Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 1

Exercise 1

Given the functions

$f(x, y) = (1 + 2x + y + 3xy - y^2, 1 - 3x^2 + 2xy + 2y^2)$

and

$g(u, v) = (-1 + 3u - 3u^2 + 3v + 3uv + 3v^2, -2 + u - u^2 + 2v - 2uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(0, 2)$.

1) 332 596.

2) 369 534.

3) 220 248.

4) 282 585.

5) 276 025.

Exercise 2

Given the system

$2xy\,u_5 == -20$
$-3xy^2 == -150$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p=(x, y, u_1, u_2, u_3, u_4, u_5) = (2, -5, -5, -1, 0, -5, 1)$. Compute if possible $\frac{\partial x}{\partial u_5}(-5, -1, 0, -5, 1)$.

1) $\frac{\partial x}{\partial u_5}(-5, -1, 0, -5, 1) = -1$

2) $\frac{\partial x}{\partial u_5}(-5, -1, 0, -5, 1) = 0$

3) $\frac{\partial x}{\partial u_5}(-5, -1, 0, -5, 1) = -3$

4) $\frac{\partial x}{\partial u_5}(-5, -1, 0, -5, 1) = -4$

5) $\frac{\partial x}{\partial u_5}(-5, -1, 0, -5, 1) = -2$

Exercise 3

Given the function

$f(x, y, z) = 3 - 2x + x^2 + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-0.749509, 0.2, ?\}$

2) We have a maximum at $\{1, ?, 1\}$

3) We have a maximum at $\{?, 0.1, -4.46875\}$

4) We have a maximum at $\{-0.249509, ?, -3.96875\}$

5) We have a maximum at $\{-0.449509, ?, -3.56875\}$

Exercise 4

Consider the domains $D_1 \equiv 8x^2 + 2xy + 6y^2 + 2xz + 4yz + 9z^2 = 1$ and $D_2 \equiv -8x - 9y + 5z = 11$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is $(*, *, ***.6***)$

2) The point of D_1 closest to D_2 is $(*, *, ***.5***)$

3) The point of D_1 closest to D_2 is $(*, *, ***.3***)$

4) The point of D_1 closest to D_2 is $(*, *, ***.1***)$

5) The point of D_1 closest to D_2 is $(*, *, ***.2***)$

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 2

Exercise 1

Given the functions

$f(x, y) = (-3 + 3x + 3x^2 + y - 3y^2, -2x + x^2 - 3y + 2xy, 2 + 2x - 3x^2 - y - xy + y^2)$

and

$g(u, v, w) = (3v + vw + 3w^2, 3v - 3v^2 + 2w + 2uw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, -2)$.

1) -4.02572×10^6

2) -5.90907×10^6

3) -4.72499×10^6

4) -7.34314×10^6

5) -1.53616×10^6

Exercise 2

Given the system

$-u^2 x + 3x^2 + 2y^2 - 2xy^2 == 123$
$3u^3 - 2y - 3u^2 y - 3xy + 3y^2 z == 34$
$-ux + xy^2 + uz^2 + 3z^3 == 355$

determine if it is possible to solve for variables x, y, z in terms of variable u arround the point $p = (x, y, z, u) = (-5, -2, 5, 0)$. Compute if possible $\frac{\partial y}{\partial u}(0)$.

1) $\frac{\partial y}{\partial u}(0) = -\frac{2278}{78\,911}$

2) $\frac{\partial y}{\partial u}(0) = -\frac{2276}{78\,911}$

3) $\frac{\partial y}{\partial u}(0) = -\frac{2279}{78\,911}$

4) $\frac{\partial y}{\partial u}(0) = -\frac{2277}{78\,911}$

5) $\frac{\partial y}{\partial u}(0) = -\frac{2280}{78\,911}$

Exercise 3

Given the function

$f(x, y, z) = 4 + 2x - x^2 - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-0.250091, ?, -0.824864\}$

2) We have a minimum at $\{?, 0, 0\}$

3) We have a minimum at $\{0.0248636, ?, 0.274955\}$

4) We have a minimum at $\{-0.8, ?, 0.\}$

5) We have a minimum at $\{-1.34991, 2.19964, ?\}$

Exercise 4

Consider the domains $D_1 \equiv 3x^2 - 2xy + 9y^2 + 8xz + 2yz + 7z^2 = 1$ and $D_2 \equiv -7x - 5y - 5z = -27$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is $(***.*4**, *, *)$

2) The point of D_1 closest to D_2 is $(***.*9**, *, *)$

3) The point of D_1 closest to D_2 is $(***.*7**, *, *)$

4) The point of D_1 closest to D_2 is $(***.*5**, *, *)$

5) The point of D_1 closest to D_2 is $(***.*1**, *, *)$

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 3

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (x_1 + 2\,x_1\,x_2 + 3\,x_2^2 - x_3^2,\ 2 + 2\,x_1\,x_3 - 2\,x_2\,x_3 + 3\,x_4 - 2\,x_3\,x_4 + x_4^2)$

and

$g(u, v) = (u + 2\,v - 2\,u\,v + 3\,v^2,\ 3 + u - 3\,v - u\,v - 3\,v^2,\ 3 - 2\,u - 2\,v - u\,v + 3\,v^2,\ -1 - 2\,u + 2\,u^2 + v - u\,v + 3\,v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 1, -2, 2)$.

1) 0.640583

2) 0.110619

3) 0.438759

4) 0.

5) −0.825543

Exercise 2

Given the system

$-3\,v^2\,x - 3\,u\,x^2 + w\,y^2 == -54$

$-2\,v - w - 3\,v\,w - w^3 - 3\,x - v^2\,x + 3\,x^2\,y == -14$

determine if it is possible to solve for variables x, y in terms of variables u, v, w arround the point $p = (x, y, u, v, w) = (1, -3, -4, -5, 1)$. Compute if possible the approximate values of (x, y) for $(u, v, w) = (-4.3, -5.3, 0.7)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (1.05734, -3.38741)$

2) $(x, y) \simeq (0.457343, -3.48741)$

3) $(x, y) \simeq (0.857343, -3.58741)$

4) $(x, y) \simeq (1.05734, -4.08741)$

5) $(x, y) \simeq (0.357343, -4.08741)$

Exercise 3

Given the function

$f(x, y, z) = 1 - x^2 + 2y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-3.94405, ?, 0.\}$

2) We have a minimum at $\{-4.24405, -0.133333, ?\}$

3) We have a minimum at $\{?, 0.0666667, -0.2\}$

4) We have a minimum at $\{0, 1, ?\}$

5) We have a minimum at $\{-4.04405, ?, 0.5\}$

Exercise 4

Consider the domain $D_1 \equiv 4x^2 + 2xy + 3y^2 = 1$ and the point $q = (1, -8)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \cup \{q\}$ is ***.*5**

2) The diameter of $D_1 \cup \{q\}$ is ***.*4**

3) The diameter of $D_1 \cup \{q\}$ is ***.*8**

4) The diameter of $D_1 \cup \{q\}$ is ***.*6**

5) The diameter of $D_1 \cup \{q\}$ is ***.*2**

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 4

Exercise 1

Given the functions

$f(x, y, z) = (1 - x + x^2 + 3y + xy, -3 + 3z)$

and

$g(u, v) = (u - 3u^2 + 3uv - 2v^2, -2 + u^2 - uv + 3v^2, 3 - 2u - 3u^2 - 3v + 3uv + v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, 2, 3)$.

1) 0.234089

2) 0.713201

3) 0.

4) 0.550274

5) 0.775037

Exercise 2

Given the system

$2xyu_3 == 36$
$-2xy^2 == 24$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-3, -2, -1, -5, 3, 2, -1)$. Compute if possible the approximate values of (x, y) for $(u_1, u_2, u_3, u_4, u_5) = (-1.3, -4.8, 2.7, 1.8, -0.8)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (-3.9, -1.7)$

2) $(x, y) \simeq (-3.3, -1.9)$

3) $(x, y) \simeq (-4., -1.5)$

4) $(x, y) \simeq (-3.2, -1.6)$

5) $(x, y) \simeq (-3.6, -1.8)$

Exercise 3

Given the function

$f(x, y, z) = -11 + 2x - x^2 - y^2 + 6z - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 1.5, 2.1}

2) We have a maximum at {1, 0, ?}

3) We have a maximum at {1.6, −1.2, ?}

4) We have a maximum at {1.9, −1.5, ?}

5) We have a maximum at {0.7, −0.6, ?}

Exercise 4

Consider the domain $D_1 \equiv 7x^2 - 6xy + 5y^2 - 10xz + 5z^2 = 1$ and the point $q = (-2, -9, -5)$.
Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The distance between D_1 and q is ***.*4**

2) The distance between D_1 and q is ***.*8**

3) The distance between D_1 and q is ***.*3**

4) The distance between D_1 and q is ***.*9**

5) The distance between D_1 and q is ***.*5**

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 5

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-1 + x_1 + x_2 + 2x_3 - 3x_1x_3 + 2x_2x_4 - x_3x_4 - x_4^2, -2x_1^2 - 3x_2, -1 - 3x_1x_3 - x_2x_3 + 2x_3^2 + x_4 - x_2x_4, -x_1 + x_1^2 - 2x_1x_3 + 3x_3x_4)$

and

$g(u_1, u_2, u_3, u_4) = (2u_1 - 3u_4^2, -u_1 - u_2^2 - u_3 + u_4 + u_3u_4 - u_4^2, u_1u_2 - u_3 - 2u_3^2 + 2u_3u_4 + u_4^2, -2u_1^2 + u_2 + u_3 + u_2u_4 - 3u_3u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, 2, -1, -2)$.

1) 8.29573×10^9

2) 3.41889×10^9

3) 5.28892×10^9

4) 1.15172×10^9

5) 1.22762×10^9

Exercise 2

Given the system

$2 + 2uvx_1 - ux_4 + x_3x_4^2 == -22$
$2uv + 3x_1 + ux_3 - 2ux_3^2 == 2$
$3ux_2^2 + ux_3^2 + x_1x_3x_4 == 0$
$3vx_2 + uvx_4 + x_2x_4^2 == -38$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (2, -2, -2, 4, 1, 3)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u, v) = (0.8, 2.8)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (1.34691, -1.62032, -1.66008, 4.00707)$

2) $(x_1, x_2, x_3, x_4) \simeq (1.54691, -1.72032, -2.36008, 4.30707)$

3) $(x_1, x_2, x_3, x_4) \simeq (1.74691, -2.02032, -1.96008, 3.80707)$

4) $(x_1, x_2, x_3, x_4) \simeq (1.94691, -2.12032, -2.36008, 3.60707)$

5) $(x_1, x_2, x_3, x_4) \simeq (2.04691, -1.82032, -1.56008, 3.30707)$

Exercise 3

Given the function

$f(x, y, z) = -4 + 2x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, -5.00006, -0.608439}

2) We have a minimum at {1, ?, 1}

3) We have a minimum at {-0.608439, -4.20006, ?}

4) We have a minimum at {-1.60844, -4.40006, ?}

5) We have a minimum at {-1.10844, -4.60006, ?}

Exercise 4

Consider the domain $D_1 \equiv 6x^2 + 2xy + 5y^2 = 1$ and the point $q = (2, -4)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 closest to q is (***.*2**, *)

2) The point of D_1 closest to q is (***.*5**, *)

3) The point of D_1 closest to q is (***.*1**, *)

4) The point of D_1 closest to q is (***.*4**, *)

5) The point of D_1 closest to q is (***.*6**, *)

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 6

Exercise 1

Given the functions

$f(x, y, z) = (-2 - 2y + 3xy + 2yz, -y^2 + xz + 2yz, -3x + y + xy - z + yz, 2x^2 + y - 3xz - 2yz)$

and

$g(u_1, u_2, u_3, u_4) = (-3 - 2u_1 + u_1^2 - 2u_2^2 - u_1 u_3 - 3u_4 + u_3 u_4 - u_4^2$
$, -u_1 - u_2 + u_1 u_3 - 2u_1 u_4 + u_2 u_4 + u_3 u_4, u_1 + 2u_3^2 + 3u_4 + 2u_3 u_4),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (3, -2, 3)$.

1) -6.87484×10^7

2) -4.02518×10^7

3) -3.58237×10^7

4) -1.73385×10^7

5) -5.03701×10^7

Exercise 2

Given the system

$2u x_2 x_4 + x_2^2 x_4 + 2x_3 x_4^2 == -14$
$2u + 2x_1 + 3u x_3 == 23$
$u x_1 x_4 - 3x_2^2 x_4 - 2x_3 x_4 == 2$
$u x_1^2 - 3u^2 x_2 + 3u^2 x_3 - 3u x_2 x_4 + 2x_3 x_4^2 == 41$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (4, 2, 1, -1, 3)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (3.2)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (5.74307, 3.07703, 0.757096, -0.595324)$

2) $(x_1, x_2, x_3, x_4) \simeq (5.74307, 3.07703, 0.157096, -0.095324)$

3) $(x_1, x_2, x_3, x_4) \simeq (6.34307, 2.37703, 0.357096, -0.495324)$

4) $(x_1, x_2, x_3, x_4) \simeq (5.94307, 2.77703, 0.457096, -0.195324)$

5) $(x_1, x_2, x_3, x_4) \simeq (5.64307, 2.97703, 0.757096, -0.395324)$

Exercise 3

Given the function

$f(x, y, z) = 6 - 6x + x^2 - 2y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-2.09554, -2.71899, ?\}$

2) We have a maximum at $\{-2.49554, ?, -0.247314\}$

3) We have a maximum at $\{-2.59554, ?, -0.347314\}$

4) We have a maximum at $\{-2.49554, ?, -0.847314\}$

5) We have a maximum at $\{?, 1, 2\}$

Exercise 4

Consider the domain $D_1 \equiv 6x^2 - 4xy + 6y^2 - 6xz - 4yz + 7z^2 = 1$ and the point $q = (1, -8, -9)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the point of D_1 where it is attained.

1) The point of D_1 furthest from q is $(**6.****, *, *)$

2) The point of D_1 furthest from q is $(**3.****, *, *)$

3) The point of D_1 furthest from q is $(**4.****, *, *)$

4) The point of D_1 furthest from q is $(**9.****, *, *)$

5) The point of D_1 furthest from q is $(**0.****, *, *)$

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 7

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-x_2^2 - 2x_3 - x_3^2 + 3x_1x_4 - 3x_2x_4 - 2x_3x_4, -2x_2 - 3x_3 - x_3^2 + 2x_4 - x_1x_4 - 3x_2x_4 + 2x_3x_4)$

and

$g(u, v) = (-1 - 3u + 3u^2 - v + 3v^2, -3 - u + u^2 - 2v, 1 + 2u + u^2 + v + 3uv - 2v^2, -1 + 2u^2 + v - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 1, 3, 1)$.

1) -0.130547

2) 0.78486

3) $0.$

4) 0.673739

5) -0.59114

Exercise 2

Given the system

$u - u^2 - 2u^3 - ux + 2x^2 - 3ux^2 - 3y + uy + 2xy - 3uy^2 + 3y^3 == 156$
$-1 + 2x - ux + u^2x - x^2 + 2y + 2u^2y + 3xy + y^2 - y^3 == -67$

determine if it is possible to solve for variables x, y in terms of variable u arround the point $p = (x, y, u) = (0, -3, -4)$. Compute if possible the approximate values of (x, y) for $(u) = (-4.3)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (2.08214, -18.1679)$

2) $(x, y) \simeq (1.88214, -17.8679)$

3) $(x, y) \simeq (2.38214, -18.3679)$

4) $(x, y) \simeq (2.28214, -18.2679)$

5) $(x, y) \simeq (2.38214, -18.5679)$

Exercise 3

Given the function

$f(x, y, z) = -10 - x^2 + 4y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, −0.666667, −0.333333}

2) We have a minimum at {−3.61184, ?, −0.233333}

3) We have a minimum at {−3.21184, ?, 0.166667}

4) We have a minimum at {0, 2, ?}

5) We have a minimum at {?, −1.16667, −0.0333333}

Exercise 4

Consider the domain $D_1 \equiv 8x^2 + 2xy + y^2 = 1$ and the point $q = (2, 7)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ***.4***

2) The distance between D_1 and q is ***.6***

3) The distance between D_1 and q is ***.8***

4) The distance between D_1 and q is ***.2***

5) The distance between D_1 and q is ***.1***

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 8

Exercise 1

Given the functions

$f(x, y, z) = (-1 - x y - 3 z, 3 + 3 x y, 3 - x - 2 y^2 + 2 x z)$

and

$g(u, v, w) = (-2 u - 3 v + 3 u w + w^2, 3 u v + 3 w + 3 u w, -3 u w - v w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, -2, 3)$.

1) -2.14787×10^6

2) $-707428.$

3) -2.92347×10^6

4) -1.17711×10^6

5) -1.69884×10^6

Exercise 2

Given the system

$3 u^2 + 2 u y^2 + y z - y z^2 == 39$
$v^2 x + 3 v x z == -40$
$-v x y == -40$

determine if it is possible to solve for variables x, y, z in terms of variables u, v arround the point $p = (x, y, z, u, v) = (5, 2, -2, 3, 4)$. Compute if possible the approximate values of (x, y, z) for $(u, v) = (2.9, 4.3)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (4.41136, 1.84545, -1.98182)$

2) $(x, y, z) \simeq (4.21136, 2.04545, -2.38182)$

3) $(x, y, z) \simeq (3.71136, 2.34545, -1.98182)$

4) $(x, y, z) \simeq (3.71136, 2.64545, -1.98182)$

5) $(x, y, z) \simeq (4.01136, 2.24545, -2.18182)$

Exercise 3

Given the function

$f(x, y, z) = -7 + 2x - x^2 + 2y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 1.5, 0.2}

2) We have a maximum at {0.7, ?, 0.5}

3) We have a maximum at {?, 0.6, 0.1}

4) We have a maximum at {1, ?, 0}

5) We have a maximum at {1.3, ?, −0.3}

Exercise 4

Consider the domain $D_1 \equiv 7x^2 + 10xy + 7y^2 + 8xz + 2yz + 8z^2 = 1$ and the point $q = (3, 1, 4)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the point of D_1 where it is attained.

1) The point of D_1 furthest from q is (*, ***.9***, *)

2) The point of D_1 furthest from q is (*, ***.4***, *)

3) The point of D_1 furthest from q is (*, ***.6***, *)

4) The point of D_1 furthest from q is (*, ***.0***, *)

5) The point of D_1 furthest from q is (*, ***.1***, *)

Further Mathematics - Degree in Engineering - 2026/2027
02-Differentiation-Training computers exam for serial number: 9

Exercise 1

Given the functions

$f(x,y)=(-1+x-3x^2-2y-xy-2y^2,\ -2x^2+y+3y^2,\ -2-x+y+y^2)$

and

$g(u,v,w)=(2u+2u^2-2v^2-3w-3vw,\ 1-uv+3w+3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3,2)$.

1) 3.1487×10^6

2) 1.4131×10^6

3) 4.61611×10^6

4) 4.00897×10^6

5) 5.36724×10^6

Exercise 2

Given the system

$v^2x+3vxy+2x^2y-2z^2==-46$
$-3u^2y-x^2y-2uxz-2uyz-vyz+3uz^2==-239$
$-ux-3y^2-xy^2==-11$

determine if it is possible to solve for variables x, y, z in terms of variables u, v arround the point $p=(x,y,z,u,v)=(-1,-2,5,-3,2)$. Compute if possible $\frac{\partial y}{\partial u}(-3,2)$.

1) $\frac{\partial y}{\partial u}(-3,2)=-\frac{59}{138}$

2) $\frac{\partial y}{\partial u}(-3,2)=-\frac{473}{1104}$

3) $\frac{\partial y}{\partial u}(-3,2)=-\frac{475}{1104}$

4) $\frac{\partial y}{\partial u}(-3,2)=-\frac{157}{368}$

5) $\frac{\partial y}{\partial u}(-3,2)=-\frac{79}{184}$

Exercise 3

Given the function

$f(x, y, z) = -10 - x^2 + 2y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{0.4, 0.8, ?\}$

2) We have a maximum at $\{-0.3, ?, 0.1\}$

3) We have a maximum at $\{0, 1, ?\}$

4) We have a maximum at $\{?, 1.5, 0.1\}$

5) We have a maximum at $\{?, 1.4, -0.2\}$

Exercise 4

Consider the domains $D_1 \equiv 7x^2 + 8xy + 8y^2 - 4xz + 3z^2 = 1$ and $D_2 \equiv 4x - 4y - 2z = -14$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The distance between both domains is ***.7***

2) The distance between both domains is ***.4***

3) The distance between both domains is ***.0***

4) The distance between both domains is ***.8***

5) The distance between both domains is ***.9***

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 10

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-2 - 2x_2 - x_3 - 3x_2x_3 - 3x_3^2 + 2x_4 + 3x_1x_4 + x_3x_4, -3x_3, 3x_1x_3 + 3x_2x_3 - x_3^2, -1 - 2x_1x_2 + 2x_1x_3 + 3x_4 + 3x_1x_4)$

and

$g(u_1, u_2, u_3, u_4) = (3u_1 + 3u_1^2 - u_1u_2 - 3u_2u_3 - 2u_4 + 2u_2u_4, 3 - u_1 - 3u_2^2 + 2u_3 - 2u_4 - 2u_1u_4 - u_2u_4, -3u_1 - 2u_3 + 3u_4, -3 - u_1 - 3u_1^2 + u_3 + 3u_3u_4 + 3u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, 1, 3, 0)$.

1) 6.68696×10^8

2) 6.94674×10^8

3) 1.61687×10^9

4) 1.11755×10^9

5) 1.33052×10^9

Exercise 2

Given the system

$-x_2x_3 + 3x_2^2x_3 == -20$

$-3ux_1^2 - x_1x_3x_4 == 205$

$-2x_1^3 + ux_2^2 - x_3 - x_2x_3^2 + x_4 == -270$

$x_1 + u^2x_1 + 3ux_2 - x_2^2 == 28$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (5, 2, -2, -2, -3)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (-2.9)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (4.61037, 2.02336, -2.68861, -0.650571)$

2) $(x_1, x_2, x_3, x_4) \simeq (4.51037, 1.52336, -2.58861, -1.05057)$

3) $(x_1, x_2, x_3, x_4) \simeq (5.31037, 1.72336, -1.98861, -0.550571)$

4) $(x_1, x_2, x_3, x_4) \simeq (5.01037, 1.82336, -2.38861, -0.950571)$

5) $(x_1, x_2, x_3, x_4) \simeq (5.21037, 2.02336, -1.98861, -1.25057)$

Exercise 3

Given the function

$f(x, y, z) = 17 - 6x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, −0.2, −0.5}

2) We have a minimum at {?, −0.2, 2.5}

3) We have a minimum at {4.2, ?, 0.4}

4) We have a minimum at {?, 1.6, 1.9}

5) We have a minimum at {?, 1, 1}

Exercise 4

Consider the domain $D_1 \equiv 9x^2 + 2xy + 7y^2 = 1$ and the point $q = (-3, -7)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is ***.*9**

2) The diameter of $D_1 \bigcup \{q\}$ is ***.*5**

3) The diameter of $D_1 \bigcup \{q\}$ is ***.*1**

4) The diameter of $D_1 \bigcup \{q\}$ is ***.*7**

5) The diameter of $D_1 \bigcup \{q\}$ is ***.*4**

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 11

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-2x_1^2 + x_1 x_2 + x_2^2 + x_3^2 - 3x_2 x_4, 1 + x_1 - x_1^2 - x_2 - 2x_3 + 2x_2 x_3 - 2x_2 x_4, 1 - 2x_1 + 2x_1 x_4 + 2x_3 x_4)$

and

$g(u, v, w) = (3u + 3uv - 3v^2 - 2vw + w^2, -3 + 3u - 2v - 3uv - 3w, -2u + uw + vw, v)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2, 3, 2, 0)$.

1) -0.583397

2) -0.194177

3) -0.60381

4) -0.335056

5) $0.$

Exercise 2

Given the system

$$-3uvx_1 - 3uvx_2 - 3v^2 x_4 - 3x_2 x_3 x_4 == -21$$
$$-2vx_2 + 2vx_2 x_3 == 16$$
$$vx_1 x_4 + x_2 x_3 x_4 == -3$$
$$-2v^2 x_2 + 2x_2^3 + 3x_2 x_4 == 3$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (-3, 1, 5, 3, 5, 2)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u, v) = (4.7, 1.9)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (-3.49737, 0.378289, 5.58684, 3.15066)$

2) $(x_1, x_2, x_3, x_4) \simeq (-2.99737, 0.478289, 6.18684, 3.65066)$

3) $(x_1, x_2, x_3, x_4) \simeq (-2.99737, 0.378289, 5.78684, 2.95066)$

4) $(x_1, x_2, x_3, x_4) \simeq (-3.09737, 0.778289, 6.08684, 3.25066)$

5) $(x_1, x_2, x_3, x_4) \simeq (-3.59737, 1.17829, 6.48684, 3.45066)$

Exercise 3

Given the function

$f(x, y, z) = -12 + 2x - x^2 + 4y - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-4.03118, ?, -0.81075\}$

2) We have a minimum at $\{-3.63118, -2.40503, ?\}$

3) We have a minimum at $\{-3.93118, ?, -0.61075\}$

4) We have a minimum at $\{1, 2, ?\}$

5) We have a minimum at $\{?, -2.40503, -1.01075\}$

Exercise 4

Consider the domains $D_1 \equiv 32 + 16x + 2x^2 - 8y - 2xy + 6y^2 \leq 1$ and $D_2 \equiv 354 + 52x + 2x^2 - 60y - 4xy + 3y^2 \leq 100$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is $(***.2***, *)$

2) The point of D_1 closest to D_2 is $(***.5***, *)$

3) The point of D_1 closest to D_2 is $(***.1***, *)$

4) The point of D_1 closest to D_2 is $(***.4***, *)$

5) The point of D_1 closest to D_2 is $(***.7***, *)$

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Exercise 1

Given the functions

$f(x, y) = (1 + 2x + 2x^2 + 3xy + 3y^2, -2 + 3x - 3x^2 + 2xy + y^2)$

and

$g(u, v) = (-3 - u + 3u^2 - 3v + uv - 2v^2, 3 + 2u - 3u^2 + v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3, 2)$.

1) 8.49242×10^6

2) 1.19156×10^7

3) 1.60084×10^7

4) 6.88856×10^6

5) 1.08381×10^7

Exercise 2

Given the system

$3x u_1 u_5 + 3y u_1 u_5 == -60$
$-x^2 y == 3$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p=(x, y, u_1, u_2, u_3, u_4, u_5) = (-1, -3, -1, 5, 5, 2, -5)$. Compute if possible $\frac{\partial y}{\partial u_3}(-1, 5, 5, 2, -5)$.

1) $\frac{\partial y}{\partial u_3}(-1, 5, 5, 2, -5) = 4$

2) $\frac{\partial y}{\partial u_3}(-1, 5, 5, 2, -5) = 2$

3) $\frac{\partial y}{\partial u_3}(-1, 5, 5, 2, -5) = 1$

4) $\frac{\partial y}{\partial u_3}(-1, 5, 5, 2, -5) = 3$

5) $\frac{\partial y}{\partial u_3}(-1, 5, 5, 2, -5) = 0$

Exercise 3

Given the function

$f(x, y, z) = 19 + x^2 - 6y + y^2 - 6z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 3, 3}

2) We have a maximum at {−0.3, −0.192191, ?}

3) We have a maximum at {?, −0.392191, −3.77698}

4) We have a maximum at {0.1, ?, −3.67698}

5) We have a maximum at {0., −0.492191, ?}

Exercise 4

Consider the domains $D_1 \equiv 6x^2 - 2xy + 9y^2 - 2xz - 10yz + 6z^2 = 1$ and $D_2 \equiv x + 2y + 9z = 11$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗0∗∗, ∗)

2) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗1∗∗, ∗)

3) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗8∗∗, ∗)

4) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗5∗∗, ∗)

5) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗6∗∗, ∗)

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 13

Exercise 1

Given the functions

$f(x, y) = (3 + 3x + 3x^2 + y - xy + 2y^2,$
$1 + 3x - 3x^2 + y, x + 3x^2 - 3y - 2xy + y^2, -3 + x - x^2 - y + 2xy + 3y^2)$

and

$g(u_1, u_2, u_3, u_4) = (3 - 2u_1 + 2u_1^2 - u_2^2 + 3u_2u_3 + 3u_1u_4, -2u_1u_4),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (3, -2)$.

1) 1.58935×10^6

2) 1.3051×10^6

3) 2.52171×10^6

4) 2.38794×10^6

5) 2.63191×10^6

Exercise 2

Given the system

$3u x_2 x_3 == 27$
$-3v x_2 x_3 - u x_1 x_4 == -61$
$-3x_1 - 2v x_2 x_3 - x_3 x_4^2 == 33$
$-u x_1 - x_2^2 x_3 == 23$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (-4, 3, -3, -5, -1, -3)$. Compute if possible $\frac{\partial x_4}{\partial u}(-1, -3)$.

1) $\frac{\partial x_4}{\partial u}(-1, -3) = \frac{2511}{671}$

2) $\frac{\partial x_4}{\partial u}(-1, -3) = \frac{2514}{671}$

3) $\frac{\partial x_4}{\partial u}(-1, -3) = \frac{2512}{671}$

4) $\frac{\partial x_4}{\partial u}(-1, -3) = \frac{2513}{671}$

5) $\frac{\partial x_4}{\partial u}(-1, -3) = \frac{2515}{671}$

Exercise 3

Given the function

$f(x, y, z) = 13 - 2x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.6, 1., ?}

2) We have a minimum at {0.6, −0.8, ?}

3) We have a minimum at {?, 0.2, 2.2}

4) We have a minimum at {?, 0.8, 3.}

5) We have a minimum at {1, ?, 2}

Exercise 4

Consider the domains $D_1 \equiv 9x^2 + 4xy + 6y^2 + 2xz - 10yz + 6z^2 = 1$ and $D_2 \equiv 9x - 3y + z = -12$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, ***.3***, *)

2) The point of D_1 closest to D_2 is (*, ***.5***, *)

3) The point of D_1 closest to D_2 is (*, ***.6***, *)

4) The point of D_1 closest to D_2 is (*, ***.9***, *)

5) The point of D_1 closest to D_2 is (*, ***.0***, *)

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Exercise 1

Given the functions

$f(x, y, z) = (3 - y^2, -2 + x)$

and

$g(u, v) = (2 + 2u + u^2 + 3v - 2v^2, -2 - 2u + u^2 - 3v - 2uv + 2v^2, -2 + u + 3u^2 + 3uv + 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, 2, -2)$.

1) 0.117083

2) −0.787796

3) 0.861243

4) 0.

5) −0.594864

Exercise 2

Given the system

$3y + 2vy - 3x^2 y - uy^2 == -260$
$-u^2 - 3u^3 + x^3 - uvy + 3v^2 y - uxy + 2vxy + 2xy^2 == 115$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (-3, 5, 4, -4)$. Compute if possible the approximate values of (x, y) for $(u, v) = (4.2, -4.1)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (-2.45473, 5.19826)$

2) $(x, y) \simeq (-2.35473, 5.59826)$

3) $(x, y) \simeq (-2.85473, 5.09826)$

4) $(x, y) \simeq (-2.45473, 4.69826)$

5) $(x, y) \simeq (-3.05473, 5.29826)$

Exercise 3

Given the function

$f(x, y, z) = 31 - 6x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {3, ?, 3}

2) We have a minimum at {2.5978, ?, 3.88205}

3) We have a minimum at {2.339, ?, 1.81162}

4) We have a minimum at {?, 1.00256, 2.58803}

5) We have a minimum at {1.30379, 1.77897, ?}

Exercise 4

Consider the domain $D_1 \equiv 3x^2 + 10xy + 9y^2 + 2xz + 2yz + 4z^2 = 1$ and the point $q = (8, -5, -9)$. Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The distance between D_1 and q is ***.9***

2) The distance between D_1 and q is ***.3***

3) The distance between D_1 and q is ***.7***

4) The distance between D_1 and q is ***.4***

5) The distance between D_1 and q is ***.1***

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (3x_1^2 + x_2 - x_1 x_3 - 3x_2 x_3 + 2x_3^2 - x_3 x_4,$
$-x_2^2 - 2x_3 + 2x_2 x_3 - 3x_4 + 3x_3 x_4 - 2x_4^2, x_1 - 2x_1 x_2 + 2x_2^2 - x_3 + 3x_1 x_4 + 3x_2 x_4)$

and

$g(u, v, w) = (2 + vw + 2w^2, 2u^2 - uw, -1 - 3u + v, -2u + 2u^2 + 2uv - 3uw),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, 0, 3, 2)$.

1) -0.385134

2) 0.730442

3) $0.$

4) 0.221014

5) 0.537297

Exercise 2

Given the system

$-y^2 z == 80$
$-uwx + 2uvz == 90$
$w + 3x^2 z == -13$

determine if it is possible to solve for variables x, y, z in terms of variables u, v, w arround the point $p = (x, y, z, u, v, w) = (1, -4, -5, 5, -2, 2)$. Compute if possible the approximate values of (x, y, z) for $(u, v, w) = (4.9, -2.3, 2.3)$ by means of the tangent affine function at p.

1) $(x, y, z) \approx (0.665238, -3.92095, -4.94762)$

2) $(x, y, z) \approx (0.865238, -4.32095, -4.34762)$

3) $(x, y, z) \approx (1.06524, -4.22095, -4.44762)$

4) $(x, y, z) \approx (1.56524, -3.72095, -4.54762)$

5) $(x, y, z) \approx (1.36524, -4.32095, -4.24762)$

Exercise 3

Given the function

$f(x, y, z) = -7 + 4x - x^2 + 4y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {2, 2, ?}

2) We have a minimum at {-1.50648, -4.07903, ?}

3) We have a minimum at {-1.60648, -4.57903, ?}

4) We have a minimum at {-1.30648, ?, -0.4}

5) We have a minimum at {-1.20648, -4.87903, ?}

Exercise 4

Consider the domain $D_1 \equiv 5x^2 + 4xy + 3y^2 = 1$ and the point $q = (-6, -2)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \cup \{q\}$ is ***.**7*

2) The diameter of $D_1 \cup \{q\}$ is ***.**5*

3) The diameter of $D_1 \cup \{q\}$ is ***.**1*

4) The diameter of $D_1 \cup \{q\}$ is ***.**9*

5) The diameter of $D_1 \cup \{q\}$ is ***.**8*

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Exercise 1

Given the functions

$f(x, y, z) = (-3x^2 - 2z^2, -3xz - 3yz)$

and

$g(u, v) = (-1 + 3u - u^2 - 3v + 3uv + 2v^2, 2 + 2u - 2u^2 - 2v, 2u - 2u^2 - v + 2uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, 0, 2)$.

1) -0.785409

2) -0.218771

3) $0.$

4) -0.227577

5) -0.215861

Exercise 2

Given the system

$$3x + vx + 2uvx + xy^2 + y^3 == 82$$
$$-3 - u - 3u^2 - 3v^2 + 3v^3 + 2uvx + 2uy + 2xy == -355$$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (-5, -2, 2, -5)$. Compute if possible the approximate values of (x, y) for $(u, v) = (2.2, -5.1)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (-5.73174, -2.88973)$

2) $(x, y) \simeq (-5.83174, -2.48973)$

3) $(x, y) \simeq (-5.73174, -2.78973)$

4) $(x, y) \simeq (-5.63174, -2.38973)$

5) $(x, y) \simeq (-5.53174, -2.68973)$

Exercise 3

Given the function

$f(x, y, z) = -5 - 4x + x^2 - 2y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {2, 1, ?}

2) We have a minimum at {?, 2., −0.4}

3) We have a minimum at {1., ?, −0.6}

4) We have a minimum at {?, 0.8, 0.4}

5) We have a minimum at {?, 0.4, 0.8}

Exercise 4

Consider the domain $D_1 \equiv 7x^2 + 2xy + y^2 - 8xz + 7z^2 = 1$ and the point $q = (7, 6, 7)$.
Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The distance between D_1 and q is ***.**1*

2) The distance between D_1 and q is ***.**3*

3) The distance between D_1 and q is ***.**0*

4) The distance between D_1 and q is ***.**2*

5) The distance between D_1 and q is ***.**6*

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 17

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-2x_1 - 2x_1^2 - x_3 - x_2 x_3 - 3x_4 + 3x_1 x_4 + x_3 x_4, 3x_2 x_3 + 3x_3 x_4 + 3x_4^2, x_2 - 3x_1 x_3)$

and

$g(u, v, w) = (u + 3u^2 - w^2, -uv - 2uw + 2w^2, -3u + 2uw, -uv + 3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, -2, -1, 3)$.

1) 0.38515

2) 0.354428

3) 0.

4) -0.41011

5) -0.789744

Exercise 2

Given the system

$-u^2 - 2u^2 x + 3x^2 z == -96$
$2yz == -4$
$3u^2 + y - 2uxy - y^3 + 2uz - 3u^2 z + 3uxz - 2x^2 z - 3xyz + 3y^2 z - z^3 == 90$

determine if it is possible to solve for variables x, y, z in terms of variable u arround the point $p = (x, y, z, u) = (4, 1, -2, 0)$. Compute if possible the approximate values of (x, y, z) for $(u) = (0.1)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (3.84, 0.52, -2.76)$

2) $(x, y, z) \simeq (3.14, 0.42, -1.86)$

3) $(x, y, z) \simeq (4.14, 0.62, -2.56)$

4) $(x, y, z) \simeq (4.14, 0.62, -2.86)$

5) $(x, y, z) \simeq (3.64, 0.82, -2.36)$

Exercise 3

Given the function

$f(x, y, z) = x^2 - 4y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.44222, −2.98349, ?}

2) We have a maximum at {2.06032, −2.57143, ?}

3) We have a maximum at {?, −3.39555, −0.618095}

4) We have a maximum at {1.85428, ?, 0.412063}

5) We have a maximum at {0, 2, ?}

Exercise 4

Consider the domain $D_1 \equiv 5x^2 - 10xy + 6y^2 = 1$ and the point $q = (1, 9)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ***.**3*

2) The distance between D_1 and q is ***.**8*

3) The distance between D_1 and q is ***.**0*

4) The distance between D_1 and q is ***.**6*

5) The distance between D_1 and q is ***.**1*

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 18

Exercise 1

Given the functions

$f(x, y, z) = (-y z, 2 - x y + 2 y^2 + z^2, -2 x + 2 z, 2 - 2 y - 3 y z - 3 z^2)$

and

$g(u_1, u_2, u_3, u_4) = (2 u_4 - 2 u_2 u_4 - 3 u_3 u_4,$
$3 u_1 u_2 + 3 u_1 u_3 + u_2 u_3 - 3 u_3^2 - u_3 u_4 + u_4^2, -1 - u_2 + 2 u_1 u_3 - 2 u_2 u_4),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, -1, -3)$.

1) $-101537.$

2) $-200424.$

3) $-257589.$

4) -47028.1

5) $-118566.$

Exercise 2

Given the system

$3 u x_2 + 3 x_1 x_2 + 2 x_3^2 + x_3^2 x_4 == 36$
$x_1 x_2 x_3 - v x_3 x_4 - x_3^2 x_4 == -24$
$-2 u x_2^2 - x_3 x_4 == -36$
$2 u^2 x_2 + v x_1 x_2 - 2 u x_2 x_4 == -24$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (-4, -4, -2, -2, 1, 0)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u, v) = (1.1, 0.1)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (-4.2103, -3.82303, -1.88303, -1.93273)$

2) $(x_1, x_2, x_3, x_4) \simeq (-4.0103, -3.52303, -2.18303, -1.63273)$

3) $(x_1, x_2, x_3, x_4) \simeq (-4.1103, -4.32303, -1.78303, -1.53273)$

4) $(x_1, x_2, x_3, x_4) \simeq (-4.3103, -3.42303, -2.28303, -1.43273)$

5) $(x_1, x_2, x_3, x_4) \simeq (-3.7103, -3.32303, -1.98303, -2.43273)$

Exercise 3

Given the function

$f(x, y, z) = 25 - 6x + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-1.06392, ?, -1.24261\}$

2) We have a maximum at $\{3, 2, ?\}$

3) We have a maximum at $\{-1.56392, -3.11759, ?\}$

4) We have a maximum at $\{-1.86392, ?, -1.34261\}$

5) We have a maximum at $\{-1.06392, ?, -1.54261\}$

Exercise 4

Consider the domain $D_1 \equiv 5x^2 + 8xy + 5y^2 + 2xz + 4yz + 5z^2 = 1$ and the point $q = (4, 2, -4)$. Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The point of D_1 closest to q is $(*, *, ***.2***)$

2) The point of D_1 closest to q is $(*, *, ***.8***)$

3) The point of D_1 closest to q is $(*, *, ***.4***)$

4) The point of D_1 closest to q is $(*, *, ***.3***)$

5) The point of D_1 closest to q is $(*, *, ***.1***)$

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Exercise 1

Given the functions

$f(x, y) = (1 - x + x^2 - y - 2xy, 2 - 3x + 3x^2 + 2y - 3xy)$

and

$g(u, v) = (3 + 3u + v + 2uv - v^2, 3 - u - 2u^2 + 3v + 3uv - 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 2)$.

1) 38 596.5

2) 18 617.8

3) 34 705.2

4) 25 620.

5) 8632.28

Exercise 2

Given the system

$3xy == -45$

$3xy^2 - 3u_1u_2u_4 == 210$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (3, -5, -1, -5, -2, 1, 5)$. Compute if possible $\frac{\partial x}{\partial u_3}(-1, -5, -2, 1, 5)$.

1) $\frac{\partial x}{\partial u_3}(-1, -5, -2, 1, 5) = 0$

2) $\frac{\partial x}{\partial u_3}(-1, -5, -2, 1, 5) = 4$

3) $\frac{\partial x}{\partial u_3}(-1, -5, -2, 1, 5) = 2$

4) $\frac{\partial x}{\partial u_3}(-1, -5, -2, 1, 5) = 3$

5) $\frac{\partial x}{\partial u_3}(-1, -5, -2, 1, 5) = 1$

Exercise 3

Given the function

$f(x, y, z) = 7 - 2x + x^2 - 6y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-0.30873, 1.11143, ?\}$

2) We have a minimum at $\{?, 2.03762, 1.4216\}$

3) We have a minimum at $\{?, 1.29666, 1.05112\}$

4) We have a minimum at $\{1, 3, ?\}$

5) We have a minimum at $\{?, 1.85238, 0.865887\}$

Exercise 4

Consider the domains $D_1 \equiv 9x^2 + 10xy + 9y^2 + 10xz - 4yz + 7z^2 = 1$ and $D_2 \equiv -4x + 3y + 2z = 14$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is $(*, *, ***.4***)$

2) The point of D_1 closest to D_2 is $(*, *, ***.3***)$

3) The point of D_1 closest to D_2 is $(*, *, ***.8***)$

4) The point of D_1 closest to D_2 is $(*, *, ***.6***)$

5) The point of D_1 closest to D_2 is $(*, *, ***.7***)$

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 20

Exercise 1

Given the functions

$f(x, y, z) = (3y - z + xz, -2z - z^2)$

and

$g(u, v) = (-3 + 3u + 2u^2 - v - 3uv + 3v^2, 1 - u - 3u^2 + 3v + 3uv - 3v^2, -2 - 3u + 2u^2 + 2v - 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, -1, 0)$.

1) -0.839589

2) 0.704858

3) -0.261359

4) $0.$

5) -0.554404

Exercise 2

Given the system

$u_1 u_2^2 + 2 u_1^2 u_3 + 2 x u_2 u_3 - 2 y u_4 == 182$
$2 x - 2 y u_2^2 - u_2 u_3 - 2 y u_1 u_4 - u_2 u_4^2 == 31$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (5, -2, 2, 3, 4, 3)$. Compute if possible the approximate values of (x, y) for $(u_1, u_2, u_3, u_4) = (1.9, 3.1, 4.1, 2.9)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (4.61441, -1.69237)$

2) $(x, y) \simeq (4.81441, -1.99237)$

3) $(x, y) \simeq (4.31441, -1.69237)$

4) $(x, y) \simeq (4.41441, -2.49237)$

5) $(x, y) \simeq (4.31441, -2.39237)$

Exercise 3

Given the function

$f(x, y, z) = -14 + 4x - x^2 + 6y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.50755, 1.72913, ?}

2) We have a maximum at {1.16172, 2.07496, ?}

3) We have a maximum at {2, 3, ?}

4) We have a maximum at {2.02629, 1.21039, ?}

5) We have a maximum at {1.16172, ?, 0.172913}

Exercise 4

Consider the domain $D_1 \equiv 4x^2 + 2xy + 5y^2 - 2yz + 8z^2 = 1$ and the point $q = (-2, 6, -8)$.
Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The distance between D_1 and q is ***.6***

2) The distance between D_1 and q is ***.1***

3) The distance between D_1 and q is ***.4***

4) The distance between D_1 and q is ***.5***

5) The distance between D_1 and q is ***.8***

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 21

Exercise 1

Given the functions

$f(x, y, z) = (-2x^2 - 3y - 3y^2 + 3z, -3 + 2y^2, -2x - 2x^2 + 2z + xz)$

and

$g(u, v, w) = (u - 2vw - 3w^2, u^2 - 2v^2, u - 2u^2 + 2uv + 2w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -3, 3)$.

1) -5.56247×10^6

2) -9.67662×10^6

3) -1.0967×10^7

4) -1.26788×10^7

5) -1.63491×10^7

Exercise 2

Given the system

$-3z - 3x^2 z == 255$
$-3 - 2u - 2u^2 + xy^2 == -127$
$3u + u^2 x - 3uz - yz == 10$

determine if it is possible to solve for variables x, y, z in terms of variable u arround the point $p = (x, y, z, u) = (-4, 4, -5, 5)$. Compute if possible the approximate values of (x, y, z) for $(u) = (4.9)$ by means of the tangent affine function at p.

1) $(x, y, z) \approx (-4.23523, 3.95114, -4.71711)$

2) $(x, y, z) \approx (-3.73523, 4.45114, -5.31711)$

3) $(x, y, z) \approx (-4.03523, 4.05114, -4.91711)$

4) $(x, y, z) \approx (-3.63523, 3.95114, -5.01711)$

5) $(x, y, z) \approx (-3.73523, 3.95114, -4.81711)$

Exercise 3

Given the function

$f(x, y, z) = -2 + 4x - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{1.92623, ?, 2.39096\}$

2) We have a maximum at $\{?, 1.22578, 2.21585\}$

3) We have a maximum at $\{1.75112, ?, 1.5154\}$

4) We have a maximum at $\{1.22578, -1.11022 \times 10^{-16}, ?\}$

5) We have a maximum at $\{?, 1, 2\}$

Exercise 4

Consider the domain $D_1 \equiv 9x^2 + 9y^2 - 6xz - 4yz + 7z^2 = 1$ and the point $q = (-7, -9, 7)$.
Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The distance between D_1 and q is ***.**3*

2) The distance between D_1 and q is ***.**2*

3) The distance between D_1 and q is ***.**8*

4) The distance between D_1 and q is ***.**5*

5) The distance between D_1 and q is ***.**4*

Further Mathematics - Degree in Engineering - 2026/2027 02-Differentiation-Training computers exam for serial number: 22

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (3 - 2x_1x_2 - x_3 + x_1x_3 + 3x_4 - 3x_1x_4 - 3x_2x_4 - 3x_4^2, x_1 + 2x_2 + x_1x_2 + 2x_3 + 3x_1x_3 - 2x_3^2 - x_3x_4 + x_4^2)$

and

$g(u, v) = (-1 - u + 2u^2 + 3v - 2uv - v^2, 3 - u^2 - 3v + 2uv, -2 + u + u^2 - 2v - 2uv + 3v^2, -1 + 3u - 3u^2 - v + 3uv + v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, -3, 3, -3)$.

1) 0.237634

2) 0.744965

3) -0.804753

4) 0.

5) -0.828997

Exercise 2

Given the system

$2u^3 - 3x_1x_2^2 + 2x_3 + x_2x_3 + 2x_3x_4 + 2x_1x_4^2 == -163$

$3u^2x_1 + x_1x_3^2 + 3ux_4 == -107$

$2u^3 + 3ux_2x_3 - 3x_1x_4 + 2ux_4^2 == -14$

$x_1x_2^2 - x_1x_3 - x_1x_2x_4 == -5$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (-5, 1, 4, -4, 1)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (1.2)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (-5.12356, 0.629123, 3.90816, -3.40871)$

2) $(x_1, x_2, x_3, x_4) \simeq (-5.42356, 1.32912, 3.90816, -4.30871)$

3) $(x_1, x_2, x_3, x_4) \simeq (-5.52356, 1.32912, 3.60816, -3.90871)$

4) $(x_1, x_2, x_3, x_4) \simeq (-5.62356, 0.929123, 3.50816, -3.80871)$

5) $(x_1, x_2, x_3, x_4) \simeq (-5.22356, 0.529123, 3.70816, -4.30871)$

Exercise 3

Given the function

$f(x, y, z) = 21 - 6x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, −0.36027, −0.310398}

2) We have a maximum at {3, 1, ?}

3) We have a maximum at {?, −0.26027, 0.389602}

4) We have a maximum at {?, −0.36027, −0.610398}

5) We have a maximum at {?, −0.66027, −0.110398}

Exercise 4

Consider the domains $D_1 \equiv 822 + 116x + 6x^2 + 152y + 8xy + 8y^2 \leq 1$ and $D_2 \equiv 519 + 86x + 8x^2 - 136y - 10xy + 9y^2 \leq 1000$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, ***.*0**)

2) The point of D_1 closest to D_2 is (*, ***.*2**)

3) The point of D_1 closest to D_2 is (*, ***.*1**)

4) The point of D_1 closest to D_2 is (*, ***.*8**)

5) The point of D_1 closest to D_2 is (*, ***.*9**)

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Exercise 1

Given the functions

$f(x,y)=(-3-x-2x^2+y+2xy+2y^2,\ -3x+3x^2+y-2xy-3y^2,\ 3x+3x^2+2y+3y^2)$

and

$g(u,v,w)=(-2-2uw-3vw,\ -1+2v-2w+uw-2vw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,-1)$.

1) $-11752.$

2) -21929.4

3) -1907.2

4) -15126.9

5) -3501.57

Exercise 2

Given the system

$u^2-u^2x+x^2+3uy^2-2uxz-2y^2z==1$
$2xyz==-4$
$1-2x^3-3y+2u^2y-yz+yz^2==-32$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point $p=(x,y,z,u)=(1,-1,2,-4)$. Compute if possible $\frac{\partial z}{\partial u}(-4)$.

1) $\frac{\partial z}{\partial u}(-4)=\frac{573}{125}$

2) $\frac{\partial z}{\partial u}(-4)=\frac{569}{125}$

3) $\frac{\partial z}{\partial u}(-4)=\frac{114}{25}$

4) $\frac{\partial z}{\partial u}(-4)=\frac{572}{125}$

5) $\frac{\partial z}{\partial u}(-4)=\frac{571}{125}$

Exercise 3

Given the function

$f(x, y, z) = 4 - 2x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, −0.841641, −2.}

2) We have a maximum at {1, 0, ?}

3) We have a maximum at {−0.8, ?, −1.6}

4) We have a maximum at {−0.7, −1.24164, ?}

5) We have a maximum at {−1.3, ?, −2.}

Exercise 4

Consider the domains $D_1 \equiv 6x^2 - 8xy + 9y^2 - 10xz + 9z^2 = 1$ and $D_2 \equiv -7x - y - 5z = 11$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The distance between both domains is **0.****

2) The distance between both domains is **5.****

3) The distance between both domains is **1.****

4) The distance between both domains is **6.****

5) The distance between both domains is **2.****

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (x_2 + 2x_2^2 + 2x_3x_4 + x_4^2, x_1 + 2x_2 - 2x_1x_4 - x_3x_4)$

and

$g(u, v) = (-3 + u - 2u^2 - 2v + 3uv - v^2, -2 + 2u - u^2 - v - uv - v^2, -3 - 2u - v + 2uv + 2v^2, -3 - 2u - 2u^2 + 2v - 2uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, 1, -2, -3)$.

1) -0.835046

2) -0.477936

3) -0.236567

4) -0.419336

5) $0.$

Exercise 2

Given the system

$3u^2 - x_2x_3 - x_1x_2x_3 + 3ux_3x_4 == -4$

$-x_2^2x_3 == 2$

$-2u^2 + 2x_1x_2^2 + 2x_3^2 + 2x_3x_4 - x_3x_4^2 == 72$

$-2ux_1 + x_1^2x_2 == 9$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (-3, 1, -2, -5, 0)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (-0.2)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (-1.52857, 1.51429, 2.15714, -12.2119)$

2) $(x_1, x_2, x_3, x_4) \simeq (-1.72857, 2.31429, 1.25714, -12.1119)$

3) $(x_1, x_2, x_3, x_4) \simeq (-1.82857, 1.91429, 1.65714, -11.9119)$

4) $(x_1, x_2, x_3, x_4) \simeq (-1.92857, 2.01429, 1.95714, -12.4119)$

5) $(x_1, x_2, x_3, x_4) \simeq (-1.72857, 1.41429, 1.25714, -12.4119)$

Exercise 3

Given the function

$f(x, y, z) = -20 + 6x - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 1, 3}

2) We have a maximum at {3.97564, ?, 0.584267}

3) We have a maximum at {2.65043, ?, 1.64444}

4) We have a maximum at {?, 0.0340022, 0.849309}

5) We have a maximum at {2.38538, 0.564088, ?}

Exercise 4

Consider the domains $D_1 \equiv 116 - 28x + 9x^2 + 44y + 2xy + 6y^2 \leq 1$ and $D_2 \equiv 7 - 4x + 3x^2 + 10y + 2xy + 6y^2 \leq 10$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, ***.4***)

2) The point of D_1 closest to D_2 is (*, ***.3***)

3) The point of D_1 closest to D_2 is (*, ***.0***)

4) The point of D_1 closest to D_2 is (*, ***.1***)

5) The point of D_1 closest to D_2 is (*, ***.8***)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-x_1^2 - 3x_1x_2 + 3x_3 + 2x_2x_3 - x_3^2 - 3x_1x_4, -2x_1 + x_1^2 + 2x_2 + x_2x_3 - 3x_4 + 3x_2x_4)$

and

$g(u, v) = (2 - 3u - v, 3 + u - 3v - 2v^2, 2 - u + 3u^2 - v - v^2, -u + 3u^2 - v + 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, -3, 0, -2)$.

1) -0.899108

2) 0.

3) 0.49729

4) 0.331774

5) 0.596375

Exercise 2

Given the system

$3 + 2u^2 - x - 3ux^2 - uy - u^2y - 3y^2 + 3uy^2 == -141$
$-u - u^3 - x - 2x^2 - y + uy - u^2y + 2y^2 == 283$

determine if it is possible to solve for variables x, y in terms of variable u arround the point $p = (x, y, u) = (1, -4, -5)$. Compute if possible the approximate values of (x, y) for $(u) = (-4.9)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (2.87227, -4.37577)$

2) $(x, y) \simeq (2.67227, -4.57577)$

3) $(x, y) \simeq (3.07227, -4.47577)$

4) $(x, y) \simeq (3.37227, -4.17577)$

5) $(x, y) \simeq (2.97227, -4.07577)$

Exercise 3

Given the function

$f(x, y, z) = 17 - 2x + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-3.75033, ?, -0.491815\}$

2) We have a maximum at $\{1, ?, 2\}$

3) We have a maximum at $\{-3.45033, ?, -0.391815\}$

4) We have a maximum at $\{-3.85033, ?, -0.591815\}$

5) We have a maximum at $\{?, -0.691815, -0.791815\}$

Exercise 4

Consider the domain $D_1 \equiv 6x^2 + 10xy + 8y^2 = 1$ and the point $q = (-1, 6)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is $(***.6***, *)$

2) The point of D_1 furthest from q is $(***.3***, *)$

3) The point of D_1 furthest from q is $(***.5***, *)$

4) The point of D_1 furthest from q is $(***.9***, *)$

5) The point of D_1 furthest from q is $(***.4***, *)$

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Exercise 1

Given the functions

$f(x, y, z) = (2x - 2x^2 - 3y + 2z - yz, -2y - 2z + yz + 2z^2)$

and

$g(u, v) = (3 + 3u + 2u^2 + 3v, 1 + v - 3uv - 3v^2, -u - 3u^2 - 3v + uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2, -1, 3)$.

1) 0.775213

2) 0.

3) -0.129001

4) 0.619537

5) -0.145474

Exercise 2

Given the system

$$u^3 - 3uxy + x^2z + 3uyz - 2xz^2 + z^3 == 99$$
$$-3u^2x + 2u^2y + 2z + xz - 2y^2z == -44$$
$$-2 + u^2 - 3xy^2 + 3xz + 2xyz - xz^2 == 49$$

determine if it is possible to solve for variables x, y, z in terms of variable u arround the point $p = (x, y, z, u) = (-1, -2, 5, 3)$. Compute if possible the approximate values of (x, y, z) for $(u) = (2.7)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (-0.925267, -1.84286, 5.08145)$

2) $(x, y, z) \simeq (-0.825267, -2.34286, 4.78145)$

3) $(x, y, z) \simeq (-1.02527, -2.04286, 4.98145)$

4) $(x, y, z) \simeq (-0.925267, -1.84286, 4.68145)$

5) $(x, y, z) \simeq (-1.12527, -2.54286, 5.38145)$

Exercise 3

Given the function

$f(x, y, z) = 4 + 2x - x^2 - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1, ?, 0}

2) We have a maximum at {0.9, ?, 0.3}

3) We have a maximum at {1.5, 0.5, ?}

4) We have a maximum at {0.7, −0.3, ?}

5) We have a maximum at {1.4, ?, −0.5}

Exercise 4

Consider the domains $D_1 \equiv 405 + 90x + 5x^2 - 108y - 12xy + 9y^2 \leq 1$ and $D_2 \equiv 24 - 24x + 6x^2 + 3y^2 \leq 300$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (***.**8*, *)

2) The point of D_1 closest to D_2 is (***.**1*, *)

3) The point of D_1 closest to D_2 is (***.**7*, *)

4) The point of D_1 closest to D_2 is (***.**9*, *)

5) The point of D_1 closest to D_2 is (***.**6*, *)

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Exercise 1

Given the functions

$f(x, y) = (-1 + 3x + 3x^2 - 3y - 2xy - 2y^2,$
$-x + 2x^2 - y + xy + 2y^2, -2 + x + 2x^2 - 3xy + y^2, 1 - 2x - x^2 + 2y - xy)$

and

$g(u_1, u_2, u_3, u_4) = (3 + u_1^2 - 2u_1u_3 - 3u_3^2 - u_4^2, -1 - 2u_2 + u_2u_3),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 1)$.

1) 2827.28

2) 2700.01

3) 2954.08

4) 2160.

5) 3242.37

Exercise 2

Given the system

$-x_3x_4^2 == -20$
$3x_1x_2^2 + x_1x_3^2 == -37$
$2x_1^3 + 3ux_2 == -20$
$vx_1^2 + vx_2 - 2x_1^2x_4 == -4$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (-1, 2, 5, 2, -3, 0)$. Compute if possible $\frac{\partial x_3}{\partial u}(-3, 0)$.

1) $\frac{\partial x_3}{\partial u}(-3, 0) = -\frac{160}{229}$

2) $\frac{\partial x_3}{\partial u}(-3, 0) = -\frac{158}{229}$

3) $\frac{\partial x_3}{\partial u}(-3, 0) = -\frac{157}{229}$

4) $\frac{\partial x_3}{\partial u}(-3, 0) = -\frac{156}{229}$

5) $\frac{\partial x_3}{\partial u}(-3, 0) = -\frac{159}{229}$

Exercise 3

Given the function

$f(x, y, z) = 4 + x^2 + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {0, ?, 0}

2) We have a maximum at {−0.3, ?, 0.1}

3) We have a maximum at {0.4, ?, 0.3}

4) We have a maximum at {?, −4.4, −0.2}

5) We have a maximum at {?, −4., 0.}

Exercise 4

Consider the domains $D_1 \equiv 7x^2 + 6y^2 + 8xz - 4yz + 5z^2 = 1$ and $D_2 \equiv -2x - 9y + 3z = 14$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (***.*6**, *, *)

2) The point of D_1 closest to D_2 is (***.*9**, *, *)

3) The point of D_1 closest to D_2 is (***.*5**, *, *)

4) The point of D_1 closest to D_2 is (***.*2**, *, *)

5) The point of D_1 closest to D_2 is (***.*7**, *, *)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (2x_2^2 + 2x_1x_3 + x_2x_3, -x_1^2 - 3x_1x_2 - x_3 + 2x_2x_4, -2x_1 - x_1^2 + 2x_2 - x_3 + x_2x_4, x_1x_2 + 3x_1x_3 - 3x_4 - 2x_2x_4 + x_3x_4 + 2x_4^2)$

and

$g(u_1, u_2, u_3, u_4) = (-u_2^2, -3u_1^2 + 3u_1u_2 + 3u_3 + 3u_1u_3, 3u_1 - 3u_4^2, -2u_2 + u_1u_2 - 3u_1u_4 - 2u_2u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, -1, 0, -3)$.

1) -4.85033×10^7

2) -6.12805×10^7

3) -1.59109×10^7

4) -2.38156×10^7

5) -4.14189×10^7

Exercise 2

Given the system

$$-u^2 - 2u^3 + x_4 - 3x_1x_4 + 2x_1x_2x_4 - 3x_2^2x_4 == 150$$
$$-2ux_1 - u^2x_1 - 3u^2x_2 - x_1x_2 + 2x_1^2x_2 - x_2^2 - ux_3^2 == -186$$
$$-x_3 + ux_1x_3 - 2x_2x_3 - x_1x_4^2 - 2x_4^3 == -71$$
$$2x_2x_3 == 24$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (5, -3, -4, -3, 3)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (2.9)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (5.40183, -3.09145, -4.11141, -2.62492)$

2) $(x_1, x_2, x_3, x_4) \simeq (5.00183, -2.99145, -4.01141, -2.92492)$

3) $(x_1, x_2, x_3, x_4) \simeq (5.40183, -2.89145, -4.31141, -3.12492)$

4) $(x_1, x_2, x_3, x_4) \simeq (5.10183, -3.09145, -3.71141, -2.82492)$

5) $(x_1, x_2, x_3, x_4) \simeq (5.50183, -3.29145, -3.71141, -3.12492)$

Exercise 3

Given the function

$f(x, y, z) = -4 - x^2 + 2y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-0.1, 1.1, ?\}$

2) We have a maximum at $\{-0.4, ?, 0.1\}$

3) We have a maximum at $\{0.3, 0.9, ?\}$

4) We have a maximum at $\{-0.4, 1.4, ?\}$

5) We have a maximum at $\{0, ?, 0\}$

Exercise 4

Consider the domain $D_1 \equiv 5x^2 + 4xy + 5y^2 = 1$ and the point $q = (-1, -9)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \cup \{q\}$ is ***.2***

2) The diameter of $D_1 \cup \{q\}$ is ***.9***

3) The diameter of $D_1 \cup \{q\}$ is ***.5***

4) The diameter of $D_1 \cup \{q\}$ is ***.3***

5) The diameter of $D_1 \cup \{q\}$ is ***.0***

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (2x_1 - 2x_1^2 + 2x_3^2 - 3x_4 + 2x_4^2,$
$3x_1^2 - 3x_3^2, -x_2 x_3 - 3x_3 x_4 + 2x_4^2, 2 + x_1^2 - 3x_2^2 + 2x_1 x_3 + x_4 + 3x_2 x_4)$

and

$g(u_1, u_2, u_3, u_4) = (2u_1 + 3u_1 u_2 + 2u_2^2 - u_3, -u_1 u_2 - 2u_1 u_4, -3 - 3u_1^2 + 2u_3^2 + u_4^2, -u_2 u_3 - u_3 u_4),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2, -3, -2, 1)$.

1) 5.05418×10^9

2) 4.75398×10^9

3) 1.81112×10^9

4) 3.35323×10^9

5) 1.019×10^9

Exercise 2

Given the system

$v x_3 x_4 == 36$
$x_1^2 - v x_2^2 == 28$
$-2x_1 - 2x_1^2 x_3 == 4$
$-2uw x_1 + 3w x_3^2 == -46$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v, w arround the point $p = (x_1, x_2, x_3, x_4, u, v, w) = (1, -3, -3, 4, 2, -3, -2)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u, v, w) = (1.8, -2.7, -2.1)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (1.01649, -3.14817, -2.91755, 4.50993)$

2) $(x_1, x_2, x_3, x_4) \simeq (0.816489, -3.04817, -3.21755, 4.30993)$

3) $(x_1, x_2, x_3, x_4) \simeq (1.11649, -3.64817, -3.21755, 4.70993)$

4) $(x_1, x_2, x_3, x_4) \simeq (0.916489, -3.54817, -3.21755, 4.30993)$

5) $(x_1, x_2, x_3, x_4) \simeq (1.11649, -3.04817, -2.61755, 5.00993)$

Exercise 3

Given the function

$f(x, y, z) = -22 + 6x - x^2 + 4y - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {2.91043, ?, 1.94029}

2) We have a maximum at {1.74626, 0.776114, ?}

3) We have a maximum at {?, 2, 2}

4) We have a maximum at {?, 0.776114, 2.23133}

5) We have a maximum at {?, 2.23133, 3.10446}

Exercise 4

Consider the domain $D_1 \equiv 8x^2 - 6xy + 4y^2 = 1$ and the point $q = (-9, 3)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is (***.*7**, *)

2) The point of D_1 furthest from q is (***.*0**, *)

3) The point of D_1 furthest from q is (***.*6**, *)

4) The point of D_1 furthest from q is (***.*1**, *)

5) The point of D_1 furthest from q is (***.*3**, *)

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Exercise 1

Given the functions

$f(x, y, z) = (-3 - 2x + 3y^2 - 3z^2, -3xy + yz)$

and

$g(u, v) = (2 - u^2 - 2v + 3uv + 3v^2, 1 + 2u - v - 3uv - 3v^2, -2 + 2u + 3u^2 + v - 2uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 2, 3)$.

1) 0.297107

2) 0.395985

3) 0.439638

4) −0.298469

5) 0.

Exercise 2

Given the system

$$-y^2 + 2y^3 + 3x^2 z + z^2 - 2uz^2 == 392$$
$$3u + uxy + 3u^2 z - 2yz - y^2 z - xz^2 == 49$$
$$-2x^2 - x^3 + 2y + xz^2 + 3yz^2 == -195$$

determine if it is possible to solve for variables x, y, z in terms of variable u arround the point $p = (x, y, z, u) = (5, -2, 4, -3)$. Compute if possible the approximate values of (x, y, z) for $(u) = (-3.3)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (10.7651, 6.47228, -3.9659)$

2) $(x, y, z) \simeq (11.3651, 6.77228, -3.1659)$

3) $(x, y, z) \simeq (11.5651, 6.57228, -3.1659)$

4) $(x, y, z) \simeq (11.2651, 6.67228, -3.6659)$

5) $(x, y, z) \simeq (11.3651, 7.07228, -3.8659)$

Exercise 3

Given the function

$f(x, y, z) = -20 - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 1, 3}

2) We have a maximum at {?, 1.76531, 1.15636}

3) We have a maximum at {?, −0.161954, 2.31272}

4) We have a maximum at {?, −0.161954, 2.11999}

5) We have a maximum at {0., ?, 1.92727}

Exercise 4

Consider the domains $D_1 \equiv 216 - 72x + 9x^2 - 6xy + 3y^2 \leq 1$ and $D_2 \equiv 25 - 2x + 9x^2 - 24y - 8xy + 8y^2 \leq 100$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, ***.*3**)

2) The point of D_1 closest to D_2 is (*, ***.*9**)

3) The point of D_1 closest to D_2 is (*, ***.*5**)

4) The point of D_1 closest to D_2 is (*, ***.*8**)

5) The point of D_1 closest to D_2 is (*, ***.*7**)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (2x_1^2 + 2x_2 - 3x_2x_4 - 3x_3x_4 - x_4^2, 1 + 2x_1x_2 + 3x_2x_3 - 2x_4^2, 3x_2 - x_2x_3 - 3x_3^2, 3 + x_1^2 - 2x_2x_4)$

and

$g(u_1, u_2, u_3, u_4) = (2u_1 + 2u_1^2 - 3u_2 + 3u_3^2 + 3u_4, 2u_2u_3 - 2u_3^2 - u_4 + 3u_2u_4 - u_3u_4 + 2u_4^2, -3 - u_3^2 + 2u_1u_4 + 3u_2u_4 - 3u_4^2, u_1 + 3u_1u_2 - 2u_3 + 2u_2u_3 + 3u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, -2, 3, 0)$.

1) -7.93623×10^{10}

2) -3.5487×10^{10}

3) -2.17421×10^{11}

4) -1.61806×10^{11}

5) -1.40482×10^{11}

Exercise 2

Given the system

$-2u^2v - 2x_2x_3^2 + x_4^3 == -31$

$3v^2x_1 + 2x_1x_3x_4 == -15$

$3v^2x_2 - 2ux_1x_4 + 3ux_4^2 - 2x_2x_4^2 == 34$

$3x_1^2 - 2x_1x_2 - 3u^2x_4 == 10$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (5, -1, -3, 1, -5, 1)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u, v) = (-4.7, 0.7)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (4.73201, -0.0527884, -1.66631, 0.918029)$

2) $(x_1, x_2, x_3, x_4) \simeq (4.43201, -0.0527884, -1.56631, 0.918029)$

3) $(x_1, x_2, x_3, x_4) \simeq (4.93201, -0.452788, -2.06631, 1.01803)$

4) $(x_1, x_2, x_3, x_4) \simeq (4.83201, -0.0527884, -2.26631, 1.51803)$

5) $(x_1, x_2, x_3, x_4) \simeq (5.43201, -0.252788, -1.96631, 0.518029)$

Exercise 3

Given the function

$f(x, y, z) = 22 - 6x + x^2 - 2y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {4.2, ?, 2.9}

2) We have a minimum at {2.1, 2.2, ?}

3) We have a minimum at {3, ?, 2}

4) We have a minimum at {2.1, 1.9, ?}

5) We have a minimum at {4.5, 0.4, ?}

Exercise 4

Consider the domain $D_1 \equiv 6x^2 + 2xy + 6y^2 = 1$ and the point $q = (6, -7)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 closest to q is (*, ***.5***)

2) The point of D_1 closest to q is (*, ***.8***)

3) The point of D_1 closest to q is (*, ***.3***)

4) The point of D_1 closest to q is (*, ***.1***)

5) The point of D_1 closest to q is (*, ***.2***)

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Exercise 1

Given the functions

$f(x, y, z) = (-2xy + 3yz - 2z^2, -1 + x^2 - y - 2xy + 3yz + 3z^2, -3x + 3y + 3xy + y^2 + z - 2yz, x^2 + 2y^2 + 3z^2)$

and

$g(u_1, u_2, u_3, u_4) = (3u_2 + 2u_1u_3 - 2u_3u_4 + 3u_4^2, -2u_2 - 3u_1u_4 - u_2u_4 - 3u_3u_4, 2u_1 - 2u_3^2 + u_4 + 3u_2u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (3, -3, 3)$.

1) -1.55663×10^{10}

2) -3.602×10^{9}

3) -1.88751×10^{10}

4) -2.23761×10^{10}

5) -1.28539×10^{10}

Exercise 2

Given the system

$-2u x_3^2 - 3x_2x_4 == 16$

$u x_1 x_2 + 3x_2^2 - x_3 == 2$

$3u^3 - x_2^2 - u x_3 + 2x_1x_3x_4 + 3x_4^3 == -215$

$-2u x_2^2 - 3u^2 x_3 - x_3^2 - 3x_2x_3^2 == -12$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (0, 1, 1, -4, -2)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (-2.1)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (-0.215502, 1.42069, 1.25517, -3.57009)$

2) $(x_1, x_2, x_3, x_4) \simeq (-0.415502, 1.12069, 0.555174, -3.77009)$

3) $(x_1, x_2, x_3, x_4) \simeq (-0.415502, 1.22069, 0.655174, -3.47009)$

4) $(x_1, x_2, x_3, x_4) \simeq (0.0844981, 1.02069, 0.955174, -3.97009)$

5) $(x_1, x_2, x_3, x_4) \simeq (0.284498, 1.22069, 0.555174, -4.17009)$

Exercise 3

Given the function

$f(x, y, z) = -6 - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {0, 1, ?}

2) We have a minimum at {0.4, ?, −4.59056}

3) We have a minimum at {0.3, −1.1323, ?}

4) We have a minimum at {0., −0.832303, ?}

5) We have a minimum at {−0.1, ?, −5.39056}

Exercise 4

Consider the domain $D_1 \equiv 8x^2 - 10xy + 8y^2 - 4xz - 4yz + 6z^2 = 1$ and the point $q = (-2, -4, 0)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the point of D_1 where it is attained.

1) The point of D_1 furthest from q is (*, *, **5.****)

2) The point of D_1 furthest from q is (*, *, **7.****)

3) The point of D_1 furthest from q is (*, *, **8.****)

4) The point of D_1 furthest from q is (*, *, **6.****)

5) The point of D_1 furthest from q is (*, *, **0.****)

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Exercise 1

Given the functions

$f(x,y)=(-2+x-2x^2+3y+xy,-3-x-2x^2-2y+3xy+3y^2,-2-x-2x^2-2y+2xy-2y^2)$

and

$g(u,v,w)=(-3uw,2v)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,1)$.

1) -101.988

2) $-78.$

3) -45.674

4) -94.8824

5) -47.0474

Exercise 2

Given the system

$3v^2-wxy==19$
$-v^2y-2vwy-3uxy==-42$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point $p=(x,y,u,v,w)=(1,-2,-2,3,-4)$. Compute if possible the approximate values of (x,y) for $(u,v,w)=(-2.2,2.8,-3.8)$ by means of the tangent affine function at p.

1) $(x,y)\simeq(0.173333,-2.35333)$

2) $(x,y)\simeq(0.773333,-2.15333)$

3) $(x,y)\simeq(0.573333,-2.55333)$

4) $(x,y)\simeq(-0.126667,-2.15333)$

5) $(x,y)\simeq(0.373333,-2.45333)$

Exercise 3

Given the function

$f(x, y, z) = -9 - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-3.90949, -0.771429, ?\}$

2) We have a minimum at $\{-4.20949, ?, -1.325\}$

3) We have a minimum at $\{?, 3, 2\}$

4) We have a minimum at $\{?, -0.371429, -1.325\}$

5) We have a minimum at $\{-4.40949, -0.571429, ?\}$

Exercise 4

Consider the domains $D_1 \equiv 80 - 20x + 8x^2 - 10y - 10xy + 5y^2 \leq 1$ and $D_2 \equiv 441 - 84x + 8x^2 - 42y - 4xy + 5y^2 \leq 10$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The distance between both domains is ***.*2**

2) The distance between both domains is ***.*9**

3) The distance between both domains is ***.*7**

4) The distance between both domains is ***.*0**

5) The distance between both domains is ***.*5**

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Exercise 1

Given the functions

$f(x,y)=(-2-3x-x^2-2xy,-3+2x+y,-1-x+3x^2-2y+2xy-2y^2)$

and

$g(u,v,w)=(-3u-2uw+w^2,u)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,-1)$.

1) $-156.$

2) -137.475

3) -137.253

4) -217.207

5) -184.197

Exercise 2

Given the system

$2-3x^3+3z+2uz+2xz+x^2z-3uyz-2y^2z==25$

$3uxy-2uz+2u^2z==88$

$uy-u^2y-z^2==-56$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point $p=(x,y,z,u)=(3,2,4,-4)$. Compute if possible $\frac{\partial z}{\partial u}(-4)$.

1) $\frac{\partial z}{\partial u}(-4)=\frac{1313}{686}$

2) $\frac{\partial z}{\partial u}(-4)=\frac{656}{343}$

3) $\frac{\partial z}{\partial u}(-4)=\frac{1315}{686}$

4) $\frac{\partial z}{\partial u}(-4)=\frac{657}{343}$

5) $\frac{\partial z}{\partial u}(-4)=\frac{1311}{686}$

Exercise 3

Given the function

$f(x, y, z) = 10 - 6x + x^2 - 6y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, 2.80203, 1.27171}

2) We have a minimum at {0.812957, 2.40174, ?}

3) We have a minimum at {3, ?, 1}

4) We have a minimum at {2.01383, ?, 1.47186}

5) We have a minimum at {1.41339, 2.00145, ?}

Exercise 4

Consider the domains $D_1 \equiv 9x^2 - 14xy + 6y^2 - 6xz + 6yz + 9z^2 = 1$ and $D_2 \equiv 9x + 5y - 4z = -21$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, *, ***.*4**)

2) The point of D_1 closest to D_2 is (*, *, ***.*1**)

3) The point of D_1 closest to D_2 is (*, *, ***.*9**)

4) The point of D_1 closest to D_2 is (*, *, ***.*5**)

5) The point of D_1 closest to D_2 is (*, *, ***.*2**)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-2 - x_1 + 2x_2 + 3x_3 + 2x_1x_3 - 2x_3^2 + 2x_4 + 2x_1x_4 - x_2x_4 - 3x_4^2, 3x_1^2 - x_2^2 + 3x_1x_4 - 2x_2x_4, -2 + 3x_1x_2 - 2x_3 + x_3^2 + x_4 + x_2x_4 - x_3x_4 + 2x_4^2, -2x_1 + 3x_2^2 + 3x_1x_3 - 3x_2x_3 - 2x_1x_4 - x_2x_4)$

and

$g(u_1, u_2, u_3, u_4) = (-3u_1u_3 - u_4^2, 2u_3, -2u_2u_4, -u_1u_2 + 2u_3 - u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(0, -1, 2, -1)$.

1) $-378\,808.$

2) -1.50509×10^6

3) -1.83382×10^6

4) -1.79974×10^6

5) -2.21054×10^6

Exercise 2

Given the system

$$-x_1^2x_4 - 3x_1x_2x_4 == -380$$
$$3 + 2ux_2x_3 + 2x_3^2x_4 == 163$$
$$-x_1 + ux_1 + 2u^2x_1 + x_1x_2^2 + 2x_2^3 == -346$$
$$-ux_1 + 3x_2 + 3x_1x_3^2 + x_2^2x_4 == -82$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p=(x_1, x_2, x_3, x_4, u)=(-4, -5, -4, 5, 0)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u)=(-0.2)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4)\simeq(-4.42746, -5.51337, -4.36032, 4.7992)$

2) $(x_1, x_2, x_3, x_4)\simeq(-3.92746, -5.01337, -4.06032, 5.0992)$

3) $(x_1, x_2, x_3, x_4)\simeq(-4.32746, -5.31337, -4.46032, 5.4992)$

4) $(x_1, x_2, x_3, x_4)\simeq(-3.52746, -4.61337, -3.56032, 4.5992)$

5) $(x_1, x_2, x_3, x_4)\simeq(-3.82746, -5.31337, -3.96032, 4.7992)$

Exercise 3

Given the function

$f(x, y, z) = -16 + 6x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, -1.46075, 0.137996}

2) We have a minimum at {?, 1, 1}

3) We have a minimum at {-4.28226, ?, -0.462004}

4) We have a minimum at {?, -1.26075, -0.162004}

5) We have a minimum at {-4.18226, -1.76075, ?}

Exercise 4

Consider the domain $D_1 \equiv 7x^2 + 2xy + 2y^2 = 1$ and the point $q = (-4, 2)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is **0.****

2) The distance between D_1 and q is **3.****

3) The distance between D_1 and q is **2.****

4) The distance between D_1 and q is **5.****

5) The distance between D_1 and q is **9.****

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (x_2 - x_2^2, -1 - x_3^2 + x_4 - 2\,x_2\,x_4)$

and

$g(u, v) = (-3 - u + 3\,u^2 + v - u\,v - 2\,v^2, 1 + 3\,u + u^2 - 2\,v - 3\,v^2, -2 + u - 3\,u^2 - 3\,v - 2\,u\,v + v^2, 2\,u + 3\,u^2 + 3\,v + 2\,u\,v + 3\,v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2, 1, -2, 2)$.

1) 0.

2) −0.458251

3) 0.613712

4) −0.679809

5) 0.762408

Exercise 2

Given the system

$v\,x^2 - 2\,x^3 - 2\,u\,y + 2\,x\,y == -16$

$3\,u^2\,x - 3\,v^2\,x - 3\,u\,x\,y - y^3 == 64$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (0, -4, -2, -5)$. Compute if possible the approximate values of (x, y) for $(u, v) = (-1.8, -5.3)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (0.104918, -4.19016)$

2) $(x, y) \simeq (-0.095082, -3.89016)$

3) $(x, y) \simeq (0.404918, -4.39016)$

4) $(x, y) \simeq (0.404918, -4.49016)$

5) $(x, y) \simeq (-0.295082, -4.59016)$

Exercise 3

Given the function

$f(x, y, z) = 23 - 6x + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, 1.4, 2.2}

2) We have a minimum at {1.8, ?, 0.1}

3) We have a minimum at {1.8, 2.6, ?}

4) We have a minimum at {1.5, ?, 0.1}

5) We have a minimum at {3, ?, 1}

Exercise 4

Consider the domain $D_1 \equiv 7x^2 + 4xy + 5y^2 = 1$ and the point $q = (1, 9)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is ***.5***

2) The diameter of $D_1 \bigcup \{q\}$ is ***.7***

3) The diameter of $D_1 \bigcup \{q\}$ is ***.9***

4) The diameter of $D_1 \bigcup \{q\}$ is ***.6***

5) The diameter of $D_1 \bigcup \{q\}$ is ***.2***

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Exercise 1

Given the functions

$f(x,y)=(1-3x+3x^2-3y+3xy+y^2,\ 2-3x-3x^2-2y+3xy)$

and

$g(u,v)=(1+3u+2u^2+v+2uv-v^2,\ 2+u+3u^2+2v)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,3)$.

1) $-743023.$

2) -1.73129×10^6

3) $-241893.$

4) -1.09859×10^6

5) -1.93628×10^6

Exercise 2

Given the system

$$-2x\,u_2\,u_4-3y\,u_4\,u_5==105$$
$$-2x\,y\,u_4+3x\,u_3\,u_4-u_1^2\,u_5==-55$$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4,u_5 arround the point $p=(x,y,u_1,u_2,u_3,u_4,u_5)=(-2,-3,-5,3,-1,5,1)$. Compute if possible $\frac{\partial y}{\partial u_1}(-5,3,-1,5,1)$.

1) $\frac{\partial y}{\partial u_1}(-5,3,-1,5,1)=-\frac{1}{5}$

2) $\frac{\partial y}{\partial u_1}(-5,3,-1,5,1)=-\frac{4}{5}$

3) $\frac{\partial y}{\partial u_1}(-5,3,-1,5,1)=-\frac{3}{5}$

4) $\frac{\partial y}{\partial u_1}(-5,3,-1,5,1)=0$

5) $\frac{\partial y}{\partial u_1}(-5,3,-1,5,1)=-\frac{2}{5}$

Exercise 3

Given the function

$f(x, y, z) = -12 + 2x - x^2 - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.4, ?, 2.2}

2) We have a maximum at {?, 0.4, 2.2}

3) We have a maximum at {1.4, ?, 1.4}

4) We have a maximum at {1, 0, ?}

5) We have a maximum at {1.2, ?, 1.4}

Exercise 4

Consider the domains $D_1 \equiv 9x^2 + 10xy + 9y^2 + 4xz + 2yz + 9z^2 = 1$ and $D_2 \equiv 2x + 6y = -8$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (***.*8**, *, *)

2) The point of D_1 closest to D_2 is (***.*0**, *, *)

3) The point of D_1 closest to D_2 is (***.*1**, *, *)

4) The point of D_1 closest to D_2 is (***.*9**, *, *)

5) The point of D_1 closest to D_2 is (***.*3**, *, *)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (3x_1 - x_3 - x_1 x_3 + 3x_3^2 - x_4 + 3x_1 x_4 - x_3 x_4, -x_1 - x_1^2 + 2x_2 + 2x_2^2 - 2x_1 x_3 + x_3^2 + 2x_4, -1 + 3x_1 - 3x_2 + x_2 x_3 - x_3^2 - 2x_1 x_4 - 2x_2 x_4, 3 + 3x_1 - 2x_2^2 - 3x_3 + x_3 x_4 + 3x_4^2)$

and

$g(u_1, u_2, u_3, u_4) = (-2u_2^2 + u_2 u_3, -3u_1^2 - u_2 - u_1 u_2 - 3u_3 - 2u_3^2 - 2u_3 u_4, 2u_1 + 3u_1 u_3 - 2u_4 - 2u_1 u_4, -3 + 3u_2 + 3u_1 u_3 - 2u_3^2 + u_4 + 2u_3 u_4 + 2u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, 0, -2, -2)$.

1) 9.17357×10^9

2) 8.49748×10^9

3) 1.49214×10^{10}

4) 1.2604×10^{10}

5) 1.88193×10^{10}

Exercise 2

Given the system

$$-3uvx_1 + 2x_1^3 - 3x_2 x_3^2 + x_3^2 x_4 + x_3 x_4^2 == 292$$
$$v^3 - 2u^2 x_3 + 3v^2 x_4 - v x_2 x_4 == 84$$
$$-2uv x_3 - 3x_2^2 x_3 == -172$$
$$-3v x_2^2 - 3x_1 x_2^2 - x_1^2 x_3 - 2x_2 x_4 == 68$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (2, -3, 4, 5, -2, -4)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u, v) = (-1.9, -3.9)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (2.3167, -3.24125, 3.77511, 4.62016)$

2) $(x_1, x_2, x_3, x_4) \simeq (2.1167, -3.14125, 3.87511, 5.12016)$

3) $(x_1, x_2, x_3, x_4) \simeq (2.5167, -3.34125, 3.97511, 5.22016)$

4) $(x_1, x_2, x_3, x_4) \simeq (2.6167, -3.24125, 3.37511, 5.62016)$

5) $(x_1, x_2, x_3, x_4) \simeq (1.9167, -3.24125, 4.37511, 5.62016)$

Exercise 3

Given the function

$f(x, y, z) = 20 - 6x + x^2 - 2y + y^2 - 6z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {−2.92857, −1.24286, ?}

2) We have a maximum at {?, −1.04286, −1.48571}

3) We have a maximum at {?, 1, 3}

4) We have a maximum at {−3.42857, ?, −1.28571}

5) We have a maximum at {−3.62857, ?, −1.58571}

Exercise 4

Consider the domain $D_1 \equiv 9x^2 - 14xy + 9y^2 = 1$ and the point $q = (3, 0)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 closest to q is (∗, ∗∗∗.∗4∗∗)

2) The point of D_1 closest to q is (∗, ∗∗∗.∗1∗∗)

3) The point of D_1 closest to q is (∗, ∗∗∗.∗7∗∗)

4) The point of D_1 closest to q is (∗, ∗∗∗.∗6∗∗)

5) The point of D_1 closest to q is (∗, ∗∗∗.∗3∗∗)

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Exercise 1

Given the functions

$f(x, y, z) = (-3 + 3x + 3y - 3xy, 3xy - 2y^2)$

and

$g(u, v) = (-3 - 3u + 2u^2 + v^2, 1 + 2u + u^2 - v + uv - 3v^2, -2 - 3u - u^2 - 2v - uv - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, 3, 1)$.

1) 0.

2) 0.824605

3) 0.79519

4) 0.863055

5) 0.796673

Exercise 2

Given the system

$3xzu_4 == 24$
$-xy^2 + 3yzu_3 == -88$
$-2y^2z - u_3 == 63$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, z, u_1, u_2, u_3, u_4) = (4, 4, -2, -5, -1, 1, -1)$. Compute if possible the approximate values of (x, y, z) for $(u_1, u_2, u_3, u_4) = (-5.2, -1.2, 0.7, -0.8)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (4.21918, 3.80022, -2.19041)$

2) $(x, y, z) \simeq (4.81918, 4.40022, -1.99041)$

3) $(x, y, z) \simeq (4.61918, 3.90022, -2.09041)$

4) $(x, y, z) \simeq (5.11918, 4.20022, -1.59041)$

5) $(x, y, z) \simeq (5.01918, 3.40022, -1.59041)$

Exercise 3

Given the function

$f(x, y, z) = -8 - x^2 - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {0., 0., ?}

2) We have a minimum at {?, 0.3, −5.3}

3) We have a minimum at {−0.1, 0.3, ?}

4) We have a minimum at {−0.3, ?, −4.7}

5) We have a minimum at {0, 0, ?}

Exercise 4

Consider the domains $D_1 \equiv 224 - 58x + 9x^2 - 20y - 10xy + 8y^2 \leq 1$ and $D_2 \equiv 36 - 16x + 2x^2 + 24y - 4xy + 6y^2 \leq 100$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗9∗∗)

2) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗2∗∗)

3) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗4∗∗)

4) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗7∗∗)

5) The point of D_1 closest to D_2 is (∗, ∗∗∗.∗0∗∗)

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Exercise 1

Given the functions

$f(x,y,z)=(3x^2+y+xy+3y^2+2xz+yz-3z^2,\ 3y+xy+3yz-3z^2,\ 2+x-3z-2yz+2z^2)$

and

$g(u,v,w)=(2v,\ 2v+2vw,\ 3v-w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,-3,1)$.

1) -0.845936

2) -0.382309

3) 0.786738

4) -0.277177

5) $0.$

Exercise 2

Given the system

$2vxy-2u^2z-xz^2==-90$
$2u^2-2y^2+v^2z==2$
$-2v-3x^3-2xy+vxy+vxz==-352$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point $p=(x,y,z,u,v)=(5,-1,4,0,1)$. Compute if possible the approximate values of (x,y,z) for $(u,v)=(0.2,1.2)$ by means of the tangent affine function at p.

1) $(x,y,z)\simeq(4.9167,-1.86294,3.75175)$

2) $(x,y,z)\simeq(4.7167,-1.86294,3.55175)$

3) $(x,y,z)\simeq(5.3167,-1.26294,4.15175)$

4) $(x,y,z)\simeq(4.8167,-1.86294,4.05175)$

5) $(x,y,z)\simeq(5.0167,-1.36294,3.85175)$

Exercise 3

Given the function

$f(x, y, z) = 2 + x^2 - 4y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {0.1, ?, 0.4}

2) We have a maximum at {?, 2, 0}

3) We have a maximum at {-0.3, -3.3, ?}

4) We have a maximum at {-0.2, ?, 0.3}

5) We have a maximum at {0., -3., ?}

Exercise 4

Consider the domain $D_1 \equiv 9x^2 + 2xy + 9y^2 + 2xz + 4yz + 5z^2 = 1$ and the point $q = (7, -3, -2)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the point of D_1 where it is attained.

1) The diameter of $D_1 \cup \{q\}$ is ***.2***

2) The diameter of $D_1 \cup \{q\}$ is ***.1***

3) The diameter of $D_1 \cup \{q\}$ is ***.9***

4) The diameter of $D_1 \cup \{q\}$ is ***.5***

5) The diameter of $D_1 \cup \{q\}$ is ***.8***

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Exercise 1

Given the functions

$f(x, y, z) = (x y - 3 y z, -3 x z, -y - y z - 3 z^2)$

and

$g(u, v, w) = (-3 w - 2 u w - v w, -u - u v - v^2 + w^2, -w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, 2, -3)$.

1) -7.24214×10^7

2) -4.14391×10^7

3) -6.18549×10^7

4) -3.91929×10^7

5) -1.03097×10^8

Exercise 2

Given the system

$2 u v + 3 x + v^2 x + 3 v x z + 2 x y z - 2 x z^2 == -94$
$2 v x + 2 u y^2 + 2 x z == -38$
$-3 v x^2 - 2 y^2 + 2 v y z + 2 z^3 == -143$

determine if it is possible to solve for variables x, y, z in terms of variables u, v arround the point $p = (x, y, z, u, v) = (-3, 2, 0, -1, 5)$. Compute if possible the approximate values of (x, y, z) for $(u, v) = (-0.7, 5.1)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (-2.95051, 2.27126, 0.0208018)$

2) $(x, y, z) \simeq (-3.25051, 1.87126, -0.0791982)$

3) $(x, y, z) \simeq (-3.45051, 2.47126, 0.420802)$

4) $(x, y, z) \simeq (-2.75051, 2.57126, -0.179198)$

5) $(x, y, z) \simeq (-3.25051, 2.17126, 0.420802)$

Exercise 3

Given the function

$f(x, y, z) = 18 - 4x + x^2 - 6y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {2, ?, 0}

2) We have a maximum at {?, −2.49615, 0.}

3) We have a maximum at {−2.1641, ?, −0.2}

4) We have a maximum at {−2.0641, ?, 0.2}

5) We have a maximum at {−1.4641, −1.99615, ?}

Exercise 4

Consider the domain $D_1 \equiv 8x^2 - 12xy + 6y^2 + 6xz - 2yz + 3z^2 = 1$ and the point $q = (-4, -9, 2)$. Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The distance between D_1 and q is ***.**0*

2) The distance between D_1 and q is ***.**8*

3) The distance between D_1 and q is ***.**7*

4) The distance between D_1 and q is ***.**5*

5) The distance between D_1 and q is ***.**4*

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Exercise 1

Given the functions

$f(x, y, z) = (y - 3xy + z^2, -x^2 - 2y - z + 3xz - yz)$

and

$g(u, v) = (-2 - u - u^2 + 3v + 3uv + 2v^2, -1 + 2u + 2v, -3 - 3u^2 + 2v + 2uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (3, 1, 2)$.

1) 0.491047

2) −0.676149

3) 0.551395

4) 0.

5) 0.843479

Exercise 2

Given the system

$-3u^2 + 2u^2 y + uxy + 3y^3 == 427$

$3 + 2u^2 v + 2v^3 - 2u^2 x + 2uvx + uy + u^2 y == 45$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (3, 5, -4, 3)$. Compute if possible the approximate values of (x, y) for $(u, v) = (-3.8, 3.2)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (3.17088, 5.04742)$

2) $(x, y) \simeq (3.67088, 4.64742)$

3) $(x, y) \simeq (3.57088, 4.94742)$

4) $(x, y) \simeq (2.97088, 5.14742)$

5) $(x, y) \simeq (3.37088, 4.94742)$

Exercise 3

Given the function

$f(x, y, z) = 13 - 4x + x^2 - 2y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {2, 1, ?}

2) We have a maximum at {-2.04712, 2.11482, ?}

3) We have a maximum at {?, -2.03472, 5.07166}

4) We have a maximum at {-1.125, ?, 4.6106}

5) We have a maximum at {-2.04712, ?, 3.68848}

Exercise 4

Consider the domain $D_1 \equiv 6x^2 + 2xy + y^2 - 4xz + z^2 = 1$ and the point $q = (4, 8, 2)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the point of D_1 where it is attained.

1) The point of D_1 furthest from q is (*, *, ***.*4**)

2) The point of D_1 furthest from q is (*, *, ***.*5**)

3) The point of D_1 furthest from q is (*, *, ***.*3**)

4) The point of D_1 furthest from q is (*, *, ***.*0**)

5) The point of D_1 furthest from q is (*, *, ***.*6**)

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Exercise 1

Given the functions

$f(x, y) = (-3 - 2x + x^2 + 3y + 3xy + 2y^2, -2 - 2x^2 - 3y + xy - y^2)$

and

$g(u, v) = (1 + u^2 - 3v + 2uv + 2v^2, -2 - u - u^2 - 2v + uv - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, -1)$.

1) $-59\,532.$

2) $-49\,147.9$

3) $-85\,717.4$

4) $-68\,127.2$

5) -6453.01

Exercise 2

Given the system

$$3 - x + 3ux - 2x^2 + 3x^3 - 3y - 2uy + 2x^2y - xy^2 - 3y^3 == 185$$
$$-u + x + 2ux - x^2 - 3ux^2 + 2x^3 + uxy + y^2 - 3xy^2 - 3y^3 == 92$$

determine if it is possible to solve for variables x, y in terms of variable u arround the point $p = (x, y, u) = (2, -4, 2)$. Compute if possible $\frac{\partial y}{\partial u}(2)$.

1) $\frac{\partial y}{\partial u}(2) = \frac{207}{1097}$

2) $\frac{\partial y}{\partial u}(2) = \frac{205}{1097}$

3) $\frac{\partial y}{\partial u}(2) = \frac{209}{1097}$

4) $\frac{\partial y}{\partial u}(2) = \frac{206}{1097}$

5) $\frac{\partial y}{\partial u}(2) = \frac{208}{1097}$

Exercise 3

Given the function

$f(x, y, z) = 15 + x^2 - 2y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {0, ?, 2}

2) We have a minimum at {-0.783943, ?, 2.7438}

3) We have a minimum at {0., ?, 1.95986}

4) We have a minimum at {-0.391972, 0.604762, ?}

5) We have a minimum at {-0.587957, 0.800748, ?}

Exercise 4

Consider the domains $D_1 \equiv 5x^2 + 2xy + 3y^2 - 6xz + 6yz + 8z^2 = 1$ and $D_2 \equiv -2x - 5y - 2z = 12$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, *, ***.**2*)

2) The point of D_1 closest to D_2 is (*, *, ***.**0*)

3) The point of D_1 closest to D_2 is (*, *, ***.**1*)

4) The point of D_1 closest to D_2 is (*, *, ***.**6*)

5) The point of D_1 closest to D_2 is (*, *, ***.**4*)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (3 x_1 x_2 + 2 x_2^2 - x_1 x_3 + 3 x_2 x_4 - 3 x_3 x_4 - 2 x_4^2, 1 + x_2 + 2 x_2^2 - 2 x_3 - 3 x_3 x_4, -1 + x_1^2 - x_1 x_2 + 3 x_2^2 - 2 x_3 x_4)$

and

$g(u, v, w) = (-2 v w, 1 + 2 u^2 + 3 v - 3 w - 3 v w, u^2 + u v + 3 w, -3 v^2 + 2 v w + 3 w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, 0, -3, 1)$.

1) 0.

2) −0.372283

3) −0.161385

4) −0.306044

5) −0.505995

Exercise 2

Given the system

$2 y^2 z == -32$

$2 w x + x^2 - 3 y - x y + 3 u y^2 == 69$

$-w^2 z + x z^2 == -32$

determine if it is possible to solve for variables x, y, z in terms of variables u, v, w arround the point $p = (x, y, z, u, v, w) = (-3, -2, -4, 4, 0, -2)$. Compute if possible the approximate values of (x, y, z) for $(u, v, w) = (3.8, -0.3, -2.3)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (-2.99773, -2.07955, -3.68182)$

2) $(x, y, z) \simeq (-3.69773, -1.77955, -4.28182)$

3) $(x, y, z) \simeq (-2.99773, -1.57955, -4.28182)$

4) $(x, y, z) \simeq (-3.19773, -1.97955, -4.08182)$

5) $(x, y, z) \simeq (-2.79773, -1.77955, -3.98182)$

Exercise 3

Given the function

$f(x, y, z) = -9 + 4x - x^2 + 4y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-1.80058, ?, 0.0665775\}$

2) We have a minimum at $\{?, -4.54844, -0.333422\}$

3) We have a minimum at $\{-2.00058, -5.04844, ?\}$

4) We have a minimum at $\{?, -4.04844, -0.733422\}$

5) We have a minimum at $\{2, 2, ?\}$

Exercise 4

Consider the domain $D_1 \equiv 2x^2 - 4xy + 4y^2 = 1$ and the point $q = (3, 7)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is **5.****

2) The distance between D_1 and q is **8.****

3) The distance between D_1 and q is **7.****

4) The distance between D_1 and q is **0.****

5) The distance between D_1 and q is **6.****

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Exercise 1

Given the functions

$f(x,y)=(-3-2x-2x^2-y+2xy,\ -2+x-x^2+2y-xy-3y^2,\ -2+x+3x^2-3y-xy)$

and

$g(u,v,w)=(2u+3v,\ 2u^2+v^2+3w-3vw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3,-2)$.

1) $-38\,698.3$

2) $-89\,069.6$

3) $-60\,237.$

4) $-30\,306.5$

5) $-32\,552.1$

Exercise 2

Given the system

$$-3uxy-y^2+2uyz+z^3=-72$$
$$ux-3uy+2xy-2y^3-2z-2yz-uz^2+z^3=42$$
$$ux-2uy^2-3xy^2-2u^2z+2x^2z-3y^2z+2yz^2=78$$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point $p=(x,y,z,u)=(-5,-2,2,2)$. Compute if possible $\frac{\partial x}{\partial u}(2)$.

1) $\frac{\partial x}{\partial u}(2)=\frac{1271}{3332}$

2) $\frac{\partial x}{\partial u}(2)=\frac{75}{196}$

3) $\frac{\partial x}{\partial u}(2)=\frac{1273}{3332}$

4) $\frac{\partial x}{\partial u}(2)=\frac{13}{34}$

5) $\frac{\partial x}{\partial u}(2)=\frac{318}{833}$

Exercise 3

Given the function

$f(x, y, z) = 7 - 2x + x^2 - 6y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, -2.63975, 0.018001}

2) We have a maximum at {?, -2.43975, 0.218001}

3) We have a maximum at {1, ?, 1}

4) We have a maximum at {?, -2.63975, -0.081999}

5) We have a maximum at {?, -2.93975, -0.281999}

Exercise 4

Consider the domains $D_1 \equiv 9x^2 + 6xy + 7y^2 + 4xz - 4yz + 8z^2 = 1$ and $D_2 \equiv 4x + 4y + 2z = 8$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (*, ***.**3*, *)

2) The point of D_1 closest to D_2 is (*, ***.**2*, *)

3) The point of D_1 closest to D_2 is (*, ***.**0*, *)

4) The point of D_1 closest to D_2 is (*, ***.**1*, *)

5) The point of D_1 closest to D_2 is (*, ***.**7*, *)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (2x_4 + 2x_1x_4, 2x_2x_4 + 2x_4^2)$

and

$g(u, v) = (3 - u + 3u^2 - v + 3v^2, -1 - 3u - 3v + 2uv + v^2, 3 + 3u - 3u^2 - 3v - 3uv, 2u - u^2 - v + 2uv + 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1, 1, -3, 0)$.

1) 0.

2) −0.480076

3) 0.846983

4) 0.543528

5) 0.18014

Exercise 2

Given the system

$3u - 2v^2 + 3vx^2 - 2x^3 - 3vy^2 == 21$
$-2v - 2x + vx - 3y == 2$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (-1, 2, 3, -2)$. Compute if possible the approximate values of (x, y) for $(u, v) = (3.3, -1.7)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (-0.753846, 1.83846)$

2) $(x, y) \simeq (-1.15385, 2.23846)$

3) $(x, y) \simeq (-1.45385, 1.63846)$

4) $(x, y) \simeq (-1.05385, 2.33846)$

5) $(x, y) \simeq (-1.25385, 2.03846)$

Exercise 3

Given the function

$f(x, y, z) = 3 - 4x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-3.56142, ?, -1.78071\}$

2) We have a maximum at $\{-3.26142, 0.309389, ?\}$

3) We have a maximum at $\{?, -0.0906108, -1.58071\}$

4) We have a maximum at $\{2, ?, 1\}$

5) We have a maximum at $\{-3.66142, -0.390611, ?\}$

Exercise 4

Consider the domain $D_1 \equiv 7x^2 - 6xy + 8y^2 = 1$ and the point $q = (-4, 4)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ***.*7**

2) The distance between D_1 and q is ***.*4**

3) The distance between D_1 and q is ***.*6**

4) The distance between D_1 and q is ***.*3**

5) The distance between D_1 and q is ***.*1**

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (2 + x_1 x_2 + x_2 x_4 + 3 x_3 x_4,$
$-3 x_1 - 2 x_2 - 2 x_1 x_3, -3 x_1 + 3 x_2 + 3 x_3 - x_2 x_3 + 3 x_4 - 3 x_2 x_4 - 2 x_4^2)$

and

$g(u, v, w) = (v + 3 w - u w, -2 u v - w - 3 u w, -2 v w, u + 2 u w),$

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (1, 0, -2, 3)$.

1) 0.

2) −0.748087

3) −0.53668

4) 0.229645

5) −0.538147

Exercise 2

Given the system

$v w x + 2 u v y + 3 v y^2 == -4$
$u x z == -10$
$-u x^2 - 3 v x z == -14$

determine if it is possible to solve for variables x, y, z in terms of variables u, v, w arround the point $p = (x, y, z, u, v, w) = (-2, -2, 1, 5, 1, -2)$. Compute if possible the approximate values of (x, y, z) for $(u, v, w) = (4.8, 1.1, -2.3)$ by means of the tangent affine function at p.

1) $(x, y, z) \simeq (-1.882, -1.918, 0.699)$

2) $(x, y, z) \simeq (-2.482, -1.918, 1.099)$

3) $(x, y, z) \simeq (-2.382, -1.118, 0.699)$

4) $(x, y, z) \simeq (-2.482, -1.718, 1.299)$

5) $(x, y, z) \simeq (-2.082, -1.418, 0.999)$

Exercise 3

Given the function

$f(x, y, z) = -16 + 4x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.4, ?, 0.1}

2) We have a maximum at {1.1, 3.9, ?}

3) We have a maximum at {0.5, ?, −0.5}

4) We have a maximum at {?, 2.4, 1.3}

5) We have a maximum at {?, 3, 1}

Exercise 4

Consider the domain $D_1 \equiv 4x^2 + 6xy + 6y^2 = 1$ and the point $q = (5, 8)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 closest to q is (**0.****, *)

2) The point of D_1 closest to q is (**1.****, *)

3) The point of D_1 closest to q is (**8.****, *)

4) The point of D_1 closest to q is (**5.****, *)

5) The point of D_1 closest to q is (**9.****, *)

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Exercise 1

Given the functions

$f(x, y, z) = (3x^2 - z^2, -3x^2)$

and

$g(u, v) = (3 + 2u + 3u^2 - v - uv - v^2, 3 + u^2 + 3v - uv + 3v^2, -3 + u - u^2 + v + uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0, -1, -2)$.

1) 0.725226

2) 0.423731

3) 0.194263

4) 0.731916

5) 0.

Exercise 2

Given the system

$2x^2 y == 100$
$-3xy == -30$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (5, 2, 3, -3, -1, 2)$. Compute if possible the approximate values of (x, y) for $(u_1, u_2, u_3, u_4) = (3.2, -3.3, -1.1, 1.7)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (5.4, 2.1)$

2) $(x, y) \simeq (5.1, 2.5)$

3) $(x, y) \simeq (5.3, 1.9)$

4) $(x, y) \simeq (5.2, 2.1)$

5) $(x, y) \simeq (5., 2.)$

Exercise 3

Given the function

$f(x, y, z) = 1 + x^2 - 6y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.6, ?, 1.}

2) We have a minimum at {0.4, 3., ?}

3) We have a minimum at {0.6, 2.2, ?}

4) We have a minimum at {0., 2., ?}

5) We have a minimum at {?, 3, 0}

Exercise 4

Consider the domain $D_1 \equiv 9x^2 - 4xy + 5y^2 + 6xz + 6yz + 5z^2 = 1$ and the point $q = (1, 5, -8)$. Compute the distance between they two, $d(D_1, q)$, and the point of D_1 where it is attained.

1) The point of D_1 closest to q is (*, *, ***.*2**)

2) The point of D_1 closest to q is (*, *, ***.*1**)

3) The point of D_1 closest to q is (*, *, ***.*9**)

4) The point of D_1 closest to q is (*, *, ***.*6**)

5) The point of D_1 closest to q is (*, *, ***.*5**)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (2x_1^2 - 3x_1x_2 + 3x_3^2 + x_2x_4 - 3x_3x_4, -2x_1 + 2x_1^2 + x_2x_3 - 3x_2x_4)$

and

$g(u, v) = (-2 - 2u + 3v + 2uv + v^2, -2 - u + 3u^2 - 3v - 3uv, -3 + 2u - u^2 + v - 2uv - 3v^2, -2 + 3u + 3u^2 - 3v - uv - 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3, -2, 0, 1)$.

1) -0.270158

2) $0.$

3) -0.669117

4) 0.366499

5) 0.835783

Exercise 2

Given the system

$-2u^2v - 3x^3 + 2vy + 2v^2y + 2x^2y == -68$

$3ux^2 + 3uvy - 2uxy + 2uy^2 + 2vy^2 + xy^2 == -108$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (-2, -5, -4, 1)$. Compute if possible the approximate values of (x, y) for $(u, v) = (-4.1, 1.1)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (-4.38723, -3.55426)$

2) $(x, y) \simeq (-4.78723, -3.55426)$

3) $(x, y) \simeq (-4.48723, -3.35426)$

4) $(x, y) \simeq (-4.68723, -3.45426)$

5) $(x, y) \simeq (-4.98723, -3.05426)$

Exercise 3

Given the function

$f(x, y, z) = -13 + 6x - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {3.08417, ?, 1.23595}

2) We have a maximum at {3, ?, 2}

3) We have a maximum at {2.84693, ?, 2.89665}

4) We have a maximum at {2.60968, 1.22042, ?}

5) We have a maximum at {2.37244, ?, 1.71043}

Exercise 4

Consider the domain $D_1 \equiv 2x^2 + 4xy + 6y^2 = 1$ and the point $q = (4, -7)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is (*, ***.3***)

2) The point of D_1 furthest from q is (*, ***.0***)

3) The point of D_1 furthest from q is (*, ***.1***)

4) The point of D_1 furthest from q is (*, ***.9***)

5) The point of D_1 furthest from q is (*, ***.4***)

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Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (-x_2 + 3 x_1 x_2 + 2 x_1 x_3, -2 x_4 - x_3 x_4)$

and

$g(u, v) = (-1 - 3 u + 3 u^2 - v + v^2, 2 - 3 u + 3 u^2 + v - u v + v^2, 1 + 3 u + 2 u^2 + 2 v + u v - 2 v^2, -3 - u^2 + 3 v + u v - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2, 3, -1, -3)$.

1) 0.

2) 0.381352

3) 0.821117

4) 0.868054

5) -0.718233

Exercise 2

Given the system

$$-2 u^2 x_1 + 2 u x_1 x_4 + 3 x_1 x_3 x_4 == 135$$
$$2 x_3 - 3 x_1 x_3^2 - 2 u x_4 - x_3 x_4^2 == -460$$
$$3 x_1^2 x_3 == 375$$
$$2 x_2^2 x_3 - x_1 x_4 + 2 x_2 x_4 + 3 x_3^2 x_4 + 3 x_4^2 == 725$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u arround the point $p = (x_1, x_2, x_3, x_4, u) = (5, 5, 5, 5, -3)$. Compute if possible the approximate values of (x_1, x_2, x_3, x_4) for $(u) = (-3.3)$ by means of the tangent affine function at p.

1) $(x_1, x_2, x_3, x_4) \simeq (5.19416, 4.45083, 4.61168, 6.26404)$

2) $(x_1, x_2, x_3, x_4) \simeq (5.29416, 4.85083, 4.81168, 6.76404)$

3) $(x_1, x_2, x_3, x_4) \simeq (5.39416, 4.05083, 4.11168, 6.56404)$

4) $(x_1, x_2, x_3, x_4) \simeq (5.29416, 4.95083, 4.21168, 6.36404)$

5) $(x_1, x_2, x_3, x_4) \simeq (5.69416, 4.35083, 4.91168, 5.76404)$

Exercise 3

Given the function

$f(x, y, z) = -11 + 4x - x^2 - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1., 1., ?}

2) We have a maximum at {1.2, 0.4, ?}

3) We have a maximum at {2.2, -1., ?}

4) We have a maximum at {?, 0.4, 2.2}

5) We have a maximum at {?, 0, 2}

Exercise 4

Consider the domains $D_1 \equiv 41 - 18x + 9x^2 - 16y + 2y^2 \leq 1$ and $D_2 \equiv 68 - 40x + 8x^2 - 24y + 12xy + 5y^2 \leq 10$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The distance between both domains is ***.8***

2) The distance between both domains is ***.2***

3) The distance between both domains is ***.7***

4) The distance between both domains is ***.4***

5) The distance between both domains is ***.6***