

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 1

Exercise 1

Consider the vectorial field $F(x, y, z) = (-6xy^2 + \frac{(-2y-1)yz}{xyz+1} - 2xy$

$$, -6x^2y - x^2 + \frac{x(-2y-1)z}{xyz+1} - 2\log(xyz+1), \frac{x(-2y-1)y}{xyz+1}$$

). Compute the potential function for this field whose potential at the origin is 1.

. Calculate the value of the potential at the point $p = (0, 4, 9)$.

- 1) $-\frac{6}{5}$ 2) $-\frac{5}{2}$ 3) 1 4) $\frac{49}{10}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(3t+5)\sin(2t) \cos(7t) + 8, (6t+8)\sin(t)\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 384.426 2) 1151.63 3) 2877.83 4) 1918.83

Exercise 3

Consider the vector field $F(x, y, z) = \{-6z^2, -xz - 5x^2y^2z, 3x\}$ and the surface

$$S \equiv \left(\frac{7+x}{6}\right)^2 + \left(\frac{1+y}{7}\right)^2 + \left(\frac{1+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -395489. 2) -1.9379×10^6 3) 474587. 4) 988723.

Further Mathematics - Degree in Engineering - 2024/2025 04-Line and Surface Integral-Computers exam for serial number: 2

Exercise 1

Consider the vectorial field $F(x, y, z) = ((-3x - 1)yz e^{xyz} - 3e^{xyz} + y^2, (-3x - 1)xze^{xyz} + 2xy + 3, (-3x - 1)xy e^{xyz})$. Compute the potential function for this field whose potential at the origin is 4.
. Calculate the value of the potential at the point $p = (10, -10, 9)$.

$$1) \quad 975 - \frac{31}{e^{900}} - \frac{12}{5} \text{ If } \left[\text{Floor} \left[975 - \frac{31}{e^{900}} \right] = 0, 1, \text{Floor}[\text{solu}] \right]$$

$$2) \quad 975 - \frac{31}{e^{900}} - \text{If} \left[\text{Floor} \left[975 - \frac{31}{e^{900}} \right] = 0, 1, \text{Floor}[\text{solu}] \right] \quad 3)$$

$$975 - \frac{31}{e^{900}} \quad 4) \quad 975 - \frac{31}{e^{900}} - \frac{1}{2} \text{ If } \left[\text{Floor} \left[975 - \frac{31}{e^{900}} \right] = 0, 1, \text{Floor}[\text{solu}] \right]$$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(5t + 5) \sin(2t) (4 \cos(12t) + 5), (2t + 4) \sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

$$1) \quad 185.027 \quad 2) \quad 430.627 \quad 3) \quad 614.827 \quad 4) \quad 307.827$$

Exercise 3

Consider the vector field $F(x, y, z) = \{-x^2 y z, 5 x z^2, -8 x^2 y + 2 x y^2 z^2\}$ and the surface

$$S \equiv \left(\frac{5+x}{4} \right)^2 + \left(\frac{1+y}{1} \right)^2 + \left(\frac{2+z}{6} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

$$1) \quad 3418.11 \quad 2) \quad -9570.29 \quad 3) \quad 6836.11 \quad 4) \quad 0.105614$$

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04-Line and Surface Integral-Computers exam for serial number: 3

Exercise 1

Consider the vectorial field $F(x, y, z) = (-3x^2y^2z^2 + 2xy^2z^2(z-3x) - 3y^2, 2x^2yz^2(z-3x) - 6xy + 2, x^2y^2z^2 + 2x^2y^2z(z-3x))$. Compute the potential function for this field whose potential at the origin is 3.
 . Calculate the value of the potential at the point $p = (-7, -4, 4)$.

- 1) $\frac{313931}{10}$ 2) $\frac{5336827}{5}$ 3) 313931 4) $\frac{4081103}{5}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right) \cos(t) (2 \cos(t) + 3)}{\sin^2(t) + 1}, \frac{\left(\frac{\sin(t)}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \cos(t) (2 \cos(t) + 3)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 11.2336 2) 12.4336 3) 20.8336 4) 2.83363

Exercise 3

Consider the vector field $F(x, y, z) = \{-7z - \sin[y^2 - z^2], -2z + \cos[x^2 - 2z^2], 3xy + \cos[2y^2]\}$ and the surface

$$S \equiv \left(\frac{8+x}{4}\right)^2 + \left(\frac{7+y}{2}\right)^2 + \left(\frac{-6+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 1.6 2) -0.9 3) 2.4 4) 0.

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04-Line and Surface Integral-Computers exam for serial number: 4

Exercise 1

Consider the vectorial field $F(x, y, z) = (y^2 z \sin(xyz) + 2x, xyz \sin(xyz) - \cos(xyz) - 4y, xy^2 \sin(xyz))$. Compute the potential function for this field whose potential at the origin is 2.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) 1.18028 2) 2.18028 3) 0.780281 4) -1.41972

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(3t + 5) \sin(2t) (2 \cos(17t) + 9), (9t + 1) \sin(t) (2 \cos(17t) + 9)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 17001.5 2) 23801.9 3) 6800.88 4) 10201.1

Exercise 3

Consider the vector field $F(x, y, z) = \{-5xy^2z - 4xyz^2, 9xyz^2, 8x\}$ and the surface
 $S \equiv \left(\frac{-9+x}{1}\right)^2 + \left(\frac{y}{6}\right)^2 + \left(\frac{5+z}{1}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 55824.8 2) -89317.6 3) 33495.2 4) 111649.

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04-Line and Surface Integral-Computers exam for serial number: 5

Exercise 1

Consider the vectorial field $F(x, y, z) = ((2y - z) \log(yz + 1) - y \frac{z(2xy - xz)}{yz + 1} + 2x \log(yz + 1) - x, \frac{y(2xy - xz)}{yz + 1} - x \log(yz + 1))$.

. Compute the potential function for this field whose potential at the origin is -3 .

. Calculate the value of the potential at the point $p = (0, 10, 3)$.

- 1) 6 2) -3 3) $\frac{57}{10}$ 4) $\frac{9}{2}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) \cos(t) (6 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left(-\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right) \cos(t) (6 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 170.103 2) 160.703 3) 94.9027 4) 76.1027

Exercise 3

Consider the vector field $F(x, y, z) = \{ 4x + \sin[y^2 - 2z^2], e^{2x^2 + z^2} - xz, e^{-x^2 - 2y^2} + 7y \}$ and the surface

$$S \equiv \left(\frac{-5 + x}{3} \right)^2 + \left(\frac{-1 + y}{3} \right)^2 + \left(\frac{-5 + z}{4} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -904.314 2) -1145.51 3) 603.186 4) -1205.81

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04-Line and Surface Integral-Computers exam for serial number: 6

Exercise 1

Consider the vectorial field $F(x, y, z) = (y^2, 2xy + 3z \sin(yz) + z(3yz + 3) \cos(yz), 3y \sin(yz) + y(3yz + 3) \cos(yz))$. Compute the potential function for this field whose potential at the origin is -4 .
 . Calculate the value of the potential at the point $p = (-2, 10, -10)$.

- 1) $-\frac{479}{2} + 297 \sin[100]$ 2) $222 + 297 \sin[100]$
 3) $\frac{2219}{2} + 297 \sin[100]$ 4) $-204 + 297 \sin[100]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(9t + 7) \sin(2t) (4 \cos(19t) + 10), (3t + 7) \sin(t) (4 \cos(19t) + 10)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 32940.9 2) 36600.9 3) 40260.9 4) 25620.9

Exercise 3

Consider the vector field $F(x, y, z) = \{-y^2 z + 8xy^2 z^2, -1, 9y^2 z - 6xy^2 z\}$ and the surface

$$S \equiv \left(\frac{-4+x}{4}\right)^2 + \left(\frac{-8+y}{2}\right)^2 + \left(\frac{3+z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -9.82214×10^6 2) -727565 3) 3.63783×10^6 4) 4.00161×10^6

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04-Line and Surface Integral-Computers exam for serial number: 7

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xy - 2\sin(yz), -2x^2 + (-2x - 2)z\cos(yz) - 1, (-2x - 2)y\cos(yz))$. Compute the potential function for this field whose potential at the origin is -2 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 1.64723 2) 7.24723 3) -13.5528 4) -3.55277

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{3}\sin(t)\right)\cos(t)(\cos(t)+3)}{\sin^2(t)+1}, \frac{\left(\frac{\sin(t)}{2} + \frac{\sqrt{3}}{2}\right)\cos(t)(\cos(t)+3)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 7.15841 2) 5.35841 3) 16.1584 4) 9.85841

Exercise 3

Consider the vector field $F(x, y, z) = \{5 + 8x + \cos[2z^2], -xy + 8xyz + \cos[x^2 + 2z^2], -xyz + \cos[x^2 + 2y^2]\}$ and the surface

$$S \equiv \left(\frac{9+x}{7}\right)^2 + \left(\frac{5+y}{4}\right)^2 + \left(\frac{-9+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 71357.8 2) 404358. 3) -237856. 4) -832499.

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04-Line and Surface Integral-Computers exam for serial number: 8

Exercise 1

Consider the vectorial field $F(x, y, z) = (z(xz - yz) \cos(xz) + z \sin(xz) - y, -z \sin(xz) - x, (x - y) \sin(xz) + x(xz - yz) \cos(xz))$. Compute the potential function for this field whose potential at the origin is 2.

. Calculate the value of the potential at the point $p = (-1, 5, 0)$.

- 1) $\frac{147}{10}$ 2) $\frac{14}{5}$ 3) $-\frac{133}{10}$ 4) 7

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(6t + 1) \sin(2t) (3 \cos(18t) + 10), (4t + 9) \sin(t) (3 \cos(18t) + 10)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 16114.5 2) 41436.5 3) 6906.54 4) 23020.5

Exercise 3

Consider the vector field $F(x, y, z) = \{-xy + 2x^2yz, -x^2y^2 + 3xz, -2x^2z\}$ and the surface

$$S \equiv \left(\frac{-1+x}{9}\right)^2 + \left(\frac{7+y}{3}\right)^2 + \left(\frac{6+z}{1}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -81956.2 2) 43135.3 3) 17254.3 4) 90583.8

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04-Line and Surface Integral-Computers exam for serial
number: 9

Exercise 1

Consider the vectorial field $F(x, y, z) = (z e^{xyz} + yz(xz - 3)e^{xyz}, xz(xz - 3)e^{xyz}, x e^{xyz} + xy(xz - 3)e^{xyz})$. Compute the potential function for this field whose potential at the origin is -6 .
Calculate the value of the potential at the point $p = (2, -5, 5)$.

- 1) $\frac{57}{10} + \frac{7}{e^{50}}$ 2) $-3 + \frac{7}{e^{50}}$ 3) $\frac{87}{10} + \frac{7}{e^{50}}$ 4) $6 + \frac{7}{e^{50}}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(6t + 2) \sin(2t) (\cos(17t) + 7), (4t + 4) \sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 1960.92 2) 1153.82 3) 346.722 4) 692.622

Exercise 3

Consider the vector field $F(x, y, z) = \{6x^2yz^2, 7y^2z^2, -8x^2y^2z^2\}$ and the surface

$$S \equiv \left(\frac{4+x}{9}\right)^2 + \left(\frac{7+y}{4}\right)^2 + \left(\frac{-3+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 1.59381×10^8 2) -7.96906×10^7 3) 1.51412×10^8 4) 2.31103×10^8

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04-Line and Surface Integral-Computers exam for serial number: 10

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xy^2 + 3ze^{xz} + ze^{xz}(3xz + 2), -4x^2y, 3xe^{xz} + xe^{xz}(3xz + 2))$. Compute the potential function for this field whose potential at the origin is 6. Calculate the value of the potential at the point $p = (-2, 4, 4)$.

- 1) $\frac{677}{2} - \frac{22}{e^8}$ 2) $-124 - \frac{22}{e^8}$ 3) $-574 - \frac{22}{e^8}$ 4) $-274 - \frac{22}{e^8}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{2} - \frac{\sqrt{3}}{2} \right) \cos(t) (9 \cos(t) + 10)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{2} - \frac{1}{2} \sqrt{3} \sin(t) \right) \cos(t) (9 \cos(t) + 10)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 203.331 2) 135.731 3) 118.831 4) 169.531

Exercise 3

Consider the vector field $F(x, y, z) = \{-\sin[y^2 + z^2], -9y + \cos[2x^2 - z^2], e^{-2x^2 + y^2} + 4y - 2xz\}$ and the surface

$$S \equiv \left(\frac{1+x}{3} \right)^2 + \left(\frac{-1+y}{3} \right)^2 + \left(\frac{-7+z}{8} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 5703.25 2) -4645.55 3) -9714.35 4) -2111.15

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 11

Exercise 1

Consider the vectorial field $F(x, y, z) = (-yz^2 \cos(xyz) - 4xy, -2x^2 - xz^2 \cos(xyz) + 2, -\sin(xyz) - xyz \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -5 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 14.5854 2) -4.41462 3) -23.4146 4) 7.08538

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (7 \cos(t) + 9) \left(\frac{\cos(t)}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right), \sin(2t) (7 \cos(t) + 9) \left(\frac{\sin(t)}{\sqrt{2}} + \frac{\cos(t)}{\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 66.4595 2) 82.8595 3) 33.6595 4) 148.46

Exercise 3

Consider the vector field $F(x, y, z) = \{xy + 8xyz + \cos[y^2], e^{x^2-z^2}, -8x + \sin[y^2]\}$ and the surface

$$S \equiv \left(\frac{-4+x}{4} \right)^2 + \left(\frac{6+y}{8} \right)^2 + \left(\frac{9+z}{1} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -74230.7 2) -159882. 3) 57101.6 4) 274085.

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04-Line and Surface Integral-Computers exam for serial number: 12

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{(-y-2)yz}{xyz+1} - 3, \frac{x(-y-2)z}{xyz+1} - \log(xyz+1) - 3, \frac{x(-y-2)y}{xyz+1} \right)$.

. Compute the potential function for this field whose potential at the origin is -4 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) -20.0931 2) -7.29309 3) -24.8931 4) -35.2931

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (3t+4) \sin(2t) (3 \cos(8t) + 10), (8t+6) \sin(t) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 1095.23 2) 2627.53 3) 2189.73 4) 3503.13

Exercise 3

Consider the vector field $F(x, y, z) = \{-3xyz, 3xyz^2 + 7x^2yz^2, 8x^2y^2 + 9xyz\}$ and the surface

$$S \equiv \left(\frac{7+x}{7} \right)^2 + \left(\frac{2+y}{1} \right)^2 + \left(\frac{-9+z}{1} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 935146 . 2) 2.43138×10^6 3) 2.99246×10^6 4) 4.20815×10^6

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04-Line and Surface Integral-Computers exam for serial number: 13

Exercise 1

Consider the vectorial field $F(x, y, z) = (y(-yz - z) \cos(xy) - 6xy^2, -6x^2y - z \sin(xy) + x(-yz - z) \cos(xy) + 3, (-y - 1) \sin(xy))$. Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the value of the potential at the point $p = (6, -8, 0)$.

- 1) $-\frac{6937}{10}$ 2) $-\frac{90181}{5}$ 3) $-\frac{55496}{5}$ 4) -6937

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(5t + 7) \sin(2t) (2 \cos(5t) + 4), (2t + 4) \sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 438.438 2) 625.938 3) 250.938 4) 938.438

Exercise 3

Consider the vector field $F(x, y, z) = \{7yz^2 - 5xy^2z^2, z - 3xy^2z^2, 6x - 3x^2\}$ and the surface

$$S \equiv \left(\frac{-2+x}{6}\right)^2 + \left(\frac{y}{5}\right)^2 + \left(\frac{-5+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 0.6 2) 1.6 3) -0.9 4) 0.

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04-Line and Surface Integral-Computers exam for serial number: 14

Exercise 1

Consider the vectorial field $F(x, y, z)$

$$= \left(-4x, \frac{yz^2}{yz+1} + z \log(yz+1) + 2, \frac{y^2z}{yz+1} + y \log(yz+1) \right)$$

. Compute the potential function for this field whose potential at the origin is -3 .

. Calculate the value of the potential at the point $p = (-4, -10, -1)$.

- 1) $-167 + 10 \log[11]$ 2) $-\frac{179}{5} + 10 \log[11]$ 3) $-\frac{787}{5} + 10 \log[11]$ 4) $-55 + 10 \log[11]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (4t+7) \sin(2t), (4 \cos(20t) + 4), (5t+9) \sin(t), (4 \cos(20t) + 4) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 7321.58 2) 1464.78 3) 4393.18 4) 6589.48

Exercise 3

Consider the vector field $F(x, y, z) = \{-8xy^2, 4xy^2z^2, -6x^2y^2z\}$ and the surface

$$S \equiv \left(\frac{-9+x}{7} \right)^2 + \left(\frac{-4+y}{3} \right)^2 + \left(\frac{-7+z}{3} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 1.27218×10^6 2) 508872. 3) 5.72481×10^6 4) 5.34315×10^6

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Exercise 1

Consider the vectorial field $F(x, y, z) = \left(-\frac{xyz^2}{xyz+1} - z \log(xyz+1) + 2, -\frac{x^2z^2}{xyz+1} - 2y, -\frac{x^2yz}{xyz+1} - x \log(xyz+1) \right)$.

. Compute the potential function for this field whose potential at the origin is -3 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) 3.01914 2) 8.71914 3) 0.0191388 4) -2.38086

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) \cos(t) (2 \cos(t) + 9)}{\sin^2(t) + 1}, \frac{\left(-\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right) \cos(t) (2 \cos(t) + 9)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 84.4336 2) 50.8336 3) 34.0336 4) 8.83363

Exercise 3

Consider the vector field $F(x, y, z) = \left\{ e^{2y^2-z^2}, -8xz + \cos[x^2-z^2], e^{-2x^2+2y^2} + 4xy \right\}$ and the surface

$$S \equiv \left(\frac{-4+x}{6} \right)^2 + \left(\frac{4+y}{7} \right)^2 + \left(\frac{2+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -0.8 2) 0. 3) -3.7 4) -1.8

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04-Line and Surface Integral-Computers exam for serial number: 16

Exercise 1

Consider the vectorial field $F(x, y, z) = (4x + z(yz + y), xz(z + 1), xyz + x(yz + y))$. Compute the potential function for this field whose potential at the origin is -2 .

. Calculate the value of the potential at the point $p = (-9, 0, -8)$.

- 1) 160 2) 592 3) 672 4) 544

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (4t + 4) \sin(2t) (2 \cos(17t) + 10), (t + 4) \sin(t) (2 \cos(17t) + 10) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 2377.81 2) 3962.41 3) 6339.31 4) 7923.91

Exercise 3

Consider the vector field $F(x, y, z) = \{6x^2y^2z^2, -x^2 - yz, -2xy^2z^2\}$ and the surface

$$S \equiv \left(\frac{-9+x}{8}\right)^2 + \left(\frac{-6+y}{9}\right)^2 + \left(\frac{9+z}{5}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 3.16201×10^9 2) 7.5286×10^8 3) 2.55972×10^9 4) 2.78558×10^9

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 17

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xy - 5e^{yz} - 3, -2x^2 - 5xz e^{yz}, -5xy e^{yz})$. Compute the potential function for this field whose potential at the origin is -2 .

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) -1.52809 2) -11.9281 3) -7.12809 4) 16.8719

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} \right) \cos(t) (5\cos(t)+6)}{\sin^2(t)+1}, \frac{\left(\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}} \right) \cos(t) (5\cos(t)+6)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 108.76 2) 57.4602 3) 68.8602 4) 97.3602

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ xy - 6xz + \cos[y^2 + z^2], 6x - 6xz - \sin[x^2 + z^2], e^{-x^2+2y^2} + 7x - 8xy \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{5+x}{3} \right)^2 + \left(\frac{5+y}{1} \right)^2 + \left(\frac{-6+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -7419.19 2) -1391.09 3) 9274.01 4) -4636.99

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 18

Exercise 1

Consider the vectorial field $F(x, y, z) = (2xy^2 - z(z - yz) \sin(xz), 2x^2y - z \cos(xz), (1 - y) \cos(xz) - x(z - yz) \sin(xz))$.
 . Compute the potential function for this field whose potential at the origin is 4.
 . Calculate the value of the potential at the point $p = (3, -1, -2)$.

- 1) $\frac{247}{10} - 4 \cos[6]$ 2) $\frac{301}{10} - 4 \cos[6]$ 3) $-\frac{16}{5} - 4 \cos[6]$ 4) $13 - 4 \cos[6]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(6t + 1) \sin(2t) (5 \cos(9t) + 6), (6t + 2) \sin(t) (5 \cos(9t) + 6)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 13195.4 2) 8247.19 3) 11546. 4) 3298.99

Exercise 3

Consider the vector field $F(x, y, z) = \{-x^2z, -5y^2z + 6xy^2z^2, -6xy^2z^2\}$ and the surface

$$S \equiv \left(\frac{x}{6}\right)^2 + \left(\frac{1+y}{2}\right)^2 + \left(\frac{-3+z}{5}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -6030.38 2) 7539.82 3) 17340.5 4) 21863.9

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 19

Exercise 1

Consider the vectorial field $F(x, y, z) = (2y^2z \cos(xyz) - 3y, 2 \sin(xyz) + 2xyz \cos(xyz) - 3x + 2y, 2xy^2 \cos(xyz))$. Compute the potential function for this field whose potential at the origin is 1.
 . Calculate the value of the potential at the point $p = (7, 1, -8)$.

- 1) $-\frac{293}{5} - 2 \sin[56]$ 2) $-19 - 2 \sin[56]$ 3) $\frac{13}{5} - 2 \sin[56]$ 4) $-\frac{437}{5} - 2 \sin[56]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(3t + 8) \sin(2t) \cos(5t) + 9, (5t + 2) \sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 2672.5 2) 713.399 3) 1247.7 4) 1782.

Exercise 3

Consider the vector field $F(x, y, z) = \{8xy^2 + 2xyz^2, -8y^2z^2, -8yz\}$ and the surface

$$S \equiv \left(\frac{7+x}{6}\right)^2 + \left(\frac{-5+y}{9}\right)^2 + \left(\frac{1+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -14801.1 2) 355249. 3) 236833. 4) 74010.9

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 20

Exercise 1

Consider the vectorial field $F(x, y, z) = (10xy^2 + \frac{yz(yz+2y)}{xyz+1},$

$$10x^2y + \frac{xz(yz+2y)}{xyz+1} + (z+2)\log(xyz+1), \frac{xy(yz+2y)}{xyz+1} + y\log(xyz+1)$$

). Compute the potential function for this field whose potential at the origin is -2 .

. Calculate the value of the potential at the point $p = (-5, -4, 0)$.

- 1) 1998 2) $\frac{45954}{5}$ 3) $\frac{999}{5}$ 4) $-\frac{2997}{5}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (4t+4) \sin(2t) (5 \cos(15t) + 5), (6t+4) \sin(t) (5 \cos(15t) + 5) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 5755.68 2) 7194.48 3) 3597.48 4) 5036.28

Exercise 3

Consider the vector field $F(x, y, z) = \{0, -4z, -7x - 2x^2yz^2\}$ and the surface

$$S \equiv \left(\frac{-5+x}{5}\right)^2 + \left(\frac{-1+y}{5}\right)^2 + \left(\frac{-4+z}{1}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -231223. 2) -50265.5 3) 15080.3 4) -5026.08

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 21

Exercise 1

Consider the vectorial field $F(x, y, z) = (2x - 3y^2 + 3yz, z(3x + y) - 6xy + yz, y(3x + y))$. Compute the potential function for this field whose potential at the origin is 0.

. Calculate the value of the potential at the point $p = (-6, -9, 6)$.

- 1) $\frac{72324}{5}$ 2) 2952 3) $-\frac{1476}{5}$ 4) $-\frac{8856}{5}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(5t + 3) \sin(2t) (\cos(14t) + 9), (9t + 5) \sin(t) (\cos(14t) + 9)\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 23772.4 2) 16640.8 3) 19018. 4) 42790.

Exercise 3

Consider the vector field $F(x, y, z) = \{-4, -7y, -5xy^2\}$ and the surface

$$S \equiv \left(\frac{1+x}{3}\right)^2 + \left(\frac{-2+y}{3}\right)^2 + \left(\frac{-7+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -5173.98 2) 634.025 3) -1055.58 4) 106.025

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 22

Exercise 1

Consider the vectorial field $F(x, y, z) = (3yz \cos(yz) + 2, 3xz \cos(yz) - z(3xyz + 2) \sin(yz), 3xy \cos(yz) - y(3xyz + 2) \sin(yz))$. Compute the potential function for this field whose potential at the origin is -5 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 1.822 2) -3.778 3) -5.378 4) 4.622

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (9 \cos(t) + 10) \left(-\frac{\sin(t)}{2} - \frac{1}{2} \sqrt{3} \cos(t) \right), \sin(2t) (9 \cos(t) + 10) \left(\frac{\cos(t)}{2} - \frac{1}{2} \sqrt{3} \sin(t) \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 88.3484 2) 132.348 3) 110.348 4) 66.3484

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 7z - 3xz - \sin[y^2], e^{-x^2-2z^2} + 9xyz, 8z - 4xyz + \cos[2x^2 + 2y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{6+x}{5} \right)^2 + \left(\frac{-1+y}{7} \right)^2 + \left(\frac{9+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 1.00675×10^6 2) -791020. 3) 719111. 4) -287643.

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 23

Exercise 1

Consider the vectorial field $F(x, y, z)$

$$= \left(\frac{(-x-1)yz}{xyz+1} - \log(xyz+1), \frac{(-x-1)xz}{xyz+1}, \frac{(-x-1)xy}{xyz+1} \right)$$

. Compute the potential function for this field whose potential at the origin is 3.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

- 1) 2.81723 2) 1.61723 3) 10.4172 4) 2.61723

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (5t+5) \sin(2t) (\cos(16t) + 4), (8t+1) \sin(t) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 880.286 2) 1466.49 3) 1564.19 4) 977.986

Exercise 3

Consider the vector field $F(x, y, z) = \{3x^2yz + 4xy^2z, 6x^2 + 8x^2yz^2, 7 - 7x^2z^2\}$ and the surface

$$S \equiv \left(\frac{5+x}{6} \right)^2 + \left(\frac{y}{1} \right)^2 + \left(\frac{-1+z}{1} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -3623.42 2) -1449.02 3) -3261.02 4) -17394.6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 24

Exercise 1

Consider the vectorial field $F(x, y, z) = (3x^3y^4z^4 + 3x^2y^3z^3(3xyz - 1) - 4xy, 3x^4y^3z^4 + 3x^3y^2z^3(3xyz - 1) - 2x^2, 3x^4y^4z^3 + 3x^3y^3z^2(3xyz - 1))$. Compute the potential function for this field whose potential at the origin is -3 .
 . Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.
 1) 8.67504 2) -3.32496 3) 10.275 4) 2.67504

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (9 \cos(t) + 10) \left(-\frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} - \frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} \right), \sin(2t) (9 \cos(t) + 10) \left(\frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} - \frac{1}{2} \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 143.348 2) 33.3484 3) 110.348 4) 187.348

Exercise 3

Consider the vector field $F(x, y, z) = \{-8x - 4xz - \sin[2y^2 + 2z^2], e^{-2x^2+z^2}, e^{-2x^2+y^2}\}$ and the surface

$$S \equiv \left(\frac{-1+x}{3} \right)^2 + \left(\frac{-6+y}{5} \right)^2 + \left(\frac{-2+z}{1} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 2415.09 2) -1005.31 3) 2012.69 4) 503.69

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 25

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2xy^2z^2 - 2y, -2x^2yz^2 - 2x - 1, -2x^2y^2z)$. Compute the potential function for this field whose potential at the origin is -5 .

. Calculate the value of the potential at the point $p = (-7, -10, -2)$.

- 1) 3947 2) -11841 3) -19735 4) -23682

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{2} - \frac{\sqrt{3}}{2}\right) \cos(t) (\cos(t)+7)}{\sin^2(t)+1}, \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{3} \sin(t)\right) \cos(t) (\cos(t)+7)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 84.1584 2) 25.3584 3) 74.3584 4) 49.8584

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ e^{-2y^2} + 3xy - 8yz, e^{-x^2+2z^2} + 3y, 5xz - \sin[x^2 + 2y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{9+x}{7}\right)^2 + \left(\frac{-4+y}{9}\right)^2 + \left(\frac{z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -31667.3 2) -38000.9 3) 6334.35 4) -19000.1

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 26

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xy^2z^2e^{xyz} - 4yz e^{xyz}, -4x^2yz^2e^{xyz} - 4xz e^{xyz}, -4x^2y^2ze^{xyz} - 4xy e^{xyz})$. Compute the potential function for this field whose potential at the origin is 3.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 5.31439 2) -4.88561 3) 6.71439 4) 2.31439

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (5 \cos(t) + 5) \left(\frac{1}{2} \sqrt{3} \cos(t) - \frac{\sin(t)}{2} \right), \sin(2t) (5 \cos(t) + 5) \left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{\cos(t)}{2} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 41.0524 2) 29.4524 3) 43.9524 4) 12.0524

Exercise 3

Consider the vector field $F(x, y, z) = \{xz + \cos[z^2], e^{2x^2-2z^2} + 8xz, 5x + \cos[x^2]\}$ and the surface

$$S \equiv \left(\frac{1+x}{9} \right)^2 + \left(\frac{2+y}{1} \right)^2 + \left(\frac{4+z}{3} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 1178.41 2) -452.389 3) -1448.99 4) -1131.89

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 27

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy^2 + 6yz^2 \sin(xyz), 6x^2y + 6xz^2 \sin(xyz), 6xyz \sin(xyz) - 6 \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -3 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 12.415 2) -5.58498 3) -25.985 4) -3.18498

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(t+4) \sin(2t) (6 \cos(16t) + 7), (9t+7) \sin(t)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 2101.55 2) 1106.15 3) 1769.75 4) 995.553

Exercise 3

Consider the vector field $F(x, y, z) = \{0, xz, y^2 - 7z^2\}$ and the surface
 $S \equiv \left(\frac{x}{4}\right)^2 + \left(\frac{9+y}{5}\right)^2 + \left(\frac{-5+z}{1}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) -3518.31 2) 11730.7 3) -25805.3 4) -5864.31

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 28

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xy^2 - 3z \sin(xyz) + yz(yz - 3xz) \cos(xyz) - 1, -4x^2y + z \sin(xyz) + xz(yz - 3xz) \cos(xyz), (y - 3x) \sin(xyz) + xy(yz - 3xz) \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -5 .

. Calculate the value of the potential at the point $p = (2, -6, 6)$.

- 1) $-\frac{313}{2} + 72 \sin[72]$ 2) $-295 + 72 \sin[72]$ 3) $\frac{2126}{5} + 72 \sin[72]$ 4) $-\frac{6551}{10} + 72 \sin[72]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{3} \sin(t)\right) \cos(t) (2 \cos(t) + 2)}{\sin^2(t) + 1}, \frac{\left(\frac{\sin(t)}{2} + \frac{\sqrt{3}}{2}\right) \cos(t) (2 \cos(t) + 2)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 8.13363 2) 7.43363 3) 13.0336 4) 3.23363

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ -8x - \sin[2y^2], e^{-x^2 - z^2} + 3xz, 5xyz + \cos[y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-2+x}{8} \right)^2 + \left(\frac{7+y}{3} \right)^2 + \left(\frac{6+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 25093. 2) -7841.33 3) -36070.7 4) -15682.8

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 29

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{yz(-xz - 2y)}{xyz + 1} - z \log(xyz + 1) + 6xy, 3x^2 + \frac{xz(-xz - 2y)}{xyz + 1} - 2 \log(xyz + 1), \frac{xy(-xz - 2y)}{xyz + 1} - x \log(xyz + 1) \right)$.

. Compute the potential function for this field whose potential at the origin is -4 .

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -10.0925 2) 3.50754 3) -3.69246 4) 7.50754

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} \right) \cos(t) (9\cos(t)+10)}{\sin^2(t)+1}, \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} + \frac{1+\sqrt{3}}{2\sqrt{2}} \right) \cos(t) (9\cos(t)+10)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

1) 135.731 2) 118.831 3) 169.531 4) 287.831

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 4 - 5yz + \cos[2y^2 + z^2], e^{2x^2} - 6xy, 8 + \cos[x^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-1+x}{4} \right)^2 + \left(\frac{7+y}{2} \right)^2 + \left(\frac{5+z}{4} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -2092.25 2) 644.752 3) 1449.75 4) -804.248

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 30

Exercise 1

Consider the vectorial field $F(x, y, z) = (-z(3x - 3xy) \sin(xz) + (3 - 3y) \cos(xz) - 1, -3x \cos(xz), -x(3x - 3xy) \sin(xz))$. Compute the potential function for this field whose potential at the origin is -2 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -2.01045 2) 3.98955 3) -1.81045 4) -8.41045

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (5 \cos(t) + 5) \left(-\frac{1}{2} \sqrt{3} \sin(t) - \frac{\cos(t)}{2} \right), \sin(2t) (5 \cos(t) + 5) \left(\frac{1}{2} \sqrt{3} \cos(t) - \frac{\sin(t)}{2} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 29.4524 2) 12.0524 3) 55.5524 4) 9.15243

Exercise 3

Consider the vector field $F(x, y, z) = \{3 + 6xy + \cos[y^2], -9xz - 6xyz + \cos[x^2 + 2z^2], y - \sin[x^2]\}$ and the surface

$$S \equiv \left(\frac{6+x}{1} \right)^2 + \left(\frac{9+y}{8} \right)^2 + \left(\frac{2+z}{7} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) $-127094.$ 2) -29556.1 3) -32511.8 4) 56159.2

Further Mathematics - Degree in Engineering - 2024/2025
04-Line and Surface Integral-Computers exam for serial
number: 31

Exercise 1

Consider the vectorial field $F(x, y, z) = (x^3 y^3 z^3 (3y + 3z) + 3x^2 y^3 z^3 (3xy + 3xz) - 2y^2, 3x^4 y^3 z^3 + 3x^3 y^2 z^3 (3xy + 3xz) - 4xy, 3x^4 y^3 z^3 + 3x^3 y^3 z^2 (3xy + 3xz))$.
 Compute the potential function for this field whose potential at the origin is -3 .
 Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -18.8733 2) 9.52667 3) -3.27333 4) -4.07333

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) \cos(t) (6 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left(-\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right) \cos(t) (6 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 104.303 2) 132.503 3) 170.103 4) 94.9027

Exercise 3

Consider the vector field $F(x, y, z) = \{e^{2y^2} - 2z + 6xyz, -7xy - 6xyz + \cos[2x^2 + z^2], -5yz + \cos[2x^2 + 2y^2]\}$ and the surface

$$S \equiv \left(\frac{-5+x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{-1+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -32021 2) -6534.51 3) 14377.5 4) -7841.51

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 32

Exercise 1

Consider the vectorial field $F(x, y, z) = (4x - 6xy^2, -6x^2y, 0)$. Compute the potential function for this field whose potential at the origin is 2.

. Calculate the value of the potential at the point $p = (3, 0, 5)$.

- 1) 100 2) -16 3) 20 4) 54

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (2t + 4) \sin(2t) (6 \cos(4t) + 6), (6t + 8) \sin(t) (6 \cos(4t) + 6) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 14077.4 2) 13071.9 3) 10055.4 4) 4022.38

Exercise 3

Consider the vector field $F(x, y, z) = \{5x^2 + 2xyz, 2xyz^2, -9y^2z\}$ and the surface

$$S \equiv \left(\frac{6+x}{1}\right)^2 + \left(\frac{5+y}{6}\right)^2 + \left(\frac{-1+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 84510.5 2) -285223. 3) -528190. 4) -105638.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 33

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2y e^{xyz} + yz e^{xyz} (3z - 2xy) - 4xy + 6x, -2x^2 - 2x e^{xyz} + xz e^{xyz} (3z - 2xy), xy e^{xyz} (3z - 2xy) + 3e^{xyz})$. Compute the potential function for this field whose potential at the origin is 0.
 . Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.
 1) 1.82972 2) 0.0297233 3) -1.57028 4) 2.02972

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (2 \cos(t) + 5) \left(-\frac{\sin(t)}{\sqrt{2}} - \frac{\cos(t)}{\sqrt{2}} \right), \sin(2t) (2 \cos(t) + 5) \left(\frac{\cos(t)}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 6.50575 2) 21.2058 3) 4.40575 4) 35.9058

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ e^{2y^2} + 9x, -7y + \cos[2x^2 + z^2], 8x + 7yz - \sin[2x^2 + y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-8+x}{3} \right)^2 + \left(\frac{2+y}{8} \right)^2 + \left(\frac{2+z}{4} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -4825.49 2) 11582.9 3) -5790.69 4) 3861.31

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 34

Exercise 1

Consider the vectorial field $F(x, y, z) = (2x^2y^2 + 2x(2x+2)y^2, 2x^2(2x+2)y, 0)$. Compute the potential function for this field whose potential at the origin is -1 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

- 1) -0.911111 2) -0.611111 3) -0.511111 4) -2.51111

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(5t+7)\sin(2t), (4\cos(9t)+10), (2t+5)\sin(t)\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 2503.62 2) 2837.42 3) 834.623 4) 1669.12

Exercise 3

Consider the vector field $F(x, y, z) = \{5xyz, -4yz^2, -6xz^2 + 8y^2z^2\}$ and the surface

$$S \equiv \left(\frac{5+x}{9}\right)^2 + \left(\frac{9+y}{7}\right)^2 + \left(\frac{2+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -4.52257×10^6 2) -1.40356×10^6 3) -1.55951×10^6 4) -2.02736×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 35

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy + 3z \cos(yz), 3x^2 - z(3xz + 2) \sin(yz), 3x \cos(yz) - y(3xz + 2) \sin(yz))$. Compute the potential function for this field whose potential at the origin is 8.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -26.0183 2) 28.8817 3) 9.08171 4) 41.4817

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{2} - \frac{\sqrt{3}}{2}\right) \cos(t) (9 \cos(t) + 10)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{2} - \frac{1}{2} \sqrt{3} \sin(t)\right) \cos(t) (9 \cos(t) + 10)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 51.231 2) 169.531 3) 220.231 4) 17.431

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ -4yz + 2xyz + \cos[2z^2], e^{2x^2 - z^2} - 9yz, 3xy + 6yz + \cos[x^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{1+x}{7} \right)^2 + \left(\frac{7+y}{6} \right)^2 + \left(\frac{-4+z}{5} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -542215. 2) -117873. 3) -377193. 4) 341832.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 36

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy + (3 - 2z) \sin(yz), 3x^2 + z(3x - 2xz) \cos(yz) + 6y, y(3x - 2xz) \cos(yz) - 2x \sin(yz))$. Compute the potential function for this field whose potential at the origin is -3 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -8.49881 2) 6.30119 3) -1.29881 4) -3.49881

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{3} \sin(t)\right) \cos(t) (8 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left(\frac{\sin(t)}{2} + \frac{\sqrt{3}}{2}\right) \cos(t) (8 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 189.738 2) 12.7381 3) 71.7381 4) 118.938

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 2xy + \cos[2y^2 + z^2], 3 + 3xz - \sin[x^2], e^{-2y^2} + 9xy + z \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-3+x}{8}\right)^2 + \left(\frac{y}{8}\right)^2 + \left(\frac{-5+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -6002.54 2) 9863.06 3) 4503.06 4) 2144.66

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 37

Exercise 1

Consider the vectorial field $F(x, y, z) = (2xy^2 - 2, 2x^2y - (-y - 3)z \sin(yz) - \cos(yz), -((-y - 3)y \sin(yz)))$. Compute the potential function for this field whose potential at the origin is -6 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -1.58684 2) -7.98684 3) 0.0131642 4) -7.18684

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(3t + 1) \sin(2t) (7 \cos(20t) + 9), (9t + 4) \sin(t)\}$
 Indication: it is necessary to represent
 the curve to check whether it has intersection points.
 1) 1441.65 2) 393.647 3) 1310.65 4) 1048.65

Exercise 3

Consider the vector field $F(x, y, z) = \{4xz, 5xy^2z, 2xy^2z^2\}$ and the surface
 $S \equiv \left(\frac{7+x}{8}\right)^2 + \left(\frac{8+y}{7}\right)^2 + \left(\frac{9+z}{2}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) 634359 . 2) 1.90308×10^6 3) 6.97794×10^6 4) 6.34358×10^6

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04-Line and Surface Integral-Computers exam for serial number: 38

Exercise 1

Consider the vectorial field $F(x, y, z) = (-y(3xz - z) \sin(xy) + 3z \cos(xy) - 3y^2 + 2y, -x(3xz - z) \sin(xy) - 6xy + 2x, (3x - 1) \cos(xy))$. Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the value of the potential at the point $p = (9, 8, 10)$.

- 1) $\frac{26978}{5} + 260 \cos[72]$ 2) $-\frac{13436}{5} + 260 \cos[72]$
 3) $-1585 + 260 \cos[72]$ 4) $-\frac{29969}{5} + 260 \cos[72]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}}{2} - \frac{\sin(t)}{2}\right) \cos(t) (9 \cos(t) + 9)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{1}{2}\right) \cos(t) (9 \cos(t) + 9)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 30.531 2) 180.531 3) 150.531 4) 255.531

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 9xy + 2z - \sin[z^2], e^{-2x^2+z^2} + 2xy - 9xyz, -5z + 5xz + \sin[x^2 - 2y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{x}{8}\right)^2 + \left(\frac{7+y}{9}\right)^2 + \left(\frac{-9+z}{5}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -164067. 2) 287118. 3) -348642. 4) -102542.

Further Mathematics - Degree in Engineering - 2024/2025
04-Line and Surface Integral-Computers exam for serial
number: 39

Exercise 1

Consider the vectorial field $F(x, y, z) = ($
 $y(-z) - 2xyz - 3xy) \sin(xyz) + (-2yz - 3y) \cos(xyz) - 3y,$
 $x(-z) - 2xyz - 3xy) \sin(xyz) + (-2xz - 3x) \cos(xyz) - 3x - 2y$
 $, -xy(-2xyz - 3xy) \sin(xyz) - 2xy \cos(xyz)$
 $). Compute the potential function for this field whose potential at the origin is 1.$
 $. Calculate the value of the potential at the point $p = (8, -10, -2)$.$

- 1) $\frac{1143}{5} - 80 \cos[160]$ 2) $-\frac{999}{10} - 80 \cos[160]$ 3) $141 - 80 \cos[160]$ 4) $-\frac{828}{5} - 80 \cos[160]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (8 \cos(t) + 8) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right), \sin(2t) (8 \cos(t) + 8) \left(\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} + \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

- 1) 75.3982 2) 60.3982 3) 82.8982 4) 105.398

Exercise 3

Consider the vector field $F(x, y, z) =$
 $\{2 + \sin[y^2], 8x + \cos[2x^2 + 2z^2], 6xz + \sin[2x^2 - 2y^2]\}$ and the surface

$$S \equiv \left(\frac{9+x}{1} \right)^2 + \left(\frac{-8+y}{5} \right)^2 + \left(\frac{7+z}{3} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 339.38 2) 1357.28 3) -3392.92 4) -4750.12

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 40

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{y(-3xz - 2y)}{xy + 1} - 3z \log(xy + 1) + 3y, \right.$

$\left. \frac{x(-3xz - 2y)}{xy + 1} - 2 \log(xy + 1) + 3x + 6y, -3x \log(xy + 1) \right)$

). Compute the potential function for this field whose potential at the origin is -5 .

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) 2.27297 2) -3.72703 3) -14.527 4) 9.07297

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (6t + 3) \sin(2t) (5 \cos(7t) + 10), (2t + 2) \sin(t) (5 \cos(7t) + 10) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 13957.1 2) 14954. 3) 9969.49 4) 17944.7

Exercise 3

Consider the vector field $F(x, y, z) = \{2xy^2z^2, 3x^2y^2z^2, -2yz\}$ and the surface

$$S \equiv \left(\frac{5+x}{6} \right)^2 + \left(\frac{5+y}{3} \right)^2 + \left(\frac{-1+z}{1} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -299267. 2) -80882.9 3) 177943. 4) 40441.6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 41

Exercise 1

Consider the vectorial field $F(x, y, z) = (z e^{xyz} + y z e^{xyz} (xz + 2y), x z e^{xyz} (xz + 2y) + 2 e^{xyz} - 3, x e^{xyz} + x y e^{xyz} (xz + 2y))$.

. Compute the potential function for this field whose potential at the origin is 1.

. Calculate the value of the potential at the point $p = (1, -8, -8)$.

- 1) $-179\,572\,293\,527\,374\,566\,227\,786\,074\,763 - 24 e^{64}$ 2) $224\,465\,366\,909\,218\,207\,784\,732\,593\,510 - 24 e^{64}$
 3) $25 - 24 e^{64}$ 4) $419\,002\,018\,230\,540\,654\,531\,500\,841\,197 - 24 e^{64}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(6t + 2) \sin(2t) (3 \cos(8t) + 9), (5t + 9) \sin(t) (3 \cos(8t) + 9)\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 31972.3 2) 36539.7 3) 13702.7 4) 22837.5

Exercise 3

Consider the vector field $F(x, y, z) = \{-8x^2z, -7yz, -8x + 7x^2y\}$ and the surface

$$S \equiv \left(\frac{2+x}{6}\right)^2 + \left(\frac{-4+y}{9}\right)^2 + \left(\frac{-7+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 609594. 2) -138544. 3) -498758. 4) 277088.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 42

Exercise 1

Consider the vectorial field $F(x, y, z) = (y e^{xy} (yz + y) + 6x + 2y, x e^{xy} (yz + y) + (z + 1) e^{xy} + 2x, y e^{xy})$. Compute the potential function for this field whose potential at the origin is 5.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) 18.0774 2) 15.9774 3) 7.57742 4) 10.3774

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(4t + 3) \sin(2t) (3 \cos(12t) + 10), (3t + 7) \sin(t) (3 \cos(12t) + 10)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 23346.5 2) 10895.3 3) 15564.5 4) 28015.7

Exercise 3

Consider the vector field $F(x, y, z) = \{3z, 8x^2y^2z + 8xy^2z^2, x^2y^2z^2\}$ and the surface

$$S \equiv \left(\frac{-9+x}{7}\right)^2 + \left(\frac{-1+y}{3}\right)^2 + \left(\frac{z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 868949. 2) 1.21653×10^6 3) 2.17237×10^6 4) 1.91169×10^6

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04-Line and Surface Integral-Computers exam for serial number: 43

Exercise 1

Consider the vectorial field $F(x, y, z) = (3 \sin(xyz) + yz(3x - 3z) \cos(xyz), xz(3x - 3z) \cos(xyz) - 4y + 3, xy(3x - 3z) \cos(xyz) - 3 \sin(xyz))$. Compute the potential function for this field whose potential at the origin is -4 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -3.16667 2) 12.0333 3) 10.0333 4) 2.43333

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(2t + 1) \sin(2t) (2 \cos(6t) + 9), (9t + 5) \sin(t)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 972.111 2) 388.911 3) 486.111 4) 1166.51

Exercise 3

Consider the vector field $F(x, y, z) = \{3y^2z, 7x^2y^2 - 5yz^2, -8x^2y^2z + 6xz^2\}$ and the surface
 $S \equiv \left(\frac{-3+x}{8}\right)^2 + \left(\frac{6+y}{5}\right)^2 + \left(\frac{-1+z}{3}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) 1.24042×10^7 2) -4.43005×10^6 3) 1.01891×10^7 4) 1.77202×10^6

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04-Line and Surface Integral-Computers exam for serial number: 44

Exercise 1

Consider the vectorial field $F(x, y, z) = (2x, 0, 0)$. Compute the potential function for this field whose potential at the origin is 3. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -6.26667 2) -6.56667 3) 3.33333 4) 9.33333

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}} \right) \cos(t) (6\cos(t)+8)}{\sin^2(t)+1}, \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} \right) \cos(t) (6\cos(t)+8)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 160.703 2) 38.5027 3) 94.9027 4) 19.7027

Exercise 3

Consider the vector field $F(x, y, z) = \{3xyz - \sin[2y^2 - z^2], -5 - 3z + \cos[x^2], 6xy - \sin[x^2 - y^2]\}$ and the surface

$$S \equiv \left(\frac{-4+x}{9} \right)^2 + \left(\frac{2+y}{7} \right)^2 + \left(\frac{9+z}{4} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -125402. 2) -131102. 3) 171003. 4) 57001.1

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 45

Exercise 1

Consider the vectorial field $F(x, y, z) = (2 \sin(xyz) + 2xyz \cos(xyz) - 6xy, 2x^2z \cos(xyz) - 3x^2 + 1, 2x^2y \cos(xyz))$. Compute the potential function for this field whose potential at the origin is 4.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) 0.962565 2) 8.16257 3) -1.43743 4) 4.16257

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (5t + 8) \sin(2t) (7 \cos(9t) + 9), (5t + 3) \sin(t) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 3992.44 2) 2883.44 3) 3548.84 4) 2218.04

Exercise 3

Consider the vector field $F(x, y, z) = \{9xz - 8y^2z^2, 2y - 7xy^2, -8y^2\}$ and the surface

$$S \equiv \left(\frac{5+x}{4}\right)^2 + \left(\frac{-5+y}{5}\right)^2 + \left(\frac{3+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 435634. 2) 217817. 3) -87126.7 4) 43563.5

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04-Line and Surface Integral-Computers exam for serial number: 46

Exercise 1

Consider the vectorial field $F(x, y, z) = (y z (x + 2 y) e^{x y z} + e^{x y z} + 3 y^2, 2 e^{x y z} + x z (x + 2 y) e^{x y z} + 6 x y, x y (x + 2 y) e^{x y z})$. Compute the potential function for this field whose potential at the origin is -2 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 0.298861 2) 0.798861 3) 3.89886 4) -0.301139

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} - \frac{1+\sqrt{3}}{2\sqrt{2}} \right) \cos(t) (3\cos(t)+7)}{\sin^2(t)+1}, \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} \right) \cos(t) (3\cos(t)+7)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 67.9257 2) 56.7257 3) 101.526 4) 73.5257

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ -9x - 3xyz - \sin[2y^2 - z^2], e^{-x^2} + 2xy - 9xz, -4x - 3y + \cos[x^2 + y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{7+x}{7} \right)^2 + \left(\frac{-9+y}{2} \right)^2 + \left(\frac{-7+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -114379. 2) -24864.7 3) -37297.2 4) -9945.66

Further Mathematics - Degree in Engineering - 2024/2025 04-Line and Surface Integral-Computers exam for serial number: 47

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xyz^2 \sin(xyz) + 4z \cos(xyz) - 1, -4x^2z^2 \sin(xyz) - 3, 4x \cos(xyz) - 4x^2yz \sin(xyz))$.
 . Compute the potential function for this field whose potential at the origin is -2 .
 . Calculate the value of the potential at the point $p = (-6, 6, 9)$.

- 1) $-\frac{2083}{5} - 216 \cos[324]$ 2) $-563 - 216 \cos[324]$
 3) $-14 - 216 \cos[324]$ 4) $\frac{2309}{5} - 216 \cos[324]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}}{2} - \frac{\sin(t)}{2}\right) \cos(t) (8 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{1}{2}\right) \cos(t) (8 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 118.938 2) 24.5381 3) 59.9381 4) 201.538

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ -5yz - \sin[2y^2], e^{x^2-z^2} - 3xy, -12yz + \cos[2x^2 - 2y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-4+x}{8}\right)^2 + \left(\frac{5+y}{1}\right)^2 + \left(\frac{6+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 4825.49 2) -9649.51 3) 17370.5 4) -5789.51

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 48

Exercise 1

Consider the vectorial field $F(x, y, z) = (-4xy^2 - yz(yz - 2y) \sin(xyz) + 1, -4x^2y - xz(yz - 2y) \sin(xyz) + (z - 2) \cos(xyz), y \cos(xyz) - xy(yz - 2y) \sin(xyz))$. Compute the potential function for this field whose potential at the origin is -3 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -10.6552 2) 3.34482 3) -3.45518 4) -1.05518

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(4t + 4) \sin(2t) (5 \cos(8t) + 9), (t + 6) \sin(t) (5 \cos(8t) + 9)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 2957.08 2) 14781.9 3) 4927.88 4) 9854.88

Exercise 3

Consider the vector field $F(x, y, z) = \{4y + 2yz^2, -5x^2z + 2xz^2, -4z^2 + 9xyz^2\}$ and the surface
 $S \equiv \left(\frac{3+x}{5}\right)^2 + \left(\frac{8+y}{4}\right)^2 + \left(\frac{-5+z}{3}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) 213126 . 2) -372970 . 3) 532814 . 4) 1.49188×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 49

Exercise 1

Consider the vectorial field $F(x, y, z) = (3yz + y + 2, z(3x + yz) + x + yz^2, y(3x + yz) + y^2z)$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the value of the potential at the point $p = (-4, -5, -5)$.

- 1) 341 2) $-\frac{7161}{10}$ 3) $\frac{13981}{10}$ 4) $\frac{341}{2}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) \cos(t) (4 \cos(t) + 10)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right) \cos(t) (4 \cos(t) + 10)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 147.635 2) 57.2345 3) 113.735 4) 181.535

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{6y + \cos[y^2 + z^2], -8xy + \cos[2z^2], 6xz + \cos[2x^2 + 2y^2]\}$$
 and the surface

$$S \equiv \left(\frac{6+x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{3+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 4944.97 2) -2532.23 3) 1206.37 4) 603.372

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 50

Exercise 1

Consider the vectorial field $F(x, y, z) = (0, -4y, 0)$. Compute the potential function for this field whose potential at the origin is -2 .

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) -13.7667 2) -5.06667 3) -2.66667 4) 6.03333

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (9 \cos(t) + 10) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right), \sin(2t) (9 \cos(t) + 10) \left(\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} + \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 198.348 2) 110.348 3) 143.348 4) 33.3484

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{3xz + 6xyz + \sin[y^2 - 2z^2], -9z + 3xz - \sin[x^2 + z^2], 6xz - \sin[x^2 + 2y^2]\}$$
 and the surface

$$S \equiv \left(\frac{-8+x}{2} \right)^2 + \left(\frac{6+y}{9} \right)^2 + \left(\frac{8+z}{5} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -223480 . 2) 552819 . 3) 211718 . 4) 117621 .

Further Mathematics - Degree in Engineering - 2024/2025 04-Line and Surface Integral-Computers exam for serial number: 51

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy^2 + \frac{yz(z+2)}{xyz+1}, 6x^2y + \frac{xz(z+2)}{xyz+1}, \frac{xy(z+2)}{xyz+1} + \log(xyz+1))$.
 . Compute the potential function for this field whose potential at the origin is 0.
 . Calculate the value of the potential at the point $p = (-3, 3, -1)$.

- 1) $-100 + \text{Log}[10]$ 2) $-\frac{837}{2} + \text{Log}[10]$ 3) $-296 + \text{Log}[10]$ 4) $243 + \text{Log}[10]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(9t+6)\sin(2t) - (9\cos(17t)+9), (7t+8)\sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 967.718 2) 2419.12 3) 8708.52 4) 4838.12

Exercise 3

Consider the vector field $F(x, y, z) = \{8xz + 6xyz, -6x^2y^2z, 8x^2y^2z\}$ and the surface

$$S \equiv \left(\frac{1+x}{4}\right)^2 + \left(\frac{-9+y}{9}\right)^2 + \left(\frac{-7+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -514564. 2) -171521. 3) -728966. 4) 428805.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 52

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy - 3y, 3x^2 - 3x - yz \sin(yz) + \cos(yz), -y^2 \sin(yz))$. Compute the potential function for this field whose potential at the origin is 4.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -4.1903 2) -7.3903 3) -9.3903 4) 4.2097

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(3t + 9) \sin(2t) (2 \cos(6t) + 5), (3t + 4) \sin(t) (2 \cos(6t) + 5)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 7504.39 2) 8387.19 3) 2648.99 4) 4414.59

Exercise 3

Consider the vector field $F(x, y, z) = \{3x^2y^2, 3x^2y^2 - 2xz^2, 0\}$ and the surface

$$S \equiv \left(\frac{-9+x}{2}\right)^2 + \left(\frac{9+y}{7}\right)^2 + \left(\frac{z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -433 207. 2) -547 209. 3) -250 804. 4) 228 004.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 53

Exercise 1

Consider the vectorial field $F(x, y, z) =$

$$\left(-\frac{y^2 z}{xyz+1} - 4xy, -2x^2 - \frac{xyz}{xyz+1} - \log(xyz+1) + 4y, -\frac{xy^2}{xyz+1} \right)$$

). Compute the potential function for this field whose potential at the origin is -3 .

. Calculate the value of the potential at the point $p = (7, -3, -6)$.

- 1) $\frac{4452}{5} + 3 \log[127]$ 2) $\frac{8581}{10} + 3 \log[127]$ 3) $309 + 3 \log[127]$ 4) $\frac{11811}{10} + 3 \log[127]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (5t+5) \sin(2t) (3 \cos(8t) + 9), (3t+7) \sin(t) (3 \cos(8t) + 9) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 22901. 2) 7046.63 3) 17616.2 4) 26424.2

Exercise 3

Consider the vector field $F(x, y, z) = \{5y^2 z^2 - 6x^2 y^2 z^2, 9y, 5z^2\}$ and the surface

$$S \equiv \left(\frac{x}{5}\right)^2 + \left(\frac{4+y}{6}\right)^2 + \left(\frac{-4+z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 205044. 2) 155168. 3) 149627. 4) 55417.7

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 54

Exercise 1

Consider the vectorial field $F(x, y, z) = (yz e^{xyz}(-xyz - 3y) - yz e^{xyz}(-xz - 3), (-xz - 3)e^{xyz} + xz e^{xyz}(-xyz - 3y) - 1, xy e^{xyz}(-xyz - 3y) - xy e^{xyz})$. Compute the potential function for this field whose potential at the origin is 1.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -5.67026 2) -1.47026 3) 4.12974 4) 6.32974

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (6 \cos(t) + 6) \left(\frac{\cos(t)}{2} - \frac{1}{2} \sqrt{3} \sin(t) \right), \sin(2t) (6 \cos(t) + 6) \left(\frac{\sin(t)}{2} + \frac{1}{2} \sqrt{3} \cos(t) \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 42.4115 2) 55.0115 3) 71.8115 4) 46.6115

Exercise 3

Consider the vector field $F(x, y, z) = \{5xy + 8xz + \sin[y^2 + z^2], e^{x^2} - 5xyz, 4 + e^{-2y^2}\}$ and the surface

$$S \equiv \left(\frac{-2+x}{1} \right)^2 + \left(\frac{-7+y}{6} \right)^2 + \left(\frac{-7+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -13774.9 2) 4750.09 3) 14725.1 4) -8549.91

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 55

Exercise 1

Consider the vectorial field $F(x, y, z) = (0, 3z^2 e^{yz}, 3yz e^{yz} + 3e^{yz})$. Compute the potential function for this field whose potential at the origin is 5.

. Calculate the value of the potential at the point $p = (3, -10, -4)$.

- 1) $5 - 12e^{40}$ 2) $-8473869606132719455 - 12e^{40}$ 3) $-1694773921226543887 - 12e^{40}$ 4) $7061558005110599555 - 12e^{40}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ -(\sin(t) \sin(2t) (8 \cos(t) + 10)), \sin(2t) \cos(t) (8 \cos(t) + 10) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 134.573 2) 144.873 3) 103.673 4) 21.2726

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 5xz + \cos[2y^2 + 2z^2], -yz + \cos[2x^2 + 2z^2], 8 + e^{-x^2 - 2y^2} - x \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-6+x}{3} \right)^2 + \left(\frac{3+y}{8} \right)^2 + \left(\frac{-6+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -7237.01 2) 19300.5 3) 4825.49 4) 7720.49

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 56

Exercise 1

Consider the vectorial field $F(x, y, z) = (xy^2 e^{xy} + y e^{xy}, x^2 y e^{xy} + x e^{xy}, 0)$. Compute the potential function for this field whose potential at the origin is -1 .

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) -3.09962 2) -0.59962 3) 2.30038 4) 2.80038

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (4 \cos(t) + 8) \left(-\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right), \sin(2t) (4 \cos(t) + 8) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 34.1487 2) 17.3487 3) 22.9487 4) 56.5487

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{4z - \sin[2y^2], -5xy - \sin[2x^2], -5y - 5xy + \cos[2x^2]\} \text{ and the surface}$$

$$S \equiv \left(\frac{-8+x}{8} \right)^2 + \left(\frac{8+y}{6} \right)^2 + \left(\frac{-7+z}{8} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 154416 . 2) -64339.8 3) 122246 . 4) -225190 .

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 57

Exercise 1

Consider the vectorial field $F(x, y, z) =$

$$\left(2xy^2 - \frac{2yz^2}{xyz+1} - 2y^2, 2x^2y - \frac{2xz^2}{xyz+1} - 4xy, -\frac{2xyz}{xyz+1} - 2\log(xyz+1) \right)$$

. Compute the potential function for this field whose potential at the origin is 3.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

- 1) -3.16716 2) -2.36716 3) 2.63284 4) -3.56716

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} \right) \cos(t) (7\cos(t)+8)}{\sin^2(t)+1}, \frac{\left(\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} + \frac{1+\sqrt{3}}{2\sqrt{2}} \right) \cos(t) (7\cos(t)+8)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 148.462 2) 201.462 3) 31.862 4) 106.062

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{-4x + 5z + \sin[2y^2 - z^2], -9 + \sin[x^2 - 2z^2], -5xyz + \cos[x^2 + y^2]\}$$
 and the surface

$$S \equiv \left(\frac{x}{5} \right)^2 + \left(\frac{-3+y}{7} \right)^2 + \left(\frac{-4+z}{6} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -3518.58 2) 6686.52 3) 7390.32 4) 5278.92

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 58

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy + ze^{yz}, 3x^2 + z(xz + 2)e^{yz} + 1, xe^{yz} + y(xz + 2)e^{yz})$. Compute the potential function for this field whose potential at the origin is 1.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -3.00505 2) 2.99495 3) 3.99495 4) -1.60505

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(6t + 7) \sin(2t) \cos(4t) + 4, (7t + 7) \sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 326.416 2) 977.416 3) 760.416 4) 1085.92

Exercise 3

Consider the vector field $F(x, y, z) = \{-7xy^2z^2, -8xy, 7y^2z + 3z^2\}$ and the surface

$$S \equiv \left(\frac{4+x}{9}\right)^2 + \left(\frac{-4+y}{9}\right)^2 + \left(\frac{-6+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -2.30674×10^7 2) 2.99876×10^7 3) -3.46011×10^7 4) 4.61348×10^7

Further Mathematics - Degree in Engineering - 2024/2025
04-Line and Surface Integral-Computers exam for serial
number: 59

Exercise 1

Consider the vectorial field $F(x, y, z) = (-3y \cos(xy) - y^2 - 2, -2xy - 3x \cos(xy), 0)$. Compute the potential function for this field whose potential at the origin is 0.
. Calculate the value of the potential at the point $p = (-10, -8, -6)$.

- 1) $-\frac{7623}{5} - 3 \sin[80]$ 2) $660 - 3 \sin[80]$ 3) $2646 - 3 \sin[80]$ 4) $-\frac{6299}{5} - 3 \sin[80]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(8t + 1) \sin(2t) (9 \cos(16t) + 10), (9t + 9) \sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 4349.97 2) 870.771 3) 4784.87 4) 1740.57

Exercise 3

Consider the vector field $F(x, y, z) = \{2xz + 8yz^2, 8xy^2 - 7y^2z^2, 2y\}$ and the surface

$$S \equiv \left(\frac{2+x}{3}\right)^2 + \left(\frac{8+y}{7}\right)^2 + \left(\frac{-9+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -1.74473×10^7 2) 3.03431×10^7 3) -1.4413×10^7 4) 7.58579×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 60

Exercise 1

Consider the vectorial field $F(x, y, z) = (6xy^2 - 2\log(yz + 1), 6x^2y - \frac{2xz}{yz+1}, -\frac{2xy}{yz+1})$. Compute the potential function for this field whose potential at the origin is -6 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -22.0754 2) -5.87543 3) -7.07543 4) -14.8754

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} - \frac{1+\sqrt{3}}{2\sqrt{2}} \right) \cos(t) (7\cos(t)+7)}{\sin^2(t)+1}, \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} \right) \cos(t) (7\cos(t)+7)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 18.262 2) 91.062 3) 100.162 4) 145.662

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 7 + 5xyz + \cos[y^2], -6yz + \cos[x^2 + 2z^2], e^{x^2+y^2} + 6xyz \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{3+x}{5} \right)^2 + \left(\frac{4+y}{2} \right)^2 + \left(\frac{-9+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -61072.6 2) -73287.2 3) -225970 4) -152682 .

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04-Line and Surface Integral-Computers exam for serial number: 61

Exercise 1

Consider the vectorial field $F(x, y, z) = (-z(2z+1)\sin(xz), 2-6y, 2\cos(xz) - x(2z+1)\sin(xz))$. Compute the potential function for this field whose potential at the origin is -4 .
 . Calculate the value of the potential at the point $p = (-5, 8, 0)$.
 1) -612 2) -180 3) -378 4) -432

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(9t+4)\sin(2t) - (5\cos(19t)+5), (9t+1)\sin(t)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 1790.38 2) 398.078 3) 2784.88 4) 1989.28

Exercise 3

Consider the vector field $F(x, y, z) = \{-9xy^2z^2, 8xy^2z + 9y^2z^2, 9xz + y^2z^2\}$ and the surface
 $S \equiv \left(\frac{1+x}{3}\right)^2 + \left(\frac{-5+y}{8}\right)^2 + \left(\frac{9+z}{3}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) -1.49301×10^7 2) -1.30638×10^7 3) -2.1151×10^7 4) -6.22087×10^6

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04-Line and Surface Integral-Computers exam for serial number: 62

Exercise 1

Consider the vectorial field $F(x, y, z) = (3y^2, 6xy + y^2z^2 + 2(y-2)yz^2, 2(y-2)y^2z)$. Compute the potential function for this field whose potential at the origin is 1.

. Calculate the value of the potential at the point $p = (-2, 6, 8)$.

- 1) $\frac{117013}{5}$ 2) $\frac{126014}{5}$ 3) $-\frac{99011}{10}$ 4) 9001

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (t+9) \sin(2t) (2 \cos(10t) + 9), (7t+9) \sin(t) (2 \cos(10t) + 9) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 42466. 2) 23592.4 3) 21233.2 4) 35388.4

Exercise 3

Consider the vector field $F(x, y, z) = \{8y - 5xy^2z, -xy^2, -yz - 2yz^2\}$ and the surface

$$S \equiv \left(\frac{-7+x}{9}\right)^2 + \left(\frac{-3+y}{2}\right)^2 + \left(\frac{3+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -91562.3 2) 233070. 3) 83239.6 4) 216422.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 63

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{y(z-2)}{xy+1} - 1, \frac{x(z-2)}{xy+1} + 6y, \log(xy+1) \right)$. Compute the potential function for this field whose potential at the origin is 4.
 . Calculate the value of the potential at the point $p = (-9, -4, 3)$.

- 1) $61 + \text{Log}[37]$ 2) $-67 + \text{Log}[37]$ 3) $-\frac{719}{5} + \text{Log}[37]$ 4) $-131 + \text{Log}[37]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (2t+4) \sin(2t) (4 \cos(4t) + 6), (3t+3) \sin(t) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 453.722 2) 383.922 3) 174.522 4) 349.022

Exercise 3

Consider the vector field $F(x, y, z) = \{-x^2 y^2 z - 9x^2 z^2, 4x^2 y z^2, 8z^2 + 5x z^2\}$ and the surface

$$S \equiv \left(\frac{8+x}{2} \right)^2 + \left(\frac{2+y}{2} \right)^2 + \left(\frac{-8+z}{3} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -2.945×10^6 2) -1.33863×10^6 3) 1.33864×10^6 4) -2.67727×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 64

Exercise 1

Consider the vectorial field $F(x, y, z) = (x^3 y^4 z^3 + 3x^2 y^3 z^3 (xy + 3y) + 2xy^2 - 2y, x^3 (x+3) y^3 z^3 + 3x^3 y^2 z^3 (xy + 3y) + 2x^2 y - 2x, 3x^3 y^3 z^2 (xy + 3y))$. Compute the potential function for this field whose potential at the origin is 1. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) 0.0586111 2) 3.65861 3) 3.75861 4) 0.658611

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (3 \cos(t) + 7) \left(\frac{\cos(t)}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right), \sin(2t) (3 \cos(t) + 7) \left(\frac{\sin(t)}{\sqrt{2}} + \frac{\cos(t)}{\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 42.0188 2) 25.2188 3) 46.2188 4) 33.6188

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{-9xy - \sin[2y^2], -xy + \cos[2x^2], -2x - 2xy + \cos[x^2 + y^2]\}$$
 and the surface

$$S \equiv \left(\frac{-9+x}{4} \right)^2 + \left(\frac{y}{7} \right)^2 + \left(\frac{-9+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -8447.15 2) -2111.15 3) -8658.35 4) -843.95

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 65

Exercise 1

Consider the vectorial field $F(x, y, z) = (2xyz e^{xyz} + 2e^{xyz} - 2xy + 2y^2, 2x^2 z e^{xyz} - x^2 + 4xy, 2x^2 y e^{xyz})$. Compute the potential function for this field whose potential at the origin is 0.
 . Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.
 1) 3.86591 2) -0.834093 3) 1.36591 4) 2.16591

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{2} - \frac{\sqrt{3}}{2}\right) \cos(t) (5 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{3} \sin(t)\right) \cos(t) (5 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 25.9602 2) 42.9602 3) 110.96 4) 85.4602

Exercise 3

Consider the vector field $F(x, y, z) = \{6xy + 4xz + \sin[y^2 + z^2], 3xy - 8xyz + \cos[2x^2 - 2z^2], e^{-2x^2 + 2y^2} + 4xyz\}$ and the surface

$$S \equiv \left(\frac{5+x}{7}\right)^2 + \left(\frac{-5+y}{7}\right)^2 + \left(\frac{z}{6}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -366373. 2) -188420. 3) -104678. 4) 125614.

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04-Line and Surface Integral-Computers exam for serial number: 66

Exercise 1

Consider the vectorial field $F(x, y, z) = ((3yz - 1)e^{xyz} + yze^{xyz}(3xyz - x) - 6x + y, 3xz e^{xyz} + xze^{xyz}(3xyz - x) + x, 3xy e^{xyz} + xye^{xyz}(3xyz - x))$. Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 1.16459 2) -8.83541 3) -2.03541 4) -1.83541

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, 2\pi] \rightarrow \mathbb{R}^2$
 $r(t) = \left\{ -\frac{\sin(t) \cos(t) (8 \cos(t) + 9)}{\sin^2(t) + 1}, \frac{\cos(t) (8 \cos(t) + 9)}{\sin^2(t) + 1} \right\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 257.438 2) 54.9381 3) 81.9381 4) 135.938

Exercise 3

Consider the vector field $F(x, y, z) = \{e^{-2y^2 + z^2} + 2x - z, xy + \cos[x^2 + z^2], -8yz + \cos[2x^2 - 2y^2]\}$ and the surface
 $S \equiv \left(\frac{2+x}{7}\right)^2 + \left(\frac{-2+y}{3}\right)^2 + \left(\frac{2+z}{9}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) -13933.6 2) -57001.4 3) -12666.9 4) 6333.6

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04-Line and Surface Integral-Computers exam for serial number: 67

Exercise 1

Consider the vectorial field $F(x, y, z) = (y e^{xyz} + y z e^{xyz} (xy - 3z) - 2, x e^{xyz} + x z e^{xyz} (xy - 3z), x y e^{xyz} (xy - 3z) - 3 e^{xyz})$. Compute the potential function for this field whose potential at the origin is 2.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 3.51904 2) 0.819041 3) 0.0190412 4) -0.480959

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(2t + 6) \sin(2t) (2 \cos(5t) + 9), (8t + 4) \sin(t)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 1940.9 2) 2716.9 3) 1746.9 4) 2910.9

Exercise 3

Consider the vector field $F(x, y, z) = \{-2xz + 3x^2yz, 6z^2, -8yz^2 + 8y^2z^2\}$ and the surface
 $S \equiv \left(\frac{-5+x}{8}\right)^2 + \left(\frac{6+y}{4}\right)^2 + \left(\frac{-5+z}{9}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) -9.79332×10^6 2) -979332. 3) 1.63222×10^6 4) 3.26444×10^6

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04-Line and Surface Integral-Computers exam for serial number: 68

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2xy^2ze^{xy} - 2yz e^{xy} + 4x, -2x^2yz e^{xy} - 2xz e^{xy} - 3, -2xy e^{xy})$. Compute the potential function for this field whose potential at the origin is -3 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 13.2663 2) -11.7337 3) -15.2337 4) -4.23371

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (8 \cos(t) + 9) \left(-\frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} - \frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} \right), \sin(2t) (8 \cos(t) + 9) \left(\frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} - \frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 88.75 2) 18.35 3) 53.55 4) 106.35

Exercise 3

Consider the vector field $F(x, y, z) = (-7x + \cos[z^2], y - \sin[2x^2 - 2z^2], 8xy + \cos[x^2 + 2y^2])$ and the surface

$$S \equiv \left(\frac{-3+x}{8} \right)^2 + \left(\frac{y}{1} \right)^2 + \left(\frac{9+z}{5} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -2514.31 2) -1005.31 3) -3218.51 4) 2515.69

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 69

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{z(1 - 3xyz)}{xz + 1} - 3yz \log(xz + 1) \right.$

$$\left. , -3xz \log(xz + 1) - 3, \frac{x(1 - 3xyz)}{xz + 1} - 3xy \log(xz + 1) \right).$$

. Compute the potential function for this field whose potential at the origin is 1.

. Calculate the value of the potential at the point $p = (9, 6, 9)$.

1) $-17 - 1457 \log[82]$ 2) $6421 - 1457 \log[82]$

3) $\frac{22448}{5} - 1457 \log[82]$ 4) $\frac{9572}{5} - 1457 \log[82]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (9 \cos(t) + 9) \left(\frac{1}{2} \sqrt{3} \cos(t) - \frac{\sin(t)}{2} \right), \sin(2t) (9 \cos(t) + 9) \left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{\cos(t)}{2} \right) \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

1) 76.4259 2) 180.926 3) 114.426 4) 95.4259

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ e^{-y^2 - z^2} + 2yz, e^{x^2 + z^2} - 7z + 5xyz, 6 - 4yz + \cos[x^2 - y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{7+x}{7} \right)^2 + \left(\frac{-6+y}{3} \right)^2 + \left(\frac{-1+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -46709.2 2) 0.800426 3) -32696.2 4) -14012.2

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04-Line and Surface Integral-Computers exam for serial number: 70

Exercise 1

Consider the vectorial field $F(x, y, z) = (-z(2 - 2xz) \sin(xz) - 2z \cos(xz) + 4y^2, 8xy, -x(2 - 2xz) \sin(xz) - 2x \cos(xz))$. Compute the potential function for this field whose potential at the origin is -2 .
 . Calculate the value of the potential at the point $p = (-5, 3, 3)$.

- 1) $-\frac{5811}{10} + 32 \cos[15]$ 2) $-184 + 32 \cos[15]$ 3) $\frac{3803}{10} + 32 \cos[15]$ 4) $-\frac{8319}{10} + 32 \cos[15]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ -\frac{\sin(t) \cos(t) (9 \cos(t) + 9)}{\sin^2(t) + 1}, \frac{\cos(t) (9 \cos(t) + 9)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 270.531 2) 150.531 3) 285.531 4) 60.531

Exercise 3

Consider the vector field $F(x, y, z) = (x - 5y - \sin[2y^2 + 2z^2], -5y - 7yz + \cos[x^2 + 2z^2], 8 - 6xy + \cos[2x^2 + 2y^2])$ and the surface

$$S \equiv \left(\frac{1+x}{3}\right)^2 + \left(\frac{3+y}{8}\right)^2 + \left(\frac{-7+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -42625.1 2) 72465.1 3) $-174766.$ 4) $-123615.$

Further Mathematics - Degree in Engineering - 2024/2025
04-Line and Surface Integral-Computers exam for serial
number: 71

Exercise 1

Consider the vectorial field $F(x, y, z) = (-6xy^2 + \frac{y(2xz - z)}{xy + 1} + 2z \log(xy + 1), \frac{x(2xz - z)}{xy + 1} - 6x^2y, (2x - 1) \log(xy + 1))$. Compute the potential function for this field whose potential at the origin is 5.
. Calculate the value of the potential at the point $p = (7, 0, 0)$.

- 1) -6 2) $\frac{29}{2}$ 3) $\frac{35}{2}$ 4) 5

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}}{2} - \frac{\sin(t)}{2}\right) \cos(t) (3 \cos(t) + 6)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{1}{2}\right) \cos(t) (3 \cos(t) + 6)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 82.4257 2) 13.6257 3) 43.7257 4) 56.6257

Exercise 3

Consider the vector field $F(x, y, z) = \left\{ \cos[y^2 - 2z^2], e^{-x^2} + 4x - 3xy, -4y + 5xyz - \sin[x^2 - y^2] \right\}$ and the surface

$$S \equiv \left(\frac{x}{1}\right)^2 + \left(\frac{4+y}{6}\right)^2 + \left(\frac{1+z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 2.5 2) -2.2 3) 0. 4) 1.9

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04-Line and Surface Integral-Computers exam for serial number: 72

Exercise 1

Consider the vectorial field $F(x, y, z) = ((-3yz - 3y)e^{xyz} + yze^{xyz}(-3xyz - 3xy) - 2x, (-3xz - 3x)e^{xyz} + xze^{xyz}(-3xyz - 3xy), xy e^{xyz}(-3xyz - 3xy) - 3xy e^{xyz})$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) 3.99875 2) 3.59875 3) 2.19875 4) 9.59875

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (8 \cos(t) + 8) \left(-\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right), \sin(2t) (8 \cos(t) + 8) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right) \right)$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 75.3982 2) 60.3982 3) 120.398 4) 112.898

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ e^{-2y^2+2z^2} - 7xz, -4x - xy + \sin[x^2], 2x - \sin[2x^2 + y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{6+x}{3} \right)^2 + \left(\frac{-7+y}{1} \right)^2 + \left(\frac{8+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 11920.4 2) 13322.8 3) 32956.4 4) 7012.03

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 73

Exercise 1

Consider the vectorial field $F(x, y, z) = (-x^2 y^3 z^2 + 2 x y^2 z^2 (-x y - 3 y) + 4, (-x - 3) x^2 y^2 z^2 + 2 x^2 y z^2 (-x y - 3 y), 2 x^2 y^2 z (-x y - 3 y))$. Compute the potential function for this field whose potential at the origin is 2.

. Calculate the value of the potential at the point $p = (0, -8, 1)$.

- 1) $\frac{24}{5}$ 2) 2 3) $\frac{48}{5}$ 4) $-\frac{27}{5}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (3 \cos(t) + 5) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right), \sin(2t) (3 \cos(t) + 5) \left(\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} + \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 2.46925 2) 18.5692 3) 23.1692 4) 25.4692

Exercise 3

Consider the vector field $F(x, y, z) = \{e^{2y^2+z^2} - 8xy, -8xy + \sin[2x^2], 5xz + \cos[x^2 + 2y^2]\}$ and the surface

$$S \equiv \left(\frac{-9+x}{4} \right)^2 + \left(\frac{1+y}{2} \right)^2 + \left(\frac{-2+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 4887.44 2) -4343.56 3) -1809.56 4) -8687.56

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 74

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2xy^2 - 2, -2x^2y, 0)$. Compute the potential function for this field whose potential at the origin is 3.

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) 5.28889 2) 3.58889 3) 0.788889 4) 1.88889

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right) \cos(t) (5 \cos(t) + 5)}{\sin^2(t) + 1}, \frac{\left(\frac{\sin(t)}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \cos(t) (5 \cos(t) + 5)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 46.4602 2) 60.2602 3) 78.6602 4) 37.2602

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ e^{2z^2} + 6xyz, 3xz + 3xyz + \cos[2z^2], -y - 5xz + \cos[2x^2 - y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{5+x}{2}\right)^2 + \left(\frac{6+y}{4}\right)^2 + \left(\frac{2+z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 38302.3 2) 103416. 3) 168529. 4) -3829.9

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 75

Exercise 1

Consider the vectorial field $F(x, y, z) = (3xy^2 \cos(xy) + 3y \sin(xy), 3x^2y \cos(xy) + 3x \sin(xy) + 2y, 0)$. Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the value of the potential at the point $p = (-1, 5, 3)$.

- 1) $24 + 15 \sin[5]$ 2) $\frac{192}{5} + 15 \sin[5]$ 3) $\frac{111}{5} + 15 \sin[5]$ 4) $\frac{141}{10} + 15 \sin[5]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (2 \cos(t) + 10) \left(-\frac{\sin(t)}{\sqrt{2}} - \frac{\cos(t)}{\sqrt{2}} \right), \sin(2t) (2 \cos(t) + 10) \left(\frac{\cos(t)}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 136.111 2) 96.1106 3) 80.1106 4) 64.1106

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 5yz - 7xyz + \cos[y^2 + 2z^2], e^{-2x^2 - z^2} - 2y + 3z, e^{x^2 - 2y^2} - 9yz \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-8+x}{3} \right)^2 + \left(\frac{-9+y}{5} \right)^2 + \left(\frac{5+z}{5} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 189499. 2) -7287.45 3) 72884.9 4) 58308.1

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04-Line and Surface Integral-Computers exam for serial number: 76

Exercise 1

Consider the vectorial field $F(x, y, z) = (-3xyz^2 \sin(xyz) + 3z \cos(xyz) + 2xy - 6x, x^2 - 3x^2z^2 \sin(xyz), 3x \cos(xyz) - 3x^2yz \sin(xyz))$.
 . Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -7.1139 2) 4.6861 3) -1.1139 4) -1.3139

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right) \cos(t) (4 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left(\frac{\sin(t)}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \cos(t) (4 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 147.035 2) 139.335 3) 100.835 4) 77.7345

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ e^{y^2 + 2z^2} - 7z - 6xz, -7y - 6yz + \cos[2x^2 - z^2], e^{-x^2} - 2x - 3xz \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{-5+x}{9}\right)^2 + \left(\frac{6+y}{9}\right)^2 + \left(\frac{6+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 271432 . 2) 74644.2 3) -128930 . 4) 67858.4

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Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{yz(3xyz - z)}{xyz + 1} + 3yz \log(xyz + 1) - 3y^2, \frac{xz(3xyz - z)}{xyz + 1} + 3xz \log(xyz + 1) - 6xy, \frac{xy(3xyz - z)}{xyz + 1} + (3xy - 1) \log(xyz + 1) \right)$. Compute the potential function for this field whose potential at the origin is 0.
 . Calculate the value of the potential at the point $p = (0, 5, 5)$.

- 1) 2 2) $-\frac{19}{5}$ 3) 0 4) $\frac{37}{10}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (6t + 9) \sin(2t) (7 \cos(12t) + 7), (8t + 7) \sin(t) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 2364.42 2) 4728.32 3) 3377.52 4) 5066.02

Exercise 3

Consider the vector field $F(x, y, z) = \{5x^2y^2z^2, 0, -8y^2 + 8xyz^2\}$ and the surface

$$S \equiv \left(\frac{-6+x}{7} \right)^2 + \left(\frac{-2+y}{2} \right)^2 + \left(\frac{-1+z}{9} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 2.59852×10^6 2) 1.22131×10^7 3) -7.27587×10^6 4) 7.79557×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 78

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{y^2 z}{xyz + 1}, \frac{xyz}{xyz + 1} + \log(xyz + 1), \frac{xy^2}{xyz + 1} \right)$. Compute the potential function for this field whose potential at the origin is -4 .
 . Calculate the value of the potential at the point $p = (-9, -4, 3)$.

- 1) $-\frac{247}{10} - 4 \log[109]$ 2) $-\frac{457}{5} - 4 \log[109]$ 3) $-\frac{411}{5} - 4 \log[109]$ 4) $-4 - 4 \log[109]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (3t + 6) \sin(2t) (8 \cos(17t) + 8), (7t + 6) \sin(t) (8 \cos(17t) + 8) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 35938.9 2) 9584.03 3) 31147.1 4) 23959.4

Exercise 3

Consider the vector field $F(x, y, z) = \{-2x^2, 3x^2z, -4x^2y^2z + 4yz^2\}$ and the surface

$$S \equiv \left(\frac{-9+x}{4} \right)^2 + \left(\frac{-9+y}{7} \right)^2 + \left(\frac{-5+z}{7} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -1.09173×10^8 2) -1.19098×10^8 3) -2.4812×10^7 4) -4.71428×10^7

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 79

Exercise 1

Consider the vectorial field $F(x, y, z) = (2xy^2z^2(2yz+z), 2x^2y^2z^3 + 2x^2yz^2(2yz+z), x^2(2y+1)y^2z^2 + 2x^2y^2z(2yz+z))$. Compute the potential function for this field whose potential at the origin is 0.
 . Calculate the value of the potential at the point $p = (9, -2, -9)$.

- 1) $\frac{11337408}{5}$ 2) $\frac{12754584}{5}$ 3) 708588 4) $-\frac{354294}{5}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(3t+5)\sin(2t)(3\cos(14t)+3), (3t+8)\sin(t)(3\cos(14t)+3)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 2489.31 2) 2263.01 3) 3847.11 4) 3394.51

Exercise 3

Consider the vector field $F(x, y, z) = \{-8x^2y^2 - 7xy^2z, 3x - 4yz, -7x^2y^2z\}$ and the surface

$$S \equiv \left(\frac{x}{3}\right)^2 + \left(\frac{8+y}{1}\right)^2 + \left(\frac{2+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 8920.89 2) -247.712 3) 2478.09 4) 7929.69

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 80

Exercise 1

Consider the vectorial field $F(x, y, z) = (2y^2z \cos(xyz) + 2y^2, 2 \sin(xyz) + 2xyz \cos(xyz) + 4xy, 2xy^2 \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -2 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -3.7041 2) 5.4959 3) -1.5041 4) 4.4959

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(5t + 4) \sin(2t) (7 \cos(13t) + 8), (4t + 5) \sin(t) (7 \cos(13t) + 8)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 16353.2 2) 11447.3 3) 4906.07 4) 21259.1

Exercise 3

Consider the vector field $F(x, y, z) = \{6y^2z^2, -5x^2yz^2, 4xy^2z\}$ and the surface

$$S \equiv \left(\frac{3+x}{4}\right)^2 + \left(\frac{-2+y}{8}\right)^2 + \left(\frac{3+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -1.22315×10^6 2) -2.56862×10^6 3) 3.05789×10^6 4) 2.44631×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 81

Exercise 1

Consider the vectorial field $F(x, y, z) = (x^2 y^3 z^2 + 2 x y^2 z^2 (x y + z), x^3 y^2 z^2 + 2 x^2 y z^2 (x y + z) + 2, x^2 y^2 z^2 + 2 x^2 y^2 z (x y + z))$. Compute the potential function for this field whose potential at the origin is 0.
 . Calculate the value of the potential at the point $p = (7, -8, -7)$.

- 1) $-\frac{203\,297\,808}{5}$ 2) $-9\,680\,848$ 3) $-43\,563\,816$ 4) $4\,840\,424$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (3t + 8) \sin(2t) \cos(17t) + 9, (9t + 7) \sin(t) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 6306.84 2) 664.535 3) 3319.74 4) 996.435

Exercise 3

Consider the vector field $F(x, y, z) = \{-5x^2y - 3x^2y^2, -6x^2 + 4x^2y^2z^2, 4y^2z + 7x^2y^2z\}$ and the surface

$$S \equiv \left(\frac{-5+x}{9}\right)^2 + \left(\frac{y}{8}\right)^2 + \left(\frac{8+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 3.55156×10^6 2) 1.5982×10^7 3) 1.49165×10^7 4) -1.77578×10^6

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 82

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{3y^2z}{xyz+1}, \frac{3xyz}{xyz+1} + 3\log(xyz+1) - 6y, \frac{3xy^2}{xyz+1} \right)$.

. Compute the potential function for this field whose potential at the origin is 2.

. Calculate the value of the potential at the point $p = (5, -7, -3)$.

- 1) $-\frac{2912}{5} - 21 \log[106]$ 2) $-\frac{2426}{5} - 21 \log[106]$
 3) $\frac{3892}{5} - 21 \log[106]$ 4) $-145 - 21 \log[106]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (t+1) \sin(2t) (4 \cos(8t) + 5), (3t+1) \sin(t) (4 \cos(8t) + 5) \}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 340.957 2) 885.757 3) 68.5567 4) 681.457

Exercise 3

Consider the vector field $F(x, y, z) = \{3z^2, -3xy - 9x^2yz, -8y + 9x^2y\}$ and the surface

$$S \equiv \left(\frac{-7+x}{8} \right)^2 + \left(\frac{9+y}{9} \right)^2 + \left(\frac{4+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -3.72204×10^6 2) 3.72204×10^6 3) 3.19032×10^6 4) 1.3293×10^6

Further Mathematics - Degree in Engineering - 2024/2025 04-Line and Surface Integral-Computers exam for serial number: 83

Exercise 1

Consider the vectorial field $F(x, y, z) = (x^2 z^2 (y z + 3) + 2 x z^2 (x y z + 3 x) + 4 x y - 2 x, x^3 z^3 + 2 x^2, x^3 y z^2 + 2 x^2 z (x y z + 3 x))$. Compute the potential function for this field whose potential at the origin is 0.
. Calculate the value of the potential at the point $p = (5, 1, -1)$.

- 1) $\frac{55}{2}$ 2) 275 3) $\frac{1815}{2}$ 4) $\frac{2585}{2}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{\sin(t)}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) \cos(t) (8 \cos(t) + 10)}{\sin^2(t) + 1}, \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right) \cos(t) (8 \cos(t) + 10)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 154.938 2) 124.138 3) 31.7381 4) 278.138

Exercise 3

Consider the vector field $F(x, y, z) = \{e^{2z^2} + y + 8xz, -6z - \sin[2x^2 + 2z^2], e^{-x^2 + 2y^2} - 4xyz\}$ and the surface

$$S \equiv \left(\frac{2+x}{4} \right)^2 + \left(\frac{y}{6} \right)^2 + \left(\frac{7+z}{8} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -211678. 2) -4503.67 3) -45037.9 4) -148625.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 84

Exercise 1

Consider the vectorial field $F(x, y, z) = (-yz(xy z - xy) \sin(xy z) + (yz - y) \cos(xy z) - 1, -xz(xy z - xy) \sin(xy z) + (xz - x) \cos(xy z) - 6y, xy \cos(xy z) - xy(xy z - xy) \sin(xy z))$. Compute the potential function for this field whose potential at the origin is 0.

. Calculate the value of the potential at the point $p = (8, 7, -6)$.

- 1) $\frac{3633}{5} - 392 \cos[336]$ 2) $-\frac{5067}{5} - 392 \cos[336]$
 3) $-\frac{1587}{5} - 392 \cos[336]$ 4) $-155 - 392 \cos[336]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (2 \cos(t) + 9) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right), \sin(2t) (2 \cos(t) + 9) \left(\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} + \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 32.688 2) 26.188 3) 65.188 4) 110.688

Exercise 3

Consider the vector field $F(x, y, z) = \{-5z - \sin[y^2], yz + \sin[x^2 - 2z^2], z - 7xz + \sin[x^2]\}$ and the surface

$$S \equiv \left(\frac{4+x}{7} \right)^2 + \left(\frac{6+y}{3} \right)^2 + \left(\frac{8+z}{4} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 2216.73 2) -19950.3 3) 7389.03 4) 35467.2

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 85

Exercise 1

Consider the vectorial field $F(x, y, z) = (yz \sin(xy) + y(xyz - 2z) \cos(xy), xz \sin(xy) + x(xyz - 2z) \cos(xy) + 6y, (xy - 2) \sin(xy))$. Compute the potential function for this field whose potential at the origin is -5 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 1.81249 2) -5.68751 3) -4.68751 4) -4.18751

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \{(7t + 2) \sin(2t) (\cos(17t) + 3), (7t + 7) \sin(t)\}$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 988.341 2) 1679.94 3) 1482.34 4) 1877.54

Exercise 3

Consider the vector field $F(x, y, z) = \{8y, -2y^2 - 6xy^2z, 4x^2y^2 + 3x^2yz^2\}$ and the surface
 $S \equiv \left(\frac{-6+x}{1}\right)^2 + \left(\frac{-2+y}{3}\right)^2 + \left(\frac{3+z}{8}\right)^2 = 1$
 Compute $\int_S F$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) -88386.8 2) -114903. 3) 17677.6 4) -97225.5

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 86

Exercise 1

Consider the vectorial field $F(x, y, z) = (-yz(2xy - yz) \sin(xyz) + 2y \cos(xyz) + 6x, (2x - z) \cos(xyz) - xz(2xy - yz) \sin(xyz), -xy(2xy - yz) \sin(xyz) - y \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -4 .

. Calculate the value of the potential at the point $p = (-7, -7, -1)$.

- 1) $-129 + 91 \cos[49]$ 2) $-163 + 91 \cos[49]$ 3) $143 + 91 \cos[49]$ 4) $126 + 91 \cos[49]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (4 \cos(t) + 9) \left(\frac{\cos(t)}{2} - \frac{1}{2} \sqrt{3} \sin(t) \right), \sin(2t) (4 \cos(t) + 9) \left(\frac{\sin(t)}{2} + \frac{1}{2} \sqrt{3} \cos(t) \right) \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 69.9004 2) 56.1004 3) 90.6004 4) 125.1

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 5xy + 9xyz + \cos[2y^2], 5 + e^{x^2 - z^2}, -5y + 6xyz - \sin[x^2 - y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{8+x}{3} \right)^2 + \left(\frac{8+y}{4} \right)^2 + \left(\frac{6+z}{2} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 78012. 2) -62409.6 3) -46807.2 4) 70210.8

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 87

Exercise 1

Consider the vectorial field $F(x, y, z) = (y \sin(xyz) + yz(xy - 2yz) \cos(xyz) + 3y, (x - 2z) \sin(xyz) + xz(xy - 2yz) \cos(xyz) + 3x + 1, xy(xy - 2yz) \cos(xyz) - 2y \sin(xyz))$. Compute the potential function for this field whose potential at the origin is -3 .

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) 1.99608 2) -0.203917 3) -1.80392 4) -5.40392

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (4 \cos(t) + 8) \left(\frac{1}{2} \sqrt{3} \cos(t) - \frac{\sin(t)}{2} \right), \sin(2t) (4 \cos(t) + 8) \left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{\cos(t)}{2} \right) \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 50.9487 2) 34.1487 3) 62.1487 4) 56.5487

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{5xy + \cos[2y^2], 6xyz + \cos[x^2 - 2z^2], -4xz + \sin[x^2 + 2y^2]\}$$
 and the surface

$$S \equiv \left(\frac{-7+x}{4} \right)^2 + \left(\frac{-5+y}{2} \right)^2 + \left(\frac{7+z}{8} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 71659.4 2) -15923.7 3) -79620.5 4) -222938.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 88

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{yz(-x-y)}{xyz+1} - \log(xyz+1) - 1, \frac{xz(-x-y)}{xyz+1} - \log(xyz+1), \frac{xy(-x-y)}{xyz+1} \right)$.

. Compute the potential function for this field whose potential at the origin is 6.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) -7.64493 2) -9.14493 3) 13.8551 4) 5.35507

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} \right) \cos(t) (3\cos(t)+5)}{\sin^2(t)+1}, \frac{\left(\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} + \frac{1+\sqrt{3}}{2\sqrt{2}} \right) \cos(t) (3\cos(t)+5)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 29.5257 2) 16.7257 3) 39.1257 4) 32.7257

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ -4 + e^{y^2+2z^2} - 9yz, 9xz - 2xyz - \sin[2x^2+z^2], e^{-x^2-y^2} - 6xz \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{7+x}{6} \right)^2 + \left(\frac{y}{6} \right)^2 + \left(\frac{-2+z}{4} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 42223. 2) -67556.8 3) -80223.7 4) 114002.

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 89

Exercise 1

Consider the vectorial field $F(x, y, z) = (4xy^2z^2e^{xyz} + 4yz e^{xyz} + 4x, 4x^2yz^2e^{xyz} + 4xz e^{xyz} - 2, 4x^2y^2ze^{xyz} + 4xy e^{xyz})$. Compute the potential function for this field whose potential at the origin is 3.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) 12.3523 2) 9.35228 3) 3.35228 4) 13.5523

Exercise 2

Compute the area of the domain whose boundary is the curve
 $r: [0, \pi] \rightarrow \mathbb{R}^2$
 $r(t) = \left(\sin(2t) (4 \cos(t) + 6) \left(-\frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} - \frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} \right), \sin(2t) (4 \cos(t) + 6) \left(\frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} - \frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} \right) \right)$
 Indication: it is necessary to represent the curve to check whether it has intersection points.
 1) 14.1575 2) 34.5575 3) 65.1575 4) 54.9575

Exercise 3

Consider the vector field $F(x, y, z) = \left\{ -8xy + \cos[y^2 - 2z^2], 9z - 4xyz + \sin[2z^2], e^{x^2} \right\}$ and the surface
 $S \equiv \left(\frac{-6+x}{8} \right)^2 + \left(\frac{-7+y}{2} \right)^2 + \left(\frac{8+z}{3} \right)^2 = 1$
 Compute $\int_S F \cdot d\mathbf{S}$.
 Indication: Use Gauss' Theorem if it is necessary.
 1) 82032.4 2) 120314. 3) 27344.4 4) 109376.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 90

Exercise 1

Consider the vectorial field $F(x, y, z) = (-6x - 3y^3z^2 + 3y^2z^2(3z - 3x) + 2yz^2(3yz - 3xy), 2y^2z(3yz - 3xy) + 3y^3z^2)$. Compute the potential function for this field whose potential at the origin is -4 .
 . Calculate the value of the potential at the point $p = (3, 3, 10)$.

- 1) 56 678 2) $\frac{340068}{5}$ 3) -56 678 4) $\frac{255051}{5}$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ (3t + 2) \sin(2t) (4 \cos(13t) + 4), (4t + 9) \sin(t) (4 \cos(13t) + 4) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 1350.32 2) 3037.82 3) 6075.32 4) 3375.32

Exercise 3

Consider the vector field $F(x, y, z) = \{7yz, -2z^2, 2x\}$ and the surface

$$S \equiv \left(\frac{1+x}{4}\right)^2 + \left(\frac{-4+y}{9}\right)^2 + \left(\frac{-3+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 0. 2) -0.1 3) 0.2 4) -2.

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 91

Exercise 1

Consider the vectorial field $F(x, y, z) = (2y^2 z^2 e^{xyz}, 2xyz^2 e^{xyz} + 2z e^{xyz}, 2xy^2 z e^{xyz} + 2y e^{xyz})$. Compute the potential function for this field whose potential at the origin is 2.
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) 8.4358 2) 2.6358 3) -3.9642 4) 7.4358

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(9t + 8) \sin(2t) (7 \cos(16t) + 8), (t + 7) \sin(t) (7 \cos(16t) + 8)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 40775.9 2) 2265.76 3) 36245.3 4) 22653.5

Exercise 3

Consider the vector field $F(x, y, z) = \{3x^2 y z, -6xy^2 z, 8y^2 - 6x^2 y^2 z\}$ and the surface

$$S \equiv \left(\frac{-9+x}{6}\right)^2 + \left(\frac{6+y}{6}\right)^2 + \left(\frac{3+z}{7}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -2.50642×10^7 2) 5.76476×10^7 3) -8.52182×10^7 4) -9.77503×10^7

Further Mathematics - Degree in Engineering - 2024/2025

04-Line and Surface Integral-Computers exam for serial number: 92

Exercise 1

Consider the vectorial field $F(x, y, z) = (-yz(1 - 3xyz) \sin(xyz) - 3yz \cos(xyz) - 1, -xz(1 - 3xyz) \sin(xyz) - 3xz \cos(xyz) - 6y, -xy(1 - 3xyz) \sin(xyz) - 3xy \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the value of the potential at the point $p = (-7, -9, -2)$.

- 1) $-238 + 379 \cos[126]$ 2) $-\frac{6069}{10} + 379 \cos[126]$
 3) $-\frac{5117}{10} + 379 \cos[126]$ 4) $\frac{238}{5} + 379 \cos[126]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \sin(2t) (9 \cos(t) + 9) \left(\frac{1}{2} \sqrt{3} \cos(t) - \frac{\sin(t)}{2} \right), \sin(2t) (9 \cos(t) + 9) \left(\frac{1}{2} \sqrt{3} \sin(t) + \frac{\cos(t)}{2} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 95.4259 2) 180.926 3) 28.9259 4) 133.426

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 6xy - 2xyz + \cos[y^2 - z^2], 1 + e^{-2x^2 - 2z^2}, e^{-2y^2} + 4xy - 5xz \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{7+x}{1} \right)^2 + \left(\frac{8+y}{1} \right)^2 + \left(\frac{6+z}{4} \right)^2 = 1$$

$$\text{Compute } \int_S F.$$

Indication: Use Gauss' Theorem if it is necessary.

- 1) -1826.31 2) 3837.39 3) 2375.79 4) -6211.11

Further Mathematics - Degree in Engineering - 2024/2025
04-Line and Surface Integral-Computers exam for serial
number: 93

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2x - 2yz e^{yz} - y, -2xyz^2 e^{yz} - 2xz e^{yz} - x, -2xy^2 z e^{yz} - 2xy e^{yz})$. Compute the potential function for this field whose potential at the origin is 2.
Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) -1.78371 2) 3.61629 3) 1.01629 4) 2.41629

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right) \cos(t) (8 \cos(t) + 9)}{\sin^2(t) + 1}, \frac{\left(\frac{\sin(t)}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \cos(t) (8 \cos(t) + 9)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 162.938 2) 230.438 3) 189.938 4) 135.938

Exercise 3

Consider the vector field $F(x, y, z) = \{6x - \sin[2y^2], 6xyz + \cos[2x^2 - z^2], -5y + 7yz + \cos[x^2 - y^2]\}$ and the surface

$$S \equiv \left(\frac{2+x}{8}\right)^2 + \left(\frac{-6+y}{7}\right)^2 + \left(\frac{-3+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 54044.1 2) 22518.9 3) -40531.5 4) 81065.7

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04-Line and Surface Integral-Computers exam for serial number: 94

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2yz^2, -2xz^2 - 2y, -4xyz)$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

- 1) 3.5 2) 1.1 3) 15.5 4) -8.5

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\cos(t)(6\cos(t)+9)}{\sin^2(t)+1}, \frac{\sin(t)\cos(t)(6\cos(t)+9)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

- 1) 100.803 2) 111.903 3) 178.503 4) 200.703

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\{-5y - \sin[2y^2 - z^2], -3x + xyz + \cos[z^2], 8yz - \sin[2y^2]\}$$
 and the surface

$$S \equiv \left(\frac{-4+x}{4}\right)^2 + \left(\frac{-2+y}{9}\right)^2 + \left(\frac{-3+z}{6}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 126 666. 2) 25 333.8 3) -75 998.2 4) 5067.4

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04-Line and Surface Integral-Computers exam for serial number: 95

Exercise 1

Consider the vectorial field $F(x, y, z) = \left(\frac{yz(2x - 2xz)}{xyz + 1} + (2 - 2z) \log(xyz + 1) + 6xy + 4x, 3x^2 + \frac{xz(2x - 2xz)}{xyz + 1}, \frac{xy(2x - 2xz)}{xyz + 1} - 2x \log(xyz + 1) \right)$.

. Compute the potential function for this field whose potential at the origin is 3.

. Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.

1) 5.81655 2) -4.98345 3) 4.21655 4) 1.01655

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} \right) \cos(t) (6\cos(t)+7)}{\sin^2(t)+1}, \frac{\left(\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} + \frac{1+\sqrt{3}}{2\sqrt{2}} \right) \cos(t) (6\cos(t)+7)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent

the curve to check whether it has intersection points.

1) 87.8027 2) 79.9027 3) 16.7027 4) 40.4027

Exercise 3

Consider the vector field $F(x, y, z) =$

$$\left\{ 9yz + \sin[y^2 + 2z^2], 3 + e^{-x^2}, -2y + \cos[x^2 - y^2] \right\} \text{ and the surface}$$

$$S \equiv \left(\frac{4+x}{4} \right)^2 + \left(\frac{-7+y}{2} \right)^2 + \left(\frac{3+z}{8} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -2.3 2) 0.4 3) 0. 4) -0.8

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04-Line and Surface Integral-Computers exam for serial number: 96

Exercise 1

Consider the vectorial field $F(x, y, z) = (2y e^{xyz} + yz e^{xyz} (2xy + z) + 4x - 2y, 2x e^{xyz} + xz e^{xyz} (2xy + z) - 2x, xy e^{xyz} (2xy + z) + e^{xyz})$.
 . Compute the potential function for this field whose potential at the origin is -1 .
 . Calculate the integral of the potential function ϕ over the domain $[0, 1]^3$.
 1) -3.59791 2) 1.10209 3) 0.402091 4) 0.702091

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \left(\sin(2t) (2 \cos(t) + 3) \left(-\frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \cos(t)}{2\sqrt{2}} \right), \sin(2t) (2 \cos(t) + 3) \left(\frac{(1+\sqrt{3}) \cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right) \right)$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 1.43938 2) 7.83938 3) 8.63938 4) 9.43938

Exercise 3

Consider the vector field $F(x, y, z) = \{-6 + \sin[z^2], e^{x^2-z^2} + 2xz - 6xyz, 7x + \cos[x^2 - y^2]\}$ and the surface

$$S \equiv \left(\frac{-8+x}{4} \right)^2 + \left(\frac{2+y}{6} \right)^2 + \left(\frac{-9+z}{7} \right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -395207 . 2) -304006 . 3) 912018 . 4) -516810 .

Further Mathematics - Degree in Engineering - 2024/2025
 04-Line and Surface Integral-Computers exam for serial
 number: 97

Exercise 1

Consider the vectorial field $F(x, y, z) = (2xy^2 + xyz \sin(xyz) - \cos(xyz), x^2z \sin(xyz) + 2x^2y - 6y, x^2y \sin(xyz))$. Compute the potential function for this field whose potential at the origin is -3 .
 . Calculate the value of the potential at the point $p = (-3, -1, -6)$.

- 1) $\frac{53}{5} + 3 \cos[18]$ 2) $-\frac{37}{5} + 3 \cos[18]$ 3) $13 + 3 \cos[18]$ 4) $3 + 3 \cos[18]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{ \sin(2t) (-\cos(t)) (7 \cos(t) + 9), -(\sin(t) \sin(2t) (7 \cos(t) + 9)) \}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 82.8595 2) 123.86 3) 66.4595 4) 74.6595

Exercise 3

Consider the vector field $F(x, y, z) = \{9yz + \cos[y^2 + z^2], e^{x^2+z^2} - 5y + 5xy, e^{-x^2-y^2} - 6x\}$ and the surface

$$S \equiv \left(\frac{-4+x}{6}\right)^2 + \left(\frac{-3+y}{5}\right)^2 + \left(\frac{3+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 5654.87 2) -9045.53 3) 16397.5 4) 19789.9

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04-Line and Surface Integral-Computers exam for serial number: 98

Exercise 1

Consider the vectorial field $F(x, y, z) = (3x^2y^3(z+2) + 6x, 3x^3y^2(z+2), x^3y^3)$. Compute the potential function for this field whose potential at the origin is -5 .
 . Calculate the value of the potential at the point $p = (-3, -3, 4)$.

- 1) $-\frac{52752}{5}$ 2) $-\frac{24178}{5}$ 3) 4396 4) -10990

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(2t+4)\sin(2t)(\cos(4t)+4), (2t+9)\sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 85.6985 2) 425.698 3) 128.198 4) 808.198

Exercise 3

Consider the vector field $F(x, y, z) = \{8 + 7y, -3x^2y^2 - xz, -7xy^2 - 7y^2z^2\}$ and the surface

$$S \equiv \left(\frac{8+x}{9}\right)^2 + \left(\frac{-6+y}{5}\right)^2 + \left(\frac{-6+z}{6}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 1.7185×10^7 2) 1.7901×10^7 3) -7.16042×10^6 4) 7.87646×10^6

Further Mathematics - Degree in Engineering - 2024/2025
04-Line and Surface Integral-Computers exam for serial
number: 99

Exercise 1

Consider the vectorial field $F(x, y, z) = (yz(-3xy - y) \cos(xyz) - 3y \sin(xyz), (-3x - 1) \sin(xyz) + xz(-3xy - y) \cos(xyz) - 10y, xy(-3xy - y) \cos(xyz))$. Compute the potential function for this field whose potential at the origin is -3 .
. Calculate the value of the potential at the point $p = (2, -5, 8)$.

- 1) $-\frac{781}{5} - 35 \sin[80]$ 2) $-\frac{1768}{5} - 35 \sin[80]$ 3) $-\frac{1298}{5} - 35 \sin[80]$ 4) $-128 - 35 \sin[80]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(7t + 5) \sin(2t) (5 \cos(6t) + 7), (t + 3) \sin(t) (5 \cos(6t) + 7)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 6211.99 2) 10559.7 3) 8696.39 4) 4969.79

Exercise 3

Consider the vector field $F(x, y, z) = \{2y, -xy^2z^2, -6z^2 - 7yz^2\}$ and the surface

$$S \equiv \left(\frac{1+x}{3}\right)^2 + \left(\frac{6+y}{3}\right)^2 + \left(\frac{8+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) -370670 . 2) -200780 . 3) -154446 . 4) -525116 .

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04-Line and Surface Integral-Computers exam for serial number: 100

Exercise 1

Consider the vectorial field $F(x, y, z) = (-2y^2 - 3, -4xy - z(2yz + 3y)\sin(yz) + (2z + 3)\cos(yz), 2y\cos(yz) - y(2yz + 3y)\sin(yz))$. Compute the potential function for this field whose potential at the origin is 5.
 . Calculate the value of the potential at the point $p = (10, 10, 3)$.

- 1) $-2025 + 90 \cos[30]$ 2) $\frac{29109}{5} + 90 \cos[30]$ 3) $\frac{3959}{5} + 90 \cos[30]$ 4) $-5043 + 90 \cos[30]$

Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, \pi] \rightarrow \mathbb{R}^2$$

$$r(t) = \{(4t + 9)\sin(2t), (4\cos(2t) + 7), (6t + 2)\sin(t), (4\cos(2t) + 7)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 598.829 2) 9579.33 3) 8381.93 4) 5987.13

Exercise 3

Consider the vector field $F(x, y, z) = \{3 - 4xy^2, 0, 8y^2\}$ and the surface

$$S \equiv \left(\frac{7+x}{1}\right)^2 + \left(\frac{-8+y}{6}\right)^2 + \left(\frac{3+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

- 1) 8590.19 2) 62275.2 3) -53684.4 4) -21473.4