

# Further Mathematics - Degree in Engineering - 2025/2026

## 5-Fourier-PDE-Handwritten-training for serial number: 1

### Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 1, 0 < t \\ u(0, t) = u(1, t) = 0 & 0 \leq t \\ u(x, 0) = -\sin(\pi x) & 0 \leq x \leq 1 \\ \frac{\partial u}{\partial t}(x, 0) = -5 \sin(6\pi x) & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x = \frac{7}{10}$  and the moment  $t = 0.007$ .

1)  $u\left(\frac{7}{10}, 0.007\right) = \text{****.4****}$

2)  $u\left(\frac{7}{10}, 0.007\right) = \text{****.5****}$

3)  $u\left(\frac{7}{10}, 0.007\right) = \text{****.8****}$

4)  $u\left(\frac{7}{10}, 0.007\right) = \text{****.0****}$

5)  $u\left(\frac{7}{10}, 0.007\right) = \text{****.1****}$

### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 1, 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(1, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x = \frac{1}{10}$

and the moment  $t = 0.002$  by means of a Fourier series of order 1.

1)  $u\left(\frac{1}{10}, 0.002\right) = \text{****.*0**}$

2)  $u\left(\frac{1}{10}, 0.002\right) = \text{****.*1**}$

3)  $u\left(\frac{1}{10}, 0.002\right) = \text{****.*9**}$

4)  $u\left(\frac{1}{10}, 0.002\right) = \text{****.*5**}$

5)  $u\left(\frac{1}{10}, 0.002\right) = \text{****.*8**}$

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### 5-Fourier-PDE-Handwritten-training for serial number: 2

#### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 4 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 5, 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(5, t) = 0 & 0 \leq t \\ u(x, 0) = -2 \cos\left(\frac{2\pi x}{5}\right) - 7 \cos(\pi x) & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$  and the moment  $t=0.008$ .

- 1)  $u(2, 0.008) = **7.*****$
- 2)  $u(2, 0.008) = **3.*****$
- 3)  $u(2, 0.008) = **0.*****$
- 4)  $u(2, 0.008) = **2.*****$
- 5)  $u(2, 0.008) = **5.*****$

#### Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 4 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 5, 0 < t \\ u(0, t) = u(5, t) = 0, \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=1$ ,  $t=0.002$ , by separation of variables by means of a Fourier series of order 1.

- 1)  $u(1, 0.002) = **3.*****$
- 2)  $u(1, 0.002) = **5.*****$
- 3)  $u(1, 0.002) = **7.*****$
- 4)  $u(1, 0.002) = **8.*****$
- 5)  $u(1, 0.002) = **2.*****$

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## 5-Fourier-PDE-Handwritten-training for serial number: 3

### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 4, 0 < t \\ u(0, t) = u(4, t) = 0 & 0 \leq t \\ u(x, 0) = 9 \sin\left(\frac{5\pi x}{4}\right) - 9 \sin(\pi x) & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=3$  and the moment  $t=0.006$ .

- 1)  $u(3, 0.006) = **7.*****$
- 2)  $u(3, 0.006) = **5.*****$
- 3)  $u(3, 0.006) = **0.*****$
- 4)  $u(3, 0.006) = **6.*****$
- 5)  $u(3, 0.006) = **8.*****$

### Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 4, 0 < t \\ u(0, t) = u(4, t) = 0, \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=1$ ,  $t=0.005$ , by separation of variables by means of a Fourier series of order 1.

- 1)  $u(1, 0.005) = **6.*****$
- 2)  $u(1, 0.005) = **4.*****$
- 3)  $u(1, 0.005) = **2.*****$
- 4)  $u(1, 0.005) = **9.*****$
- 5)  $u(1, 0.005) = **7.*****$

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### 5-Fourier-PDE-Handwritten-training for serial number: 4

#### Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 5, \ 0 < t \\ u(0, t) = u(5, t) = 0 & 0 \leq t \\ u(x, 0) = -7 \sin\left(\frac{2\pi x}{5}\right) & 0 \leq x \leq 5 \\ \frac{\partial}{\partial t} u(x, 0) = -9 \sin\left(\frac{3\pi x}{5}\right) & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=3$  and the moment  $t=0.005$ .

- 1)  $u(3, 0.005) = **2.*****$
- 2)  $u(3, 0.005) = **0.*****$
- 3)  $u(3, 0.005) = **6.*****$
- 4)  $u(3, 0.005) = **1.*****$
- 5)  $u(3, 0.005) = **4.*****$

#### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 5, \ 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(5, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=4$

and the moment  $t=0.006$  by means of a Fourier series of order 1.

- 1)  $u(4, 0.006) = *3*.*****$
- 2)  $u(4, 0.006) = *1*.*****$
- 3)  $u(4, 0.006) = *8*.*****$
- 4)  $u(4, 0.006) = *6*.*****$
- 5)  $u(4, 0.006) = *2*.*****$

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### 5-Fourier-PDE-Handwritten-training for serial number: 5

#### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = 2 \sin(2x) - 6 \sin(4x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$  and the moment  $t=0.004$ .

- 1)  $u(2, 0.004) = **8.****$
- 2)  $u(2, 0.004) = **7.****$
- 3)  $u(2, 0.004) = **4.****$
- 4)  $u(2, 0.004) = **2.****$
- 5)  $u(2, 0.004) = **9.****$

#### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$  and the moment  $t=0.007$  by means of a Fourier series of order 1.

- 1)  $u(2, 0.007) = **3.****$
- 2)  $u(2, 0.007) = **8.****$
- 3)  $u(2, 0.007) = **5.****$
- 4)  $u(2, 0.007) = **6.****$
- 5)  $u(2, 0.007) = **9.****$

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### 5-Fourier-PDE-Handwritten-training for serial number: 6

#### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 1, 0 < t \\ u(0, t) = u(1, t) = 0 & 0 \leq t \\ u(x, 0) = -5 \sin(2\pi x) - 5 \sin(7\pi x) & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x = \frac{7}{10}$  and the moment  $t = 0.001$ .

1)  $u\left(\frac{7}{10}, 0.001\right) = **9.*****$

2)  $u\left(\frac{7}{10}, 0.001\right) = **1.*****$

3)  $u\left(\frac{7}{10}, 0.001\right) = **7.*****$

4)  $u\left(\frac{7}{10}, 0.001\right) = **5.*****$

5)  $u\left(\frac{7}{10}, 0.001\right) = **3.*****$

#### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 1, 0 < t \\ u(0, t) = u(1, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x = \frac{9}{10}$

and the moment  $t = 0.01$  by means of a Fourier series of order 2.

1)  $u\left(\frac{9}{10}, 0.01\right) = ***.7***$

2)  $u\left(\frac{9}{10}, 0.01\right) = ***.3***$

3)  $u\left(\frac{9}{10}, 0.01\right) = ***.6***$

4)  $u\left(\frac{9}{10}, 0.01\right) = ***.5***$

5)  $u\left(\frac{9}{10}, 0.01\right) = ***.4***$

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## 5-Fourier-PDE-Handwritten-training for serial number: 7

### Exercise 1

$$\begin{cases} (1+5t+2t^2) \frac{\partial u}{\partial t}(x,t) = 25(5 + 4t) \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin(4x) + \sin(5x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=1$ ,  $t=0.009$ , by separation of variables.

- 1)  $u(1, 0.009) = \text{****.*****4}$
- 2)  $u(1, 0.009) = \text{****.*****3}$
- 3)  $u(1, 0.009) = \text{****.*****2}$
- 4)  $u(1, 0.009) = \text{****.*****8}$
- 5)  $u(1, 0.009) = \text{****.*****5}$

### Exercise 2

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = -9 \sin(x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=2$  and the moment  $t=0.005$  by means of a Fourier series of order 1.

- 1)  $u(2, 0.005) = \text{**6.****}$
- 2)  $u(2, 0.005) = \text{**2.****}$
- 3)  $u(2, 0.005) = \text{**0.****}$
- 4)  $u(2, 0.005) = \text{**4.****}$
- 5)  $u(2, 0.005) = \text{**8.****}$

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## 5-Fourier-PDE-Handwritten-training for serial number: 8

### Exercise 1

$$\begin{cases} (-1 + 2x)(1+2t+t^2) \frac{\partial u}{\partial t}(x,t) = (2 + 2t) \frac{\partial u}{\partial x}(x,t) & 0 < x < \pi, 0 < t \\ u(x,0) = e^3(x^2-x) - 3e^{x-x^2} & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution for this boundary problem at the points  $x=1$ ,  $t=0.005$ , by means of the technique of separation of variables.

- 1)  $u(1, 0.005) = **2.*****$
- 2)  $u(1, 0.005) = **5.*****$
- 3)  $u(1, 0.005) = **4.*****$
- 4)  $u(1, 0.005) = **0.*****$
- 5)  $u(1, 0.005) = **1.*****$

### Exercise 2

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = -5 \sin(2x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=1$  and the moment  $t=0.004$  by means of a Fourier series of order 1.

- 1)  $u(1, 0.004) = **9.*****$
- 2)  $u(1, 0.004) = **3.*****$
- 3)  $u(1, 0.004) = **7.*****$
- 4)  $u(1, 0.004) = **1.*****$
- 5)  $u(1, 0.004) = **0.*****$

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## 5-Fourier-PDE-Handwritten-training for serial number: 9

### Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = 7 \sin(2x) - 5 \sin(x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x = 2, t = 0.002$ , by separation of variables.

- 1)  $u(2, 0.002) = **0.*****$
- 2)  $u(2, 0.002) = **1.*****$
- 3)  $u(2, 0.002) = **3.*****$
- 4)  $u(2, 0.002) = **9.*****$
- 5)  $u(2, 0.002) = **4.*****$

### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x = 2$  and the moment  $t = 0.009$  by means of a Fourier series of order 1.

- 1)  $u(2, 0.009) = **6.*****$
- 2)  $u(2, 0.009) = **1.*****$
- 3)  $u(2, 0.009) = **7.*****$
- 4)  $u(2, 0.009) = **4.*****$
- 5)  $u(2, 0.009) = **9.*****$

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## 5-Fourier-PDE-Handwritten-training for serial number: 10

### Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = -9 \sin(3x) - 2 \sin(7x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x = 2, t = 0.001$ , by separation of variables.

- 1)  $u(2, 0.001) = \text{****.9****}$
- 2)  $u(2, 0.001) = \text{****.6****}$
- 3)  $u(2, 0.001) = \text{****.7****}$
- 4)  $u(2, 0.001) = \text{****.2****}$
- 5)  $u(2, 0.001) = \text{****.0****}$

### Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x = 2, t = 0.001$ , by separation of variables by means of a Fourier series of order 2.

- 1)  $u(2, 0.001) = \text{**0.*****}$
- 2)  $u(2, 0.001) = \text{**3.*****}$
- 3)  $u(2, 0.001) = \text{**5.*****}$
- 4)  $u(2, 0.001) = \text{**8.*****}$
- 5)  $u(2, 0.001) = \text{**2.*****}$

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### 5-Fourier-PDE-Handwritten-training for serial number: 11

#### Exercise 1

$$\begin{cases} (1+3t+2t^2) \frac{\partial u}{\partial t}(x,t) = 25(3+4t) \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, 0 < t \\ u(0,t) = u(4,t) = 0 & 0 \leq t \\ u(x,0) = 8 \sin\left(\frac{3\pi x}{4}\right) - 4 \sin\left(\frac{\pi x}{2}\right) & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=1$ ,  $t=0.006$ , by separation of variables.

- 1)  $u(1, 0.006) = \text{****.9****}$
- 2)  $u(1, 0.006) = \text{****.8****}$
- 3)  $u(1, 0.006) = \text{****.6****}$
- 4)  $u(1, 0.006) = \text{****.0****}$
- 5)  $u(1, 0.006) = \text{****.1****}$

#### Exercise 2

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, 0 < t \\ u(0,t) = u(4,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin\left(\frac{\pi x}{2}\right) & 0 \leq x \leq 4 \\ \frac{\partial}{\partial t} u(x,0) = x^2 & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=2$  and the moment  $t=0.003$  by means of a Fourier series of order 1.

- 1)  $u(2, 0.003) = \text{**9.*****}$
- 2)  $u(2, 0.003) = \text{**8.*****}$
- 3)  $u(2, 0.003) = \text{**4.*****}$
- 4)  $u(2, 0.003) = \text{**6.*****}$
- 5)  $u(2, 0.003) = \text{**7.*****}$

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### Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = \sin(x) & 0 \leq x \leq \pi \\ \frac{\partial u}{\partial t}(x, 0) = -5 \sin(6x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=1$  and the moment  $t=0.01$ .

- 1)  $u(1, 0.01) = \text{****.7****}$
- 2)  $u(1, 0.01) = \text{****.8****}$
- 3)  $u(1, 0.01) = \text{****.6****}$
- 4)  $u(1, 0.01) = \text{****.4****}$
- 5)  $u(1, 0.01) = \text{****.3****}$

### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$  and the moment  $t=0.005$  by means of a Fourier series of order 2.

- 1)  $u(2, 0.005) = \text{**8.****}$
- 2)  $u(2, 0.005) = \text{**5.****}$
- 3)  $u(2, 0.005) = \text{**9.****}$
- 4)  $u(2, 0.005) = \text{**1.****}$
- 5)  $u(2, 0.005) = \text{**2.****}$

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### 5-Fourier-PDE-Handwritten-training for serial number: 13

#### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = 9 \sin(8x) - \sin(7x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=1$  and the moment  $t=0.01$ .

- 1)  $u(1, 0.01) = **8.*****$
- 2)  $u(1, 0.01) = **4.*****$
- 3)  $u(1, 0.01) = **9.*****$
- 4)  $u(1, 0.01) = **1.*****$
- 5)  $u(1, 0.01) = **5.*****$

#### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$

and the moment  $t=0.001$  by means of a Fourier series of order 2.

- 1)  $u(2, 0.001) = **2.*****$
- 2)  $u(2, 0.001) = **0.*****$
- 3)  $u(2, 0.001) = **6.*****$
- 4)  $u(2, 0.001) = **7.*****$
- 5)  $u(2, 0.001) = **1.*****$

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## 5-Fourier-PDE-Handwritten-training for serial number: 14

### Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 4, 0 < t \\ u(0, t) = u(4, t) = 0, \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = 4 \sin\left(\frac{5\pi x}{4}\right) - 2 \sin\left(\frac{3\pi x}{2}\right) & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=2$ ,  $t=0.004$ , by separation of variables.

- 1)  $u(2, 0.004) = **4.*****$
- 2)  $u(2, 0.004) = **8.*****$
- 3)  $u(2, 0.004) = **3.*****$
- 4)  $u(2, 0.004) = **0.*****$
- 5)  $u(2, 0.004) = **6.*****$

### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 4, 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(4, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$  and the moment  $t=0.004$  by means of a Fourier series of order 2.

- 1)  $u(2, 0.004) = **4.*****$
- 2)  $u(2, 0.004) = **8.*****$
- 3)  $u(2, 0.004) = **1.*****$
- 4)  $u(2, 0.004) = **3.*****$
- 5)  $u(2, 0.004) = **2.*****$

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### 5-Fourier-PDE-Handwritten-training for serial number: 15

#### Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = -9 \sin(6x) & 0 \leq x \leq \pi \\ \frac{\partial u}{\partial t}(x, 0) = -8 \sin(4x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=1$  and the moment  $t=0.005$ .

- 1)  $u(1, 0.005) = **2.*****$
- 2)  $u(1, 0.005) = **9.*****$
- 3)  $u(1, 0.005) = **3.*****$
- 4)  $u(1, 0.005) = **1.*****$
- 5)  $u(1, 0.005) = **7.*****$

#### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=2$  and the moment  $t=0.001$  by means of a Fourier series of order 2.

- 1)  $u(2, 0.001) = **4.*****$
- 2)  $u(2, 0.001) = **2.*****$
- 3)  $u(2, 0.001) = **8.*****$
- 4)  $u(2, 0.001) = **9.*****$
- 5)  $u(2, 0.001) = **6.*****$

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## 5-Fourier-PDE-Handwritten-training for serial number: 16

### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 4 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 2, 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(2, t) = 0 & 0 \leq t \\ u(x, 0) = 7 \cos\left(\frac{5\pi x}{2}\right) + 3 \cos\left(\frac{7\pi x}{2}\right) & 0 \leq x \leq 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x = \frac{9}{5}$  and the moment  $t = 0.001$ .

1)  $u\left(\frac{9}{5}, 0.001\right) = **3.*****$

2)  $u\left(\frac{9}{5}, 0.001\right) = **9.*****$

3)  $u\left(\frac{9}{5}, 0.001\right) = **1.*****$

4)  $u\left(\frac{9}{5}, 0.001\right) = **7.*****$

5)  $u\left(\frac{9}{5}, 0.001\right) = **4.*****$

### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 4 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 2, 0 < t \\ u(0, t) = u(2, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x = \frac{3}{2}$  and the moment  $t = 0.008$  by means of a Fourier series of order 2.

1)  $u\left(\frac{3}{2}, 0.008\right) = **8.*****$

2)  $u\left(\frac{3}{2}, 0.008\right) = **2.*****$

3)  $u\left(\frac{3}{2}, 0.008\right) = **6.*****$

4)  $u\left(\frac{3}{2}, 0.008\right) = **4.*****$

5)  $u\left(\frac{3}{2}, 0.008\right) = **1.*****$

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### 5-Fourier-PDE-Handwritten-training for serial number: 17

#### Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = 7 \sin(3x) + 3 \sin(8x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x = 1, t = 0.01$ , by separation of variables.

- 1)  $u(1, 0.01) = **9.****$
- 2)  $u(1, 0.01) = **3.****$
- 3)  $u(1, 0.01) = **1.****$
- 4)  $u(1, 0.01) = **4.****$
- 5)  $u(1, 0.01) = **7.****$

#### Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x = 1, t = 0.006$ , by separation of variables by means of a Fourier series of order 2.

- 1)  $u(1, 0.006) = ***.9***$
- 2)  $u(1, 0.006) = ***.3***$
- 3)  $u(1, 0.006) = ***.1***$
- 4)  $u(1, 0.006) = ***.6***$
- 5)  $u(1, 0.006) = ***.8***$

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## 5-Fourier-PDE-Handwritten-training for serial number: 18

### Exercise 1

$$\begin{cases} (1+9t+t^2) \frac{\partial u}{\partial t}(x,t) = 25(9 + 2t) \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = -5 \sin(6x) - 7 \sin(7x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=1$ ,  $t=0.002$ , by separation of variables.

- 1)  $u(1, 0.002) = \text{****.*****1}$
- 2)  $u(1, 0.002) = \text{****.*****7}$
- 3)  $u(1, 0.002) = \text{****.*****3}$
- 4)  $u(1, 0.002) = \text{****.*****8}$
- 5)  $u(1, 0.002) = \text{****.*****2}$

### Exercise 2

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin(2x) & 0 \leq x \leq \pi \\ \frac{\partial u}{\partial t}(x,0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=1$  and the moment  $t=0.01$  by means of a Fourier series of order 1.

- 1)  $u(1, 0.01) = \text{**2.****}$
- 2)  $u(1, 0.01) = \text{**3.****}$
- 3)  $u(1, 0.01) = \text{**0.****}$
- 4)  $u(1, 0.01) = \text{**5.****}$
- 5)  $u(1, 0.01) = \text{**7.****}$

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## 5-Fourier-PDE-Handwritten-training for serial number: 19

### Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 4 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = 3 \cos(4x) - 9 \cos(5x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=1$  and the moment  $t=0.002$ .

- 1)  $u(1, 0.002) = \text{**0.*****}$
- 2)  $u(1, 0.002) = \text{**5.*****}$
- 3)  $u(1, 0.002) = \text{**3.*****}$
- 4)  $u(1, 0.002) = \text{**8.*****}$
- 5)  $u(1, 0.002) = \text{**4.*****}$

### Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 4 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point  $x=1$ ,  $t=0.002$ , by separation of variables by means of a Fourier series of order 1.

- 1)  $u(1, 0.002) = \text{**2.*****}$
- 2)  $u(1, 0.002) = \text{**9.*****}$
- 3)  $u(1, 0.002) = \text{**0.*****}$
- 4)  $u(1, 0.002) = \text{**3.*****}$
- 5)  $u(1, 0.002) = \text{**4.*****}$

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### 5-Fourier-PDE-Handwritten-training for serial number: 20

#### Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \ 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = -5 \sin(5x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x, 0) = \sin(x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at  $x=2$  and the moment  $t=0.008$ .

- 1)  $u(2, 0.008) = **4.*****$
- 2)  $u(2, 0.008) = **6.*****$
- 3)  $u(2, 0.008) = **3.*****$
- 4)  $u(2, 0.008) = **2.*****$
- 5)  $u(2, 0.008) = **9.*****$

#### Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \ 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point  $x=1$   
and the moment  $t=0.006$  by means of a Fourier series of order 1.

- 1)  $u(1, 0.006) = **7.*****$
- 2)  $u(1, 0.006) = **2.*****$
- 3)  $u(1, 0.006) = **8.*****$
- 4)  $u(1, 0.006) = **1.*****$
- 5)  $u(1, 0.006) = **6.*****$