

Further Mathematics - Degree in Engineering - 2025/2026

5-Fourier-PDE-Handwritten-training for serial number: 1

Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 1, 0 < t \\ u(0,t) = u(1,t) = 0 & 0 \leq t \\ u(x,0) = -\sin(\pi x) & 0 \leq x \leq 1 \\ \frac{\partial}{\partial t} u(x,0) = -5 \sin(6\pi x) & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{7}{10}$ and the moment $t = 0.007$.

$$1) u\left(\frac{7}{10}, 0.007\right) = ***.4***$$

$$2) u\left(\frac{7}{10}, 0.007\right) = ***.5***$$

$$3) u\left(\frac{7}{10}, 0.007\right) = ***.8***$$

$$4) u\left(\frac{7}{10}, 0.007\right) = ***.0***$$

$$5) u\left(\frac{7}{10}, 0.007\right) = ***.1***$$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 1, 0 < t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \leq t \\ u(x,0) = x^2 & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{1}{10}$

and the moment $t = 0.002$ by means of a Fourier series of order 1.

$$1) u\left(\frac{1}{10}, 0.002\right) = ***.*0**$$

$$2) u\left(\frac{1}{10}, 0.002\right) = ***.*1**$$

$$3) u\left(\frac{1}{10}, 0.002\right) = ***.*9**$$

$$4) u\left(\frac{1}{10}, 0.002\right) = ***.*5**$$

$$5) u\left(\frac{1}{10}, 0.002\right) = ***.*8**$$

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5-Fourier-PDE-Handwritten-training for serial number: 2

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 5, \ 0 < t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(5,t) = 0 & 0 \leq t \\ u(x,0) = -2 \cos\left(\frac{2\pi x}{5}\right) - 7 \cos(\pi x) & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.008$.

- 1) $u(2, 0.008) = **7.****$
- 2) $u(2, 0.008) = **3.****$
- 3) $u(2, 0.008) = **0.****$
- 4) $u(2, 0.008) = **2.****$
- 5) $u(2, 0.008) = **5.****$

Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 5, \ 0 < t \\ u(0,t) = u(5,t) = 0, \ \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = x^2 & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.002$, by separation of variables by means of a Fourier series of order 1.

- 1) $u(1, 0.002) = **3.****$
- 2) $u(1, 0.002) = **5.****$
- 3) $u(1, 0.002) = **7.****$
- 4) $u(1, 0.002) = **8.****$
- 5) $u(1, 0.002) = **2.****$

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5-Fourier-PDE-Handwritten-training for serial number: 3

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, \ 0 < t \\ u(0,t) = u(4,t) = 0 & 0 \leq t \\ u(x,0) = 9 \sin\left(\frac{5\pi x}{4}\right) - 9 \sin(\pi x) & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=3$ and the moment $t=0.006$.

- 1) $u(3, 0.006) = **7.****$
- 2) $u(3, 0.006) = **5.****$
- 3) $u(3, 0.006) = **0.****$
- 4) $u(3, 0.006) = **6.****$
- 5) $u(3, 0.006) = **8.****$

Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, \ 0 < t \\ u(0,t) = u(4,t) = 0, \ \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = x^2 & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.005$, by separation of variables by means of a Fourier series of order 1.

- 1) $u(1, 0.005) = **6.****$
- 2) $u(1, 0.005) = **4.****$
- 3) $u(1, 0.005) = **2.****$
- 4) $u(1, 0.005) = **9.****$
- 5) $u(1, 0.005) = **7.****$

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5-Fourier-PDE-Handwritten-training for serial number: 4

Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 5, \ 0 < t \\ u(0, t) = u(5, t) = 0 & 0 \leq t \\ u(x, 0) = -7 \sin\left(\frac{2\pi x}{5}\right) & 0 \leq x \leq 5 \\ \frac{\partial}{\partial t} u(x, 0) = -9 \sin\left(\frac{3\pi x}{5}\right) & 0 \leq x \leq 5 \end{cases}$$

Compute the position of the string at $x=3$ and the moment $t=0.005$.

- 1) $u(3, 0.005) = **2.****$
- 2) $u(3, 0.005) = **0.****$
- 3) $u(3, 0.005) = **6.****$
- 4) $u(3, 0.005) = **1.****$
- 5) $u(3, 0.005) = **4.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < 5, \ 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(5, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq 5 \end{cases}$$

Compute the temperature of the bar at the point $x=4$
and the moment $t=0.006$ by means of a Fourier series of order 1.

- 1) $u(4, 0.006) = *3*.****$
- 2) $u(4, 0.006) = *1*.****$
- 3) $u(4, 0.006) = *8*.****$
- 4) $u(4, 0.006) = *6*.****$
- 5) $u(4, 0.006) = *2*.****$

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5-Fourier-PDE-Handwritten-training for serial number: 5

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \ 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin(2x) - 6 \sin(4x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.004$.

- 1) $u(2, 0.004) = **8.****$
- 2) $u(2, 0.004) = **7.****$
- 3) $u(2, 0.004) = **4.****$
- 4) $u(2, 0.004) = **2.****$
- 5) $u(2, 0.004) = **9.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \ 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$
and the moment $t=0.007$ by means of a Fourier series of order 1.

- 1) $u(2, 0.007) = **3.****$
- 2) $u(2, 0.007) = **8.****$
- 3) $u(2, 0.007) = **5.****$
- 4) $u(2, 0.007) = **6.****$
- 5) $u(2, 0.007) = **9.****$

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5-Fourier-PDE-Handwritten-training for serial number: 6

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 1, \ 0 < t \\ u(0,t) = u(1,t) = 0 & 0 \leq t \\ u(x,0) = -5 \sin(2\pi x) - 5 \sin(7\pi x) & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{7}{10}$ and the moment $t = 0.001$.

$$1) \ u\left(\frac{7}{10}, 0.001\right) = **9.***$$

$$2) \ u\left(\frac{7}{10}, 0.001\right) = **1.***$$

$$3) \ u\left(\frac{7}{10}, 0.001\right) = **7.***$$

$$4) \ u\left(\frac{7}{10}, 0.001\right) = **5.***$$

$$5) \ u\left(\frac{7}{10}, 0.001\right) = **3.***$$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 1, \ 0 < t \\ u(0,t) = u(1,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{9}{10}$

and the moment $t = 0.01$ by means of a Fourier series of order 2.

$$1) \ u\left(\frac{9}{10}, 0.01\right) = ***.7***$$

$$2) \ u\left(\frac{9}{10}, 0.01\right) = ***.3***$$

$$3) \ u\left(\frac{9}{10}, 0.01\right) = ***.6***$$

$$4) \ u\left(\frac{9}{10}, 0.01\right) = ***.5***$$

$$5) \ u\left(\frac{9}{10}, 0.01\right) = ***.4***$$

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5-Fourier-PDE-Handwritten-training for serial number: 7

Exercise 1

$$\left\{ \begin{array}{ll} (1+5t+2t^2) \frac{\partial u}{\partial t}(x,t) = 25(5 + 4t) \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin(4x) + \sin(5x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{array} \right.$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.009$, by separation of variables.

- 1) $u(1, 0.009) = \text{***.*****}4$
- 2) $u(1, 0.009) = \text{***.*****}3$
- 3) $u(1, 0.009) = \text{***.*****}2$
- 4) $u(1, 0.009) = \text{***.*****}8$
- 5) $u(1, 0.009) = \text{***.*****}5$

Exercise 2

$$\left\{ \begin{array}{ll} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = -9 \sin(x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{array} \right.$$

Compute the position of the string at $x=2$ and the moment $t=0.005$ by means of a Fourier series of order 1.

- 1) $u(2, 0.005) = \text{**}6.\text{****}$
- 2) $u(2, 0.005) = \text{**}2.\text{****}$
- 3) $u(2, 0.005) = \text{**}0.\text{****}$
- 4) $u(2, 0.005) = \text{**}4.\text{****}$
- 5) $u(2, 0.005) = \text{**}8.\text{****}$

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5-Fourier-PDE-Handwritten-training for serial number: 8

Exercise 1

$$\begin{cases} (-1 + 2x)(1+2t+t^2) \frac{\partial u}{\partial t}(x,t) = (2 + 2t) \frac{\partial u}{\partial x}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(x,0) = e^{3(x^2-x)} - 3e^{x-x^2} & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution for this boundary problem at the points $x=1$, $t=0.005$, by means of the technique of separation of variables.

- 1) $u(1, 0.005) = **2.****$
- 2) $u(1, 0.005) = **5.****$
- 3) $u(1, 0.005) = **4.****$
- 4) $u(1, 0.005) = **0.****$
- 5) $u(1, 0.005) = **1.****$

Exercise 2

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = -5 \sin(2x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=1$ and the moment $t=0.004$ by means of a Fourier series of order 1.

- 1) $u(1, 0.004) = **9.****$
- 2) $u(1, 0.004) = **3.****$
- 3) $u(1, 0.004) = **7.****$
- 4) $u(1, 0.004) = **1.****$
- 5) $u(1, 0.004) = **0.****$

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5-Fourier-PDE-Handwritten-training for serial number: 9

Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0, \quad \lim_{t \rightarrow \infty} u(x, t) = 0 & 0 \leq t \\ u(x, 0) = 7 \sin(2x) - 5 \sin(x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.002$, by separation of variables.

- 1) $u(2, 0.002) = **0.****$
- 2) $u(2, 0.002) = **1.****$
- 3) $u(2, 0.002) = **3.****$
- 4) $u(2, 0.002) = **9.****$
- 5) $u(2, 0.002) = **4.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 16 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.009$ by means of a Fourier series of order 1.

- 1) $u(2, 0.009) = **6.****$
- 2) $u(2, 0.009) = **1.****$
- 3) $u(2, 0.009) = **7.****$
- 4) $u(2, 0.009) = **4.****$
- 5) $u(2, 0.009) = **9.****$

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5-Fourier-PDE-Handwritten-training for serial number: 10

Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \ 0 < t \\ u(0,t) = u(\pi,t) = 0, \ \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = -9 \sin(3x) - 2 \sin(7x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.001$, by separation of variables.

- 1) $u(2, 0.001) = \text{***.9***}$
- 2) $u(2, 0.001) = \text{***.6***}$
- 3) $u(2, 0.001) = \text{***.7***}$
- 4) $u(2, 0.001) = \text{***.2***}$
- 5) $u(2, 0.001) = \text{***.0***}$

Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \ 0 < t \\ u(0,t) = u(\pi,t) = 0, \ \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.001$, by separation of variables by means of a Fourier series of order 2.

- 1) $u(2, 0.001) = \text{**0.***}$
- 2) $u(2, 0.001) = \text{**3.***}$
- 3) $u(2, 0.001) = \text{**5.***}$
- 4) $u(2, 0.001) = \text{**8.***}$
- 5) $u(2, 0.001) = \text{**2.***}$

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5-Fourier-PDE-Handwritten-training for serial number: 11

Exercise 1

$$\left\{ \begin{array}{ll} (1+3t+2t^2) \frac{\partial u}{\partial t}(x,t) = 25(3+4t) \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, \quad 0 < t \\ u(0,t) = u(4,t) = 0 & 0 \leq t \\ u(x,0) = 8 \sin\left(\frac{3\pi x}{4}\right) - 4 \sin\left(\frac{\pi x}{2}\right) & 0 \leq x \leq 4 \\ 0 & \text{True} \end{array} \right.$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.006$, by separation of variables.

- 1) $u(1, 0.006) = \text{***.9***}$
- 2) $u(1, 0.006) = \text{***.8***}$
- 3) $u(1, 0.006) = \text{***.6***}$
- 4) $u(1, 0.006) = \text{***.0***}$
- 5) $u(1, 0.006) = \text{***.1***}$

Exercise 2

$$\left\{ \begin{array}{ll} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, \quad 0 < t \\ u(0,t) = u(4,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin\left(\frac{\pi x}{2}\right) & 0 \leq x \leq 4 \\ \frac{\partial}{\partial t} u(x,0) = x^2 & 0 \leq x \leq 4 \\ 0 & \text{True} \end{array} \right.$$

Compute the position of the string at $x=2$ and the moment $t=0.003$ by means of a Fourier series of order 1.

- 1) $u(2, 0.003) = \text{**9.***}$
- 2) $u(2, 0.003) = \text{**8.***}$
- 3) $u(2, 0.003) = \text{**4.***}$
- 4) $u(2, 0.003) = \text{**6.***}$
- 5) $u(2, 0.003) = \text{**7.***}$

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5-Fourier-PDE-Handwritten-training for serial number: 12

Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = \sin(x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x, 0) = -5 \sin(6x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=1$ and the moment $t=0.01$.

- 1) $u(1, 0.01) = \text{***.7***}$
- 2) $u(1, 0.01) = \text{***.8***}$
- 3) $u(1, 0.01) = \text{***.6***}$
- 4) $u(1, 0.01) = \text{***.4***}$
- 5) $u(1, 0.01) = \text{***.3***}$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$
and the moment $t=0.005$ by means of a Fourier series of order 2.

- 1) $u(2, 0.005) = \text{**8.***}$
- 2) $u(2, 0.005) = \text{**5.***}$
- 3) $u(2, 0.005) = \text{**9.***}$
- 4) $u(2, 0.005) = \text{**1.***}$
- 5) $u(2, 0.005) = \text{**2.***}$

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5-Fourier-PDE-Handwritten-training for serial number: 13

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 9 \sin(8x) - \sin(7x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$ and the moment $t=0.01$.

- 1) $u(1, 0.01) = **8.****$
- 2) $u(1, 0.01) = **4.****$
- 3) $u(1, 0.01) = **9.****$
- 4) $u(1, 0.01) = **1.****$
- 5) $u(1, 0.01) = **5.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$

and the moment $t=0.001$ by means of a Fourier series of order 2.

- 1) $u(2, 0.001) = **2.****$
- 2) $u(2, 0.001) = **0.****$
- 3) $u(2, 0.001) = **6.****$
- 4) $u(2, 0.001) = **7.****$
- 5) $u(2, 0.001) = **1.****$

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5-Fourier-PDE-Handwritten-training for serial number: 14

Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, \ 0 < t \\ u(0,t) = u(4,t) = 0, \ \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = 4 \sin\left(\frac{5\pi x}{4}\right) - 2 \sin\left(\frac{3\pi x}{2}\right) & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.004$, by separation of variables.

- 1) $u(2, 0.004) = **4.****$
- 2) $u(2, 0.004) = **8.****$
- 3) $u(2, 0.004) = **3.****$
- 4) $u(2, 0.004) = **0.****$
- 5) $u(2, 0.004) = **6.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 4, \ 0 < t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.004$ by means of a Fourier series of order 2.

- 1) $u(2, 0.004) = **4.****$
- 2) $u(2, 0.004) = **8.****$
- 3) $u(2, 0.004) = **1.****$
- 4) $u(2, 0.004) = **3.****$
- 5) $u(2, 0.004) = **2.****$

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5-Fourier-PDE-Handwritten-training for serial number: 15

Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = -9 \sin(6x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x, 0) = -8 \sin(4x) & 0 \leq x \leq \pi \end{cases}$$

Compute the position of the string at $x=1$ and the moment $t=0.005$.

- 1) $u(1, 0.005) = **2.****$
- 2) $u(1, 0.005) = **9.****$
- 3) $u(1, 0.005) = **3.****$
- 4) $u(1, 0.005) = **1.****$
- 5) $u(1, 0.005) = **7.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x & 0 \leq x \leq \pi \end{cases}$$

Compute the temperature of the bar at the point $x=2$
and the moment $t=0.001$ by means of a Fourier series of order 2.

- 1) $u(2, 0.001) = **4.****$
- 2) $u(2, 0.001) = **2.****$
- 3) $u(2, 0.001) = **8.****$
- 4) $u(2, 0.001) = **9.****$
- 5) $u(2, 0.001) = **6.****$

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5-Fourier-PDE-Handwritten-training for serial number: 16

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 2, \quad 0 < t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(2,t) = 0 & 0 \leq t \\ u(x,0) = 7 \cos\left(\frac{5\pi x}{2}\right) + 3 \cos\left(\frac{7\pi x}{2}\right) & 0 \leq x \leq 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{9}{5}$ and the moment $t = 0.001$.

$$1) u\left(\frac{9}{5}, 0.001\right) = **3.***$$

$$2) u\left(\frac{9}{5}, 0.001\right) = **9.***$$

$$3) u\left(\frac{9}{5}, 0.001\right) = **1.***$$

$$4) u\left(\frac{9}{5}, 0.001\right) = **7.***$$

$$5) u\left(\frac{9}{5}, 0.001\right) = **4.***$$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < 2, \quad 0 < t \\ u(0,t) = u(2,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{3}{2}$

and the moment $t = 0.008$ by means of a Fourier series of order 2.

$$1) u\left(\frac{3}{2}, 0.008\right) = **8.***$$

$$2) u\left(\frac{3}{2}, 0.008\right) = **2.***$$

$$3) u\left(\frac{3}{2}, 0.008\right) = **6.***$$

$$4) u\left(\frac{3}{2}, 0.008\right) = **4.***$$

$$5) u\left(\frac{3}{2}, 0.008\right) = **1.***$$

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5-Fourier-PDE-Handwritten-training for serial number: 17

Exercise 1

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0, \quad \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = 7 \sin(3x) + 3 \sin(8x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.01$, by separation of variables.

- 1) $u(1, 0.01) = \text{**9.***}$
- 2) $u(1, 0.01) = \text{**3.***}$
- 3) $u(1, 0.01) = \text{**1.***}$
- 4) $u(1, 0.01) = \text{**4.***}$
- 5) $u(1, 0.01) = \text{**7.***}$

Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \quad 0 < t \\ u(0,t) = u(\pi,t) = 0, \quad \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = x & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.006$, by separation of variables by means of a Fourier series of order 2.

- 1) $u(1, 0.006) = \text{***.9***}$
- 2) $u(1, 0.006) = \text{***.3***}$
- 3) $u(1, 0.006) = \text{***.1***}$
- 4) $u(1, 0.006) = \text{***.6***}$
- 5) $u(1, 0.006) = \text{***.8***}$

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5-Fourier-PDE-Handwritten-training for serial number: 18

Exercise 1

$$\begin{cases} (1+9t+t^2) \frac{\partial u}{\partial t}(x,t) = 25(9+2t) \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = -5 \sin(6x) - 7 \sin(7x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.002$, by separation of variables.

- 1) $u(1, 0.002) = \text{***.*****}1$
- 2) $u(1, 0.002) = \text{***.*****}7$
- 3) $u(1, 0.002) = \text{***.*****}3$
- 4) $u(1, 0.002) = \text{***.*****}8$
- 5) $u(1, 0.002) = \text{***.*****}2$

Exercise 2

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, 0 < t \\ u(0,t) = u(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 2 \sin(2x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x,0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=1$ and the moment $t=0.01$ by means of a Fourier series of order 1.

- 1) $u(1, 0.01) = \text{**}2.\text{****}$
- 2) $u(1, 0.01) = \text{**}3.\text{****}$
- 3) $u(1, 0.01) = \text{**}0.\text{****}$
- 4) $u(1, 0.01) = \text{**}5.\text{****}$
- 5) $u(1, 0.01) = \text{**}7.\text{****}$

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5-Fourier-PDE-Handwritten-training for serial number: 19

Exercise 1

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \ 0 < t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \leq t \\ u(x,0) = 3 \cos(4x) - 9 \cos(5x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$ and the moment $t=0.002$.

- 1) $u(1, 0.002) = **0.****$
- 2) $u(1, 0.002) = **5.****$
- 3) $u(1, 0.002) = **3.****$
- 4) $u(1, 0.002) = **8.****$
- 5) $u(1, 0.002) = **4.****$

Exercise 2

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi, \ 0 < t \\ u(0,t) = u(\pi,t) = 0, \ \lim_{t \rightarrow \infty} u(x,t) = 0 & 0 \leq t \\ u(x,0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.002$, by separation of variables by means of a Fourier series of order 1.

- 1) $u(1, 0.002) = **2.****$
- 2) $u(1, 0.002) = **9.****$
- 3) $u(1, 0.002) = **0.****$
- 4) $u(1, 0.002) = **3.****$
- 5) $u(1, 0.002) = **4.****$

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5-Fourier-PDE-Handwritten-training for serial number: 20

Exercise 1

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ u(0, t) = u(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = -5 \sin(5x) & 0 \leq x \leq \pi \\ \frac{\partial}{\partial t} u(x, 0) = \sin(x) & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=2$ and the moment $t=0.008$.

- 1) $u(2, 0.008) = **4.****$
- 2) $u(2, 0.008) = **6.****$
- 3) $u(2, 0.008) = **3.****$
- 4) $u(2, 0.008) = **2.****$
- 5) $u(2, 0.008) = **9.****$

Exercise 2

$$\begin{cases} \frac{\partial u}{\partial t}(x, t) = 9 \frac{\partial^2 u}{\partial x^2}(x, t) & 0 < x < \pi, \quad 0 < t \\ \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0 & 0 \leq t \\ u(x, 0) = x^2 & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$
and the moment $t=0.006$ by means of a Fourier series of order 1.

- 1) $u(1, 0.006) = **7.****$
- 2) $u(1, 0.006) = **2.****$
- 3) $u(1, 0.006) = **8.****$
- 4) $u(1, 0.006) = **1.****$
- 5) $u(1, 0.006) = **6.****$