[[n]]]];, {n, 1, Length[A]}];
If[minimo == {}, Print["No tiene mínimo"], Print["Mínimo: ", minimo[[1]]]]
```

```
No tiene mínimo
```

3. Elementos maximales y minimales

■ Ejemplo 8.3.

Sea A el mismo conjunto ordenado del ejemplo 8.2., calcular sus elementos maximales y minimales.

```
A = {a, b, c, d, e, f, g, h, i};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
maximales = {}; Do[maximal = True;
 Do[If[Intersection[{{A[[n]], A[[m]]}}, R] ≠ {} && n ≠ m, maximal = False],
  {m, 1, Length[A]}]; If[maximal, AppendTo[maximales, A[[n]]]];,
 {n, 1, Length[A]}]; Print["Maximales: ", maximales]
```

```
Maximales: {c, i}
```

```
A = {a, b, c, d, e, f, g, h, i};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
minimales = {}; Do[minimal = True;
 Do[If[Intersection[{{A[[m]], A[[n]]}}, R] ≠ {} && n ≠ m, minimal = False],
  {m, 1, Length[A]}]; If[minimal, AppendTo[minimales, A[[n]]]];,
 {n, 1, Length[A]}]; Print["Minimales: ", minimales]
```

```
Minimales: {a, g}
```

4. Cotas superiores e inferiores. Supremo e ínfimo

■ Ejemplo 8.4.

Consideremos el mismo conjunto A ordenado del ejemplo 8.2. y sea B el subconjunto de A dado por B = {a, e, f, g, h}. Calculamos todas las cotas superiores e inferiores y el supremo e ínfimo, si existen.

```
A = {a, b, c, d, e, f, g, h, i};
B = {a, e, f, g, h};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
cotassuperiores = {}; Do[csuper = True;
 Do[If[Intersection[{{B[[m]], A[[n]]}}, R] == {}, csuper = False],
  {m, 1, Length[B]}];
 If[csuper, AppendTo[cotassuperiores, A[[n]]]];, {n, 1, Length[A]}];
If[cotassuperiores == {}, Print["No hay cotas superiores"],
 Print["Cotas superiores: ", cotassuperiores]; supremo = {}; Do[mini = True;
  Do[If[Intersection[{{cotassuperiores[[n]], cotassuperiores[[m]]}}, R] ==
      {}, mini = False], {m, 1, Length[cotassuperiores]}];
  If[mini, AppendTo[supremo, cotassuperiores[[n]]]];,
  {n, 1, Length[cotassuperiores]}]; If[supremo == {},
  Print["No tiene supremo"], Print["Supremo: ", supremo[[1]]]]];
```

```
Cotas superiores: {i}
```

```
Supremo: i
```

```
A = {a, b, c, d, e, f, g, h, i};
B = {a, e, f, g, h};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
cotasinferiores = {}; Do[cinfer = True;
 Do[If[Intersection[{{A[[n]], B[[m]]}}, R] == {}, cinfer = False],
  {m, 1, Length[B]}];
 If[cinfer, AppendTo[cotasinferiores, A[[n]]]];, {n, 1, Length[A]}];
If[cotasinferiores == {}, Print["No hay cotas inferiores"],
 Print["Cotas inferiores: ", cotasinferiores]; infimo = {}; Do[maxi = True;
  Do[If[Intersection[{{cotasinferiores[[m]], cotasinferiores[[n]]}}, R] ==
      {}, maxi = False], {m, 1, Length[cotasinferiores]}];
  If[maxi, AppendTo[infimo, cotasinferiores[[n]]]];,
  {n, 1, Length[cotasinferiores]}];
 If[infimo == Print["No tiene ínfimo"], Print["Ínfimo: ", infimo[[1]]]]]];
```

```
No hay cotas inferiores
```

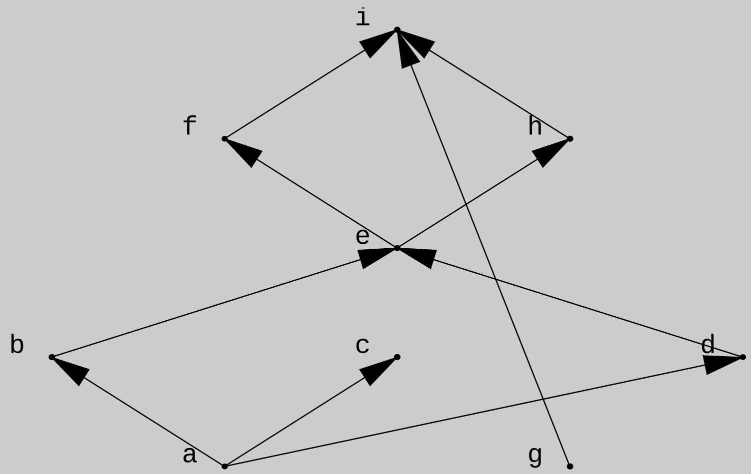
5. Diagramas de orden

■ Ejemplo 8.5.

Consideremos el mismo conjunto A ordenado del ejemplo 8.2. Determinar su diagrama de orden.

```
<< Graphics`Arrow`
A = {a, b, c, d, e, f, g, h, i};
R = {{a, a}, {a, b}, {a, c}, {a, d}, {a, e}, {a, f},
   {a, h}, {a, i}, {b, b}, {b, e}, {b, f}, {b, h}, {b, i}, {c, c},
   {d, d}, {d, e}, {d, f}, {d, h}, {d, i}, {e, e}, {e, f}, {e, h},
   {e, i}, {f, f}, {f, i}, {g, g}, {g, i}, {h, h}, {h, i}, {i, i}};
Clear[Coord];
tabla = Table[0, {i1, Length[A]}, {j1, 3}]; B = A; t1 = 1; nivel = 0;
While[B ≠ {}, minimales = {}; nivel++; Do[minimal = True;
  Do[If[Intersection[{{B[[m1]], B[[n1]]}}, R] ≠ {} && n1 ≠ m1,
    minimal = False], {m1, 1, Length[B]}];
  If[minimal, AppendTo[minimales, B[[n1]]];
   tabla[[t1]] = {nivel, B[[n1]], 0};
   t1++;];,
  {n1, 1, Length[B]}];
 B = Complement[B, minimales];
]
R1 = {};
Do[AppendTo[R1, {A[[i1]], A[[i1]]}], {i1, 1, Length[A]}];
R = Complement[R, R1]; R1 = {}; Do[
 Do[Do[If[Intersection[R, {{R[[k1, 1]], A[[j1]]}, {A[[j1]], R[[k1, 2]]}}] ==
     {{R[[k1, 1]], A[[j1]]}, {A[[j1]], R[[k1, 2]]}},
    R1 = Union[R1, {R[[k1]]}]]], {j1, 1, Length[A]}], {k1, 1, Length[R]}];
R = Complement[R, R1];
puntos = {}; t1 = 0;
Do[cont = 0;
 Do[If[tabla[[i1, 1]] == j1, cont = cont + 1], {i1, 1, Length[A]}];
 Do[t1++; puntos = Union[puntos, {Text[tabla[[t1, 2]],
      {k1 - (cont / 2) - .1, j1 + .1}], Point[{k1 - (cont / 2), j1}]}];
  tabla[[t1, 3]] = k1 - (cont / 2), {k1, 1, cont}],
 {j1, 1, tabla[[Length[A], 1]]}];
Coord[elem_] := Do[If[elem == tabla[[h1, 2]],
   Coord[elem] = {tabla[[h1, 3]], tabla[[h1, 1]]}], {h1, 1, Length[A]}];
Do[Coord[A[[i1]]], {i1, 1, Length[A]}];
Do[AppendTo[puntos, Arrow[Coord[R[[t1, 1]]], Coord[R[[t1, 2]]]]];
 , {t1, 1, Length[R]}];
Print["Diagrama de orden:"]
Show[Graphics[puntos]]
```

```
Diagrama de orden:
```



- Graphics -