

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 1

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-2 + x_2 x_4 + 2 x_3 x_4 + 2 x_4^2, -3 x_1^2 - x_1 x_2 + 3 x_3 + x_3^2 - 2 x_4)$$

and

$$g(u, v) = (-u - 3 u^2 + v - 3 u v, -1 - u^2 - 2 v + 3 u v + v^2, 3 - 3 u - u^2 + 3 v + 3 u v + v^2, -2 + 2 u^2 - v - u v - 2 v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (0, 1, 3, 3).$$

- 1) -0.473363
- 2) -0.800147
- 3) 0.213585
- 4) $0.$
- 5) -0.247397

Exercise 2

Given the system

$$-2 u^2 x_3 + 2 u x_4^2 = -42$$

$$3 u x_2 x_3 + x_2^2 x_3 = -16$$

$$u x_1 x_2 - u x_1 x_3 = -40$$

$$-3 u x_3 + 2 x_2^2 x_3 = -140$$

determine if it is possible to solve for variables $x_1,$

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (5, 4, -4, -5, -1)$. Compute if possible $\frac{\partial x_2}{\partial u}(-1)$.

- 1) $\frac{\partial x_2}{\partial u}(-1) = -\frac{144}{37}$
- 2) $\frac{\partial x_2}{\partial u}(-1) = -\frac{141}{37}$
- 3) $\frac{\partial x_2}{\partial u}(-1) = -\frac{143}{37}$
- 4) $\frac{\partial x_2}{\partial u}(-1) = -\frac{140}{37}$
- 5) $\frac{\partial x_2}{\partial u}(-1) = -\frac{142}{37}$

Exercise 3

Given the function

$f(x, y, z) = -7 + 4x - x^2 + 4y - y^2 - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 1.8, 1.\}$
- 2) We have a maximum at $\{?, 2.8, 1.\}$
- 3) We have a maximum at $\{?, 1.4, -0.4\}$
- 4) We have a maximum at $\{1.2, 1., ?\}$
- 5) We have a maximum at $\{2, ?, 0\}$

Further Mathematics - Degree in Engineering - 2024/2025
01-Multivariate Functions-Training computers exam for for
serial number: 2

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-1 - 3x_1 + 2x_3 + 2x_4^2, -x_1 + x_1^2 - 3x_1x_3 - 2x_4)$$

and

$$g(u, v) = (2u + 3u^2 - uv + 2v^2, -1 + u - 3u^2 + 2v + 2uv - 3v^2, -1 - u + v - 2uv + v^2, u + u^2 + 3v + 2uv + 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (1, 2, -1, 3).$$

- 1) 0.354536
- 2) 0.436994
- 3) 0.543751
- 4) -0.775218
- 5) 0.

Exercise 2

Given the system

$$3x_2 - ux_2^2 + 2x_1x_2x_4 = 65$$

$$-x_1x_2 + x_1^2x_3 + 3x_1x_3^2 - 3x_3x_4 = 105$$

$$2x_2^2 + 3x_1x_2^2 - 2x_1x_3x_4 = 145$$

$$2x_1x_3^2 = 50$$

determine if it is possible to solve for variables x_1 ,

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (1, -5, -5, 2, -4)$. Compute if possible $\frac{\partial x_2}{\partial u}(-4)$.

- 1) $\frac{\partial x_2}{\partial u}(-4) = -\frac{141}{289}$
- 2) $\frac{\partial x_2}{\partial u}(-4) = -\frac{144}{289}$
- 3) $\frac{\partial x_2}{\partial u}(-4) = -\frac{142}{289}$
- 4) $\frac{\partial x_2}{\partial u}(-4) = -\frac{145}{289}$
- 5) $\frac{\partial x_2}{\partial u}(-4) = -\frac{143}{289}$

Exercise 3

Given the function

$f(x, y, z) = -16 + 2x - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -5.06525, -0.255256\}$
- 2) We have a minimum at $\{?, 1, 3\}$
- 3) We have a minimum at $\{?, -5.06525, -0.355256\}$
- 4) We have a minimum at $\{?, -5.16525, -0.955256\}$
- 5) We have a minimum at $\{-1.1143, ?, -0.455256\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 3

Exercise 1

Given the functions

$$f(x, y, z) = (-2x - 2z, -2y + 2y^2 - xz - 3yz)$$

and

$$g(u, v) = (-2 - 3u^2 - v + 3uv - 3v^2, 1 + 2u + 3u^2 + v + uv, -3 - 3u + 2u^2 + 2v - 3uv + 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, -2, -2)$.

- 1) -0.164022
- 2) -0.877694
- 3) 0.572457
- 4) -0.19994
- 5) 0 .

Exercise 2

Given the system

$$2z + 2yz - 3uyz = -40$$

$$2u^2y - 2xz + 2yz^2 - z^3 = -32$$

$$3u^3 - 3uxy = 0$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (-2, -2, -4, 2)$. Compute if possible $\frac{\partial x}{\partial u}(2)$.

- 1) $\frac{\partial x}{\partial u}(2) = -\frac{47}{16}$
- 2) $\frac{\partial x}{\partial u}(2) = -\frac{49}{16}$
- 3) $\frac{\partial x}{\partial u}(2) = -\frac{25}{8}$
- 4) $\frac{\partial x}{\partial u}(2) = -3$
- 5) $\frac{\partial x}{\partial u}(2) = -\frac{51}{16}$

Exercise 3

Given the function

$f(x, y, z) = 7 - x^2 - y^2 + 2z - z^2$ defined over the domain $D = \left\{ (x, y, z) \mid \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0.2, 0.1, ?\}$
- 2) We have a maximum at $\{0.3, -0.5, ?\}$
- 3) We have a maximum at $\{-0.4, ?, 1.4\}$
- 4) We have a maximum at $\{0, 0, ?\}$
- 5) We have a maximum at $\{?, -0.5, 0.6\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for

serial number: 4

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-2 + 2x_2 - 3x_2x_3, 3x_1^2 + 3x_3 - 3x_1x_4 - 3x_2x_4 - 3x_4^2, x_2 + 3x_1x_3 - 3x_2x_3 + x_3^2, 2x_1 - 3x_1x_2 + 3x_1x_3 + 2x_4 - 2x_1x_4 - x_3x_4)$$

and

$$g(u_1, u_2, u_3, u_4) = (2 - 3u_2 + 2u_2u_4 + 2u_3u_4 - u_4^2, -u_1u_2 + u_1u_3 + 3u_3u_4, -u_2 + 3u_2^2 - 3u_2u_3 - 3u_1u_4, -3u_1 + u_2 - u_1u_2 - 3u_1u_3 + u_3^2 + u_4)$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, -2, 1, 0)$.

- 1) -537423 .
- 2) -3.71312×10^6
- 3) -2.01447×10^6
- 4) -3.84555×10^6
- 5) -2.41919×10^6

Exercise 2

Given the system

$$3wx_2x_4 = -9$$

$$-3wx_1^2 + 2vx_2^2 - 2u^2x_4 = 235$$

$$-3x_2x_3 + 3x_1x_3^2 + x_1x_2x_4 = 71$$

$$3x_1x_4^2 = 15$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u, v, w) = (5, -1, 2, -1, 2, 1, -3)$. Compute if possible $\frac{\partial x_3}{\partial w} (2, 1, -3)$.

$v, w) = (5, -1, 2, -1, 2, 1, -3)$. Compute if possible $\frac{\partial x_3}{\partial w} (2, 1, -3)$.

- 1) $\frac{\partial x_3}{\partial w} (2, 1, -3) = -\frac{1663}{7056}$
- 2) $\frac{\partial x_3}{\partial w} (2, 1, -3) = -\frac{79}{336}$
- 3) $\frac{\partial x_3}{\partial w} (2, 1, -3) = -\frac{1661}{7056}$
- 4) $\frac{\partial x_3}{\partial w} (2, 1, -3) = -\frac{277}{1176}$
- 5) $\frac{\partial x_3}{\partial w} (2, 1, -3) = -\frac{415}{1764}$

Exercise 3

Given the function

$f(x, y, z) = -26 + 6x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -2.9217, -0.437879\}$
- 2) We have a minimum at $\{?, -3.0217, -0.537879\}$
- 3) We have a minimum at $\{-2.4217, ?, -0.237879\}$
- 4) We have a minimum at $\{3, ?, 1\}$
- 5) We have a minimum at $\{-2.8217, -2.8217, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 5

Exercise 1

Given the functions

$$f(x, y) = (2x - 2x^2 + 3y + 3xy - 3y^2, -2x + x^2 + xy + y^2, 1 + 3x + x^2 + 3xy + y^2, 2x + 2x^2 - 3y - 3xy - 2y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (2u_1^2 + 2u_2 + 3u_1u_2 - u_3u_4, -u_2 - 2u_1u_2 - 3u_1u_3 - u_2u_3 + 3u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, -3).$$

- 1) -1.03439×10^6
- 2) -2.25482×10^6
- 3) $-400898.$
- 4) -1.54075×10^6
- 5) -2.53851×10^6

Exercise 2

Given the system

$$-2xy^2 = -32$$

$$-xu_1u_2 + 3xyu_4 = 56$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 around the point $p = (x, y, u_1, u_2, u_3, u_4,$

$$u_5) = (1, 4, 2, 2, 4, 5, 4). \text{ Compute if possible } \frac{\partial x}{\partial u_4} (2, 2, 4, 5, 4).$$

- 1) $\frac{\partial x}{\partial u_4} (2, 2, 4, 5, 4) = -\frac{4}{13}$
- 2) $\frac{\partial x}{\partial u_4} (2, 2, 4, 5, 4) = -\frac{5}{13}$
- 3) $\frac{\partial x}{\partial u_4} (2, 2, 4, 5, 4) = -\frac{6}{13}$
- 4) $\frac{\partial x}{\partial u_4} (2, 2, 4, 5, 4) = -\frac{2}{13}$
- 5) $\frac{\partial x}{\partial u_4} (2, 2, 4, 5, 4) = -\frac{3}{13}$

Exercise 3

Given the function

$f(x, y, z) = -12 + 6x - x^2 + 4y - y^2 - z^2$ defined over the domain $D =$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{3, 2, ?\}$
- 2) We have a maximum at $\{1.98714, 1.35519, ?\}$
- 3) We have a maximum at $\{2.52909, ?, 0.361299\}$
- 4) We have a maximum at $\{2.70974, 1.17454, ?\}$
- 5) We have a maximum at $\{?, 1.71649, 0.\}$

Further Mathematics - Degree in Engineering - 2024/2025
01-Multivariate Functions-Training computers exam for serial number: 6

Exercise 1

Given the functions

$$f(x, y) = (-3 - 3x + 3x^2 - 3y^2, 2 - 2x^2 + 2y - 2xy + 3y^2, 3 - 2x - 2x^2 - xy)$$

and

$$g(u, v, w) = (-2 - 2u^2 - 2uv + 2v^2, u + 3u^2 - 3v - uv - w + w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, -2)$.

- 1) -301284.
- 2) -166684.
- 3) -547890.
- 4) -114833.
- 5) -98917.4

Exercise 2

Given the system

$$-uv^2 - 2ux - vx^2 - wy^2 = -20$$

$$vx - 2y = 2$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u, v$

, $w) = (0, -1, 1, -5, -5)$. Compute if possible $\frac{\partial x}{\partial u}(1, -5, -5)$.

- 1) $\frac{\partial x}{\partial u}(1, -5, -5) = \frac{29}{23}$
- 2) $\frac{\partial x}{\partial u}(1, -5, -5) = \frac{28}{23}$
- 3) $\frac{\partial x}{\partial u}(1, -5, -5) = \frac{26}{23}$
- 4) $\frac{\partial x}{\partial u}(1, -5, -5) = \frac{27}{23}$
- 5) $\frac{\partial x}{\partial u}(1, -5, -5) = \frac{25}{23}$

Exercise 3

Given the function

$f(x, y, z) = 23 - 2x + x^2 - 6y + y^2 - 6z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-1.21914, -4.35741, ?\}$
- 2) We have a maximum at $\{-1.51914, ?, -0.831877\}$
- 3) We have a maximum at $\{-1.91914, -4.75741, ?\}$
- 4) We have a maximum at $\{?, 3, 3\}$
- 5) We have a maximum at $\{-1.11914, ?, -1.03188\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 7

Exercise 1

Given the functions

$$f(x, y, z) = (-1 - 2x^2, -3z + xz)$$

and

$$g(u, v) = (1 - u + 3u^2 - 2v + 3uv + 2v^2, -1 - u - 3u^2 - 3v - 2uv - 2v^2, 2 + 3u^2 + 3v - uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, -1, 0).$$

- 1) 0.786646
- 2) -0.552676
- 3) 0.
- 4) 0.144858
- 5) -0.687152

Exercise 2

Given the system

$$xz = 20$$

$$-3yu_3 + 3zu_4^2 = -90$$

$$3z + 3yzu_1 = -315$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4)$

$$p = (-4, 5, -5, 4, -1, 5, -1). \text{ Compute if possible } \frac{\partial x}{\partial u_1} (4, -1, 5, -1).$$

- 1) $\frac{\partial x}{\partial u_1} (4, -1, 5, -1) = -\frac{19}{17}$
- 2) $\frac{\partial x}{\partial u_1} (4, -1, 5, -1) = -1$
- 3) $\frac{\partial x}{\partial u_1} (4, -1, 5, -1) = -\frac{20}{17}$
- 4) $\frac{\partial x}{\partial u_1} (4, -1, 5, -1) = -\frac{18}{17}$
- 5) $\frac{\partial x}{\partial u_1} (4, -1, 5, -1) = -\frac{16}{17}$

Exercise 3

Given the function

$f(x, y, z) = 5 + x^2 - 6y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.552966, 3.04132, ?\}$
- 2) We have a minimum at $\{0., ?, 1.94058\}$
- 3) We have a minimum at $\{-1.10593, 3.59428, ?\}$
- 4) We have a minimum at $\{1.10593, ?, 2.77003\}$
- 5) We have a minimum at $\{0, ?, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 8

Exercise 1

Given the functions

$$f(x, y, z) = (2x - xy - xz, 2y + 3y^2 - 2xz)$$

and

$$g(u, v) = (3 - 3u + u^2 - 2v - 3uv - v^2, 2u - 3u^2 + 2v - 3uv - 3v^2, 3u + v + 3uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-2, -2, 3)$.

- 1) 0.553221
- 2) 0.
- 3) -0.812341
- 4) -0.448928
- 5) 0.829352

Exercise 2

Given the system

$$-3ux^2 - uy^2 + u^2z + 3z^2 = 127$$

$$-3u - 2y - 2y^2 + z^2 + 3z^3 = 402$$

$$-2uy^2 + xy^2 - 2uyz = -16$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (2, -2, 5, -2)$. Compute if possible $\frac{\partial y}{\partial u}(-2)$.

- 1) $\frac{\partial y}{\partial u}(-2) = \frac{6284}{817}$
- 2) $\frac{\partial y}{\partial u}(-2) = \frac{12569}{1634}$
- 3) $\frac{\partial y}{\partial u}(-2) = \frac{6285}{817}$
- 4) $\frac{\partial y}{\partial u}(-2) = \frac{12571}{1634}$
- 5) $\frac{\partial y}{\partial u}(-2) = \frac{12567}{1634}$

Exercise 3

Given the function

$f(x, y, z) = -19 + 2x - x^2 + 6y - y^2 - z^2$ defined over the domain $D =$

$\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{1, ?, 0\}$
- 2) We have a minimum at $\{-4.54108, -1.2555, ?\}$
- 3) We have a minimum at $\{-4.74108, ?, -0.3\}$
- 4) We have a minimum at $\{-4.64108, ?, 0.1\}$
- 5) We have a minimum at $\{-4.34108, -1.0555, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
01-Multivariate Functions-Training computers exam for for
serial number: 9

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (x_1 x_4 + 2 x_2 x_4, -x_1^2 - 2 x_3 + 2 x_2 x_3 - x_4 + x_1 x_4)$$

and

$$g(u, v) = (2 + 3u - 2v - 2uv - 3v^2, 3u - 2u^2 - v - 3uv + 3v^2, 3 + u + 2u^2 - v + 3uv + v^2, -2u + u^2 - 3v - 3uv),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (3, -1, 2, 1).$$

- 1) 0.648598
- 2) 0.453256
- 3) -0.362701
- 4) 0.
- 5) -0.403714

Exercise 2

Given the system

$$-3x_2^2 x_4 = 3$$

$$u^2 + u x_2^2 - 2x_2^3 - 2u x_1 x_3 - 2x_2^2 x_3 - 2x_1 x_3 x_4 - 2x_2 x_4^2 = -6$$

$$2u^2 - 2u x_2^2 + x_1 x_2^2 + 3u x_2 x_4 + x_3 x_4 + 2x_2 x_4^2 + x_4^3 = -5$$

$$-2x_1^2 x_2 - 3x_3^3 = 5$$

determine if it is possible to solve for variables x_1, x_2

, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2, x_3$

, $x_4, u) = (-2, -1, 1, -1, -1)$. Compute if possible $\frac{\partial x_4}{\partial u}(-1)$.

- 1) $\frac{\partial x_4}{\partial u}(-1) = \frac{134}{331}$
- 2) $\frac{\partial x_4}{\partial u}(-1) = \frac{136}{331}$
- 3) $\frac{\partial x_4}{\partial u}(-1) = \frac{135}{331}$
- 4) $\frac{\partial x_4}{\partial u}(-1) = \frac{137}{331}$
- 5) $\frac{\partial x_4}{\partial u}(-1) = \frac{133}{331}$

Exercise 3

Given the function

$f(x, y, z) = 19 + x^2 + y^2 - 6z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0.4, ?, -5.4\}$
- 2) We have a maximum at $\{0.2, ?, -5.4\}$
- 3) We have a maximum at $\{-0.5, -0.4, ?\}$
- 4) We have a maximum at $\{0., 0., ?\}$
- 5) We have a maximum at $\{0, 0, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 10

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-2x_2^2 - 3x_3 + 3x_2x_3 + 3x_2x_4 - 2x_3x_4, \\ 2 - 2x_1^2 - 2x_3^2 + x_1x_4 - 2x_2x_4, -x_1^2 + 3x_2 - 3x_1x_3 - 2x_1x_4)$$

and

$$g(u, v, w) = (-3uv - 2w^2, -u + 3uw + 2vw, 3u^2 - 2uv - 3w, 2 + 3v - 3v^2 + 2uw + 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (0, -1, -1, 0).$$

- 1) 0.
- 2) -0.16867
- 3) -0.856047
- 4) 0.362694
- 5) 0.315744

Exercise 2

Given the system

$$-3v^2 + 3vx_3 + vx_4 = -57$$

$$2x_1x_2 - 2vx_3 + 2ux_1x_4 = 56$$

$$-3x_1^3 + 3x_4^3 = -378$$

$$3ux_1^2 + 2uvx_4 = -66$$

determine if it is possible to solve for variables $x_1, x_2, x_3,$

x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$u, v) = (1, 3, 5, -5, -2, -3)$. Compute if possible $\frac{\partial x_1}{\partial v}(-2, -3)$.

- 1) $\frac{\partial x_1}{\partial v}(-2, -3) = \frac{127}{72}$
- 2) $\frac{\partial x_1}{\partial v}(-2, -3) = \frac{125}{72}$
- 3) $\frac{\partial x_1}{\partial v}(-2, -3) = \frac{7}{4}$
- 4) $\frac{\partial x_1}{\partial v}(-2, -3) = \frac{43}{24}$
- 5) $\frac{\partial x_1}{\partial v}(-2, -3) = \frac{16}{9}$

Exercise 3

Given the function

$f(x, y, z) = -6 + 2x - x^2 + 4y - y^2 + 2z - z^2$ defined over the domain $D = \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0.8, 2.4, ?\}$
- 2) We have a maximum at $\{2., 3., ?\}$
- 3) We have a maximum at $\{1, 2, ?\}$
- 4) We have a maximum at $\{1.2, ?, 0.2\}$
- 5) We have a maximum at $\{1.8, ?, 0.6\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 11

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (3x_1 - 2x_2 + 3x_1x_3 - 3x_3^2 + 2x_4 + 2x_3x_4, \\ 2x_1x_2 - 2x_2x_3 + 3x_1x_4 - 2x_4^2, 3x_2 - 3x_1x_2 + 3x_3)$$

and

$$g(u, v, w) = (1 - w, 3v^2 + uw + 3w^2, -uv - w + 3vw - 2w^2, -uv - w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (1, 2, 3, -2).$$

- 1) 0.141403
- 2) -0.701602
- 3) 0.
- 4) 0.349957
- 5) -0.688989

Exercise 2

Given the system

$$2x_1^2x_3 = -32$$

$$-3x_2^2 + vx_3 - 3vx_1x_4 + 2vx_4^2 = -56$$

$$-2x_2^2 + 3x_1x_2x_4 = 40$$

$$2ux_1x_2 - 2ux_3^2 - 2x_3x_4^2 = 120$$

determine if it is possible to solve for variables x_1, x_2, x_3

, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4$

, $u, v) = (2, 4, -4, 3, -3, 2)$. Compute if possible $\frac{\partial x_1}{\partial u}(-3, 2)$.

- 1) $\frac{\partial x_1}{\partial u}(-3, 2) = -\frac{50}{1069}$
- 2) $\frac{\partial x_1}{\partial u}(-3, 2) = -\frac{48}{1069}$
- 3) $\frac{\partial x_1}{\partial u}(-3, 2) = -\frac{46}{1069}$
- 4) $\frac{\partial x_1}{\partial u}(-3, 2) = -\frac{47}{1069}$
- 5) $\frac{\partial x_1}{\partial u}(-3, 2) = -\frac{49}{1069}$

Exercise 3

Given the function

$f(x, y, z) = 20 - 4x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, -1.15324, -0.878188\}$
- 2) We have a maximum at $\{?, 2, 3\}$
- 3) We have a maximum at $\{?, -1.65324, -0.678188\}$
- 4) We have a maximum at $\{?, -1.75324, -1.07819\}$
- 5) We have a maximum at $\{?, -1.55324, -0.978188\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 12

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (x_1^2 - 3x_2, -2 - 2x_1 + 3x_1^2 - x_1x_3 - 2x_3^2)$$

and

$$g(u, v) = (-3 - 2u - 2u^2 + 2v - uv - v^2, 3 - u^2 + 2v - 3v^2, 2 - u^2 - 2v - uv - 3v^2, -3 - u^2 - 2v + 3uv + v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, -2, 3, 0).$$

- 1) 0.746848
- 2) 0.
- 3) 0.384822
- 4) -0.480347
- 5) -0.685713

Exercise 2

Given the system

$$-u x_3^2 + 3 x_3^2 x_4 = -162$$

$$x_2 x_3^2 - 3 x_2 x_3 x_4 - x_3^2 x_4 = 81$$

$$2 x_1 - 3 x_2 x_3 - u^2 x_4 + x_1 x_4 + u x_1 x_4 = 18$$

$$x_1^2 + x_3^3 = -26$$

determine if it is possible to solve for variables x_1 ,

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (1, -1, -3, -5, 3)$. Compute if possible $\frac{\partial x_3}{\partial u}(3)$.

- 1) $\frac{\partial x_3}{\partial u}(3) = -\frac{259}{3291}$
- 2) $\frac{\partial x_3}{\partial u}(3) = -\frac{86}{1097}$
- 3) $\frac{\partial x_3}{\partial u}(3) = -\frac{87}{1097}$
- 4) $\frac{\partial x_3}{\partial u}(3) = -\frac{262}{3291}$
- 5) $\frac{\partial x_3}{\partial u}(3) = -\frac{260}{3291}$

Exercise 3

Given the function

$f(x, y, z) = -10 - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 0.8, 2.2\}$
- 2) We have a maximum at $\{-0.8, 2., ?\}$
- 3) We have a maximum at $\{?, 2., 1.6\}$
- 4) We have a maximum at $\{-1., ?, 1.4\}$
- 5) We have a maximum at $\{0, 1, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 13

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (3x_2 + 3x_2^2 + x_1x_3 + 2x_3^2, 3 + 2x_1 + 3x_1^2 + 3x_2^2 - x_3^2 + x_1x_4 - 3x_2x_4, 2x_1^2 - x_2x_3 - 2x_4^2)$$

and

$$g(u, v, w) = (-3u^2 + 2v, 3 - 2v - uw, 3u^2 + w - 3uw, -v - 3uv + 3uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, -1, -3, -2).$$

- 1) 0.343582
- 2) -0.291923
- 3) 0.200508
- 4) 0.860313
- 5) 0.

Exercise 2

Given the system

$$3ux_3 + 3u^2x_4 = -78$$

$$2uvx_1 + 3vx_1^2 + vx_3 - ux_3^2 - 3x_2x_4 = -158$$

$$2ux_4 + 3vx_1x_4 = 208$$

$$2x_1^3 - 2x_2^2x_4 = 328$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$$u, v) = (4, -5, 5, -4, -2, -4). \text{ Compute if possible } \frac{\partial x_2}{\partial u}(-2, -4).$$

- 1) $\frac{\partial x_2}{\partial u}(-2, -4) = \frac{4433}{1894}$
- 2) $\frac{\partial x_2}{\partial u}(-2, -4) = \frac{8863}{3788}$
- 3) $\frac{\partial x_2}{\partial u}(-2, -4) = \frac{8867}{3788}$
- 4) $\frac{\partial x_2}{\partial u}(-2, -4) = \frac{8865}{3788}$
- 5) $\frac{\partial x_2}{\partial u}(-2, -4) = \frac{2216}{947}$

Exercise 3

Given the function

$f(x, y, z) = 15 - 6x + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D =$

$\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{2.66176, ?, 2.51474\}$
- 2) We have a minimum at $\{3, 2, ?\}$
- 3) We have a minimum at $\{4.14051, 3.13909, ?\}$
- 4) We have a minimum at $\{1.7745, 3.43484, ?\}$
- 5) We have a minimum at $\{2.95751, 1.95609, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for

serial number: 14

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-3x_1 + 2x_2 + x_1x_2 - 2x_3 - 3x_3^2 - 3x_2x_4 - 3x_3x_4, -2 - x_2^2 - 3x_1x_3 - 3x_2x_3 - 3x_1x_4, -1 - x_1^2 - 3x_1x_2 - 2x_3 + x_2x_3 - 2x_3^2 - 2x_4 + 2x_2x_4 - x_4^2, 2x_2 + 3x_4 - 3x_1x_4 + 3x_3x_4 - 3x_4^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (3u_1 - u_2 + u_1u_4 + 2u_4^2, 2u_1 + u_1u_2 + u_3 - 2u_2u_4, 3u_1^2 + 2u_2 - u_2u_3 - 3u_3^2 + 2u_4, u_1 + 2u_1^2 + 3u_2^2 - u_2u_3 + 3u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, 3, 3, -1).$$

- 1) -1.62579×10^{12}
- 2) -1.47145×10^{12}
- 3) -4.55039×10^{11}
- 4) -4.40679×10^{11}
- 5) -2.12292×10^{11}

Exercise 2

Given the system

$$v x_1 x_3 = 30$$

$$-v^2 w + 2 x_3^2 x_4 = 20$$

$$2 u x_3^2 - 2 x_1 x_4 = -20$$

$$3 v^2 x_2 + 2 v x_1 x_3 + x_2 x_3 x_4 = 60$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u, v,$

$w) = (5, 0, -2, -2, -5, -3, -4)$. Compute if possible $\frac{\partial x_1}{\partial v}(-5, -3, -4)$.

- 1) $\frac{\partial x_1}{\partial v}(-5, -3, -4) = \frac{5}{2}$
- 2) $\frac{\partial x_1}{\partial v}(-5, -3, -4) = 3$
- 3) $\frac{\partial x_1}{\partial v}(-5, -3, -4) = \frac{9}{2}$
- 4) $\frac{\partial x_1}{\partial v}(-5, -3, -4) = 4$
- 5) $\frac{\partial x_1}{\partial v}(-5, -3, -4) = \frac{7}{2}$

Exercise 3

Given the function

$f(x, y, z) = 2 + 2x - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2., ?, 1.8\}$
- 2) We have a maximum at $\{?, 1.6, 1.6\}$
- 3) We have a maximum at $\{1, ?, 2\}$
- 4) We have a maximum at $\{1.8, ?, 2.4\}$
- 5) We have a maximum at $\{0.4, ?, 3.\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 15

Exercise 1

Given the functions

$$f(x, y, z) = (2y^2 - z + 2xz + yz, x - 2y + 3xy + 2xz, x^2 - xy - 2xz - yz, 3x^2 + 3y^2 + z + 3xz)$$

and

$$g(u_1, u_2, u_3, u_4) = (-3u_2^2 + u_3 - u_1u_3 - u_2u_3, 2 - 2u_1 + u_2^2 - 2u_3 + 3u_1u_3, 2u_1 - 3u_2 - 3u_2u_3 - 3u_3u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (3, -2, 3).$$

1) 6.84277×10^6

2) 4.27916×10^6

3) 6.95813×10^6

4) 3.75064×10^6

5) 6.04812×10^6

Exercise 2

Given the system

$$y^2 z = 16$$

$$-3v^2 + 2v^2 x + 2y = -283$$

$$u + vy - 2wx - y - uwz = -126$$

determine if it is possible to solve for variables x, y, z

in terms of variables u, v, w around the point $p = (x, y, z, u,$

$v, w) = (-4, -4, 1, 5, 5, 3)$. Compute if possible $\frac{\partial x}{\partial w}(5, 5, 3)$.

1) $\frac{\partial x}{\partial w}(5, 5, 3) = -\frac{73}{1027}$

2) $\frac{\partial x}{\partial w}(5, 5, 3) = -\frac{70}{1027}$

3) $\frac{\partial x}{\partial w}(5, 5, 3) = -\frac{72}{1027}$

4) $\frac{\partial x}{\partial w}(5, 5, 3) = -\frac{74}{1027}$

5) $\frac{\partial x}{\partial w}(5, 5, 3) = -\frac{71}{1027}$

Exercise 3

Given the function

$f(x, y, z) = 16 + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-0.1, -1.30648, ?\}$
- 2) We have a maximum at $\{0., -1.60648, ?\}$
- 3) We have a maximum at $\{-0.2, ?, -4.67903\}$
- 4) We have a maximum at $\{-0.3, -1.10648, ?\}$
- 5) We have a maximum at $\{0, 2, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 16

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (3x_1x_3 + x_4 + 2x_2x_4 + 2x_3x_4, x_1x_3 + 2x_2x_3, 3x_1x_2 + 3x_3 - x_3^2 - 3x_4)$$

and

$$g(u, v, w) = (-2w - 3vw, -3v + 3uv, 3uv, 3 - u + 3w + 2uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (0, 1, -3, -3).$$

- 1) 0.
- 2) 0.107575
- 3) -0.321766
- 4) 0.686089
- 5) -0.858098

Exercise 2

Given the system

$$3v^3 + 2x_2x_3^2 = -8$$

$$-3x_1^2x_2 = 48$$

$$-x_1x_2 - 2x_2x_3^2 = 28$$

$$-3x_1x_3 - 3x_3^2x_4 = 48$$

determine if it is possible to solve for variables x_1, x_2, x_3

, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3,$

$x_4, u, v) = (-4, -1, 4, 0, 5, 2)$. Compute if possible $\frac{\partial x_1}{\partial v}(5, 2)$.

- 1) $\frac{\partial x_1}{\partial v}(5, 2) = 37$
- 2) $\frac{\partial x_1}{\partial v}(5, 2) = 39$
- 3) $\frac{\partial x_1}{\partial v}(5, 2) = 36$
- 4) $\frac{\partial x_1}{\partial v}(5, 2) = 38$
- 5) $\frac{\partial x_1}{\partial v}(5, 2) = 40$

Exercise 3

Given the function

$f(x, y, z) = 8 - 2x + x^2 - 6y + y^2 - 2z + z^2$ defined over the domain $D =$

$\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-3.35825, -2.91894, ?\}$
- 2) We have a maximum at $\{?, -2.41894, 0.0161122\}$
- 3) We have a maximum at $\{-3.75825, -2.51894, ?\}$
- 4) We have a maximum at $\{1, ?, 1\}$
- 5) We have a maximum at $\{?, -3.31894, 0.0161122\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 17

Exercise 1

Given the functions

$$f(x, y) = (-2 + 3x - 2x^2 + y - xy - 2y^2, -2 + 2x - 3x^2 - 2y - 3xy - 3y^2)$$

and

$$g(u, v) = (-2u - 3u^2 + 2v - 3uv + 2v^2, 3u^2 + 3v + 2uv + v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, -2)$.

- 1) -12740.3
- 2) -17553.5
- 3) -31429.
- 4) -48747.1
- 5) -26048.

Exercise 2

Given the system

$$-3u^3 + 2v^2 + vx + v^2x - 3x^2 - 2u^2y - 2x^2y = -309$$

$$-3u + u^3 - v^2x + 2x^2 - 3xy + 2vxy = 124$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-3, 1, 4, 5)$. Compute if possible $\frac{\partial y}{\partial v}(4, 5)$.

- 1) $\frac{\partial y}{\partial v}(4, 5) = \frac{35}{92}$
- 2) $\frac{\partial y}{\partial v}(4, 5) = \frac{37}{92}$
- 3) $\frac{\partial y}{\partial v}(4, 5) = \frac{39}{92}$
- 4) $\frac{\partial y}{\partial v}(4, 5) = \frac{19}{46}$
- 5) $\frac{\partial y}{\partial v}(4, 5) = \frac{9}{23}$

Exercise 3

Given the function

$f(x, y, z) = 15 + x^2 - 6y + y^2 - 4z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 3.47409, 1.34113\}$
- 2) We have a minimum at $\{0.289507, 3.18458, ?\}$
- 3) We have a minimum at $\{0, 3, ?\}$
- 4) We have a minimum at $\{0., 2.89507, ?\}$
- 5) We have a minimum at $\{-0.579015, 3.7636, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for serial number: 18

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 + 2x_4, 3x_1 - 3x_1^2 + 3x_3 - x_1x_3 - 3x_2x_3 - x_3^2 - 2x_1x_4 + 2x_4^2)$$

and

$$g(u, v) = (-2 + u^2 - 2v + 2v^2, 3 - 3u + 3u^2 + 2v + uv - v^2, 3 - u + 2u^2 + v + uv - v^2, -3 + 2u + 3u^2 - 3v - 2uv - 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, 3, 2, -1).$$

- 1) 0.491201
- 2) 0.420936
- 3) -0.371159
- 4) -0.514105
- 5) 0.

Exercise 2

Given the system

$$-3u - 3ux_2^2 - 2ux_1x_3 - 2x_2^2x_4 = 4$$

$$-ux_3x_4 + x_3^2x_4 + 2x_4^3 = 60$$

$$-2x_3^2 + x_3^3 + 3u^2x_4 - ux_4^2 = 2$$

$$u - 2x_1x_2x_3 = -5$$

determine if it is possible to solve for variables x_1 ,

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (-1, 1, -2, 3, -1)$. Compute if possible $\frac{\partial x_2}{\partial u}(-1)$.

- 1) $\frac{\partial x_2}{\partial u}(-1) = -\frac{1104}{1201}$
- 2) $\frac{\partial x_2}{\partial u}(-1) = -\frac{2211}{2402}$
- 3) $\frac{\partial x_2}{\partial u}(-1) = -\frac{2209}{2402}$
- 4) $\frac{\partial x_2}{\partial u}(-1) = -\frac{1105}{1201}$
- 5) $\frac{\partial x_2}{\partial u}(-1) = -\frac{2207}{2402}$

Exercise 3

Given the function

$f(x, y, z) = 9 - 4x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-2.14149, -0.148462, ?\}$
- 2) We have a maximum at $\{-1.74149, ?, -3.90674\}$
- 3) We have a maximum at $\{-2.34149, ?, -4.00674\}$
- 4) We have a maximum at $\{2, 1, ?\}$
- 5) We have a maximum at $\{-2.04149, -0.0484624, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 19

Exercise 1

Given the functions

$$f(x, y) = (-2 + x - x^2 + 3y - 3y^2, 2 - x^2 - 3y + y^2, 1 - 3x + x^2 - 2y - 2xy - y^2)$$

and

$$g(u, v, w) = (v^2 - uw, 2u^2 + v),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-2, 3)$.

- 1) -499439.
- 2) -327431.
- 3) -613758.
- 4) -538148.
- 5) -329455.

Exercise 2

Given the system

$$vx^2 - w^2y = -48$$

$$-v - w^3 - 3ux - wx^2 - 3x^3 - wy + 2vwy - 2y^2 = -149$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u, v$

, $w) = (4, 0, 0, -3, -2)$. Compute if possible $\frac{\partial y}{\partial u}(0, -3, -2)$.

- 1) $\frac{\partial y}{\partial u}(0, -3, -2) = \frac{19}{53}$
- 2) $\frac{\partial y}{\partial u}(0, -3, -2) = \frac{22}{53}$
- 3) $\frac{\partial y}{\partial u}(0, -3, -2) = \frac{18}{53}$
- 4) $\frac{\partial y}{\partial u}(0, -3, -2) = \frac{20}{53}$
- 5) $\frac{\partial y}{\partial u}(0, -3, -2) = \frac{21}{53}$

Exercise 3

Given the function

$f(x, y, z) = -21 - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain $D =$

$\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0., 1.72913, ?\}$
- 2) We have a maximum at $\{0.345827, ?, 2.37212\}$
- 3) We have a maximum at $\{-0.172913, ?, 1.68047\}$
- 4) We have a maximum at $\{?, 3, 2\}$
- 5) We have a maximum at $\{0.51874, 1.38331, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for serial number: 20

Exercise 1

Given the functions

$$f(x, y) = (-3 - 3x + 3x^2 + 3xy + 3y^2, 3 + x + 2x^2 + y + 2xy + 2y^2, 3 - 2x + 2x^2 + 3y + 3xy - 3y^2)$$

and

$$g(u, v, w) = (-3u^2 - 2w + vw - w^2, uv + 3w - uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 0)$.

- 1) 252.571
- 2) 3284.37
- 3) 3230.66
- 4) 886.537
- 5) 2034.

Exercise 2

Given the system

$$w^2x - 2u^2y + 3y^3 = 378$$

$$3v^2 + 2u^2w - 2vw + 3v^2x - 3uwx + 3vwx + 3uxy - 3wy^2 + y^3 = 364$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u,$

$v, w) = (3, 5, 0, 4, -1)$. Compute if possible $\frac{\partial x}{\partial v}(0, 4, -1)$.

- 1) $\frac{\partial x}{\partial v}(0, 4, -1) = -\frac{1335}{533}$
- 2) $\frac{\partial x}{\partial v}(0, 4, -1) = -\frac{1332}{533}$
- 3) $\frac{\partial x}{\partial v}(0, 4, -1) = -\frac{1331}{533}$
- 4) $\frac{\partial x}{\partial v}(0, 4, -1) = -\frac{1334}{533}$
- 5) $\frac{\partial x}{\partial v}(0, 4, -1) = -\frac{1333}{533}$

Exercise 3

Given the function

$f(x, y, z) = 22 + x^2 - 6y + y^2 - 6z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-1.02262, ?, 0.645075\}$
- 2) We have a minimum at $\{0.409049, ?, 0.645075\}$
- 3) We have a minimum at $\{-0.409049, 1.22715, ?\}$
- 4) We have a minimum at $\{0., ?, 1.46317\}$
- 5) We have a minimum at $\{?, 3, 3\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 21

Exercise 1

Given the functions

$$f(x, y, z) = (3y - 3y^2 + 2z, 2 + 3x^2, 3y^2 + 2z)$$

and

$$g(u, v, w) = (3 - uv, 2v + 3w, -2w + 2vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 2, -1)$.

- 1) -0.563772
- 2) 0 .
- 3) 0.511831
- 4) -0.842005
- 5) -0.760106

Exercise 2

Given the system

$$x^2 y + 3u_1^2 u_2 = -32$$

$$2z u_3 - x^2 u_4 = 32$$

$$-2x u_1^2 - z^2 u_3 + y u_1 u_4 = 108$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4$

$) = (-4, 1, 2, 4, -1, 4, -1)$. Compute if possible $\frac{\partial z}{\partial u_4} (4, -1, 4, -1)$.

- 1) $\frac{\partial z}{\partial u_4} (4, -1, 4, -1) = \frac{38}{25}$
- 2) $\frac{\partial z}{\partial u_4} (4, -1, 4, -1) = \frac{36}{25}$
- 3) $\frac{\partial z}{\partial u_4} (4, -1, 4, -1) = \frac{39}{25}$
- 4) $\frac{\partial z}{\partial u_4} (4, -1, 4, -1) = \frac{8}{5}$
- 5) $\frac{\partial z}{\partial u_4} (4, -1, 4, -1) = \frac{37}{25}$

Exercise 3

Given the function

$f(x, y, z) = 15 + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 1.94058, 2.76483\}$
- 2) We have a minimum at $\{?, 2, 3\}$
- 3) We have a minimum at $\{?, 2.21706, 1.38242\}$
- 4) We have a minimum at $\{?, 1.11113, 2.48835\}$
- 5) We have a minimum at $\{0.276483, ?, 1.93538\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 22

Exercise 1

Given the functions

$$f(x, y) = (1 - 3x + 2x^2 - y - xy - 2y^2, 3 + 2x - 3x^2 + 2y - y^2, 2x - 3x^2 + 2y - xy + y^2)$$

and

$$g(u, v, w) = (3v - 3uv - w + w^2, 2v^2 - 3w),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 0)$.

- 1) 58.6481
- 2) 436.879
- 3) 324.
- 4) 526.747
- 5) 517.258

Exercise 2

Given the system

$$\begin{aligned} x^2 u_4 + 2y u_1 u_4 + 3x u_4^2 &= 160 \\ -3xy &= -18 \end{aligned}$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4)$

$= (3, 2, 1, -5, -2, -5)$. Compute if possible $\frac{\partial x}{\partial u_3} (1, -5, -2, -5)$.

- 1) $\frac{\partial x}{\partial u_3} (1, -5, -2, -5) = 1$
- 2) $\frac{\partial x}{\partial u_3} (1, -5, -2, -5) = 4$
- 3) $\frac{\partial x}{\partial u_3} (1, -5, -2, -5) = 2$
- 4) $\frac{\partial x}{\partial u_3} (1, -5, -2, -5) = 0$
- 5) $\frac{\partial x}{\partial u_3} (1, -5, -2, -5) = 3$

Exercise 3

Given the function

$f(x, y, z) = 2 - 6x + x^2 + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{2.35881, 0.589703, ?\}$
- 2) We have a minimum at $\{2.55538, -0.196568, ?\}$
- 3) We have a minimum at $\{2.16224, -0.786271, ?\}$
- 4) We have a minimum at $\{3, 0, ?\}$
- 5) We have a minimum at $\{1.96568, 0., ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 23

Exercise 1

Given the functions

$$f(x, y) = (x + 3y - y^2, -1 - 3x - y - xy + y^2, -3 - x + x^2 + 3y + 2xy - 3y^2, 3x - x^2 + y - 2xy)$$

and

$$g(u_1, u_2, u_3, u_4) = (-2u_1^2 - u_2 - 2u_1u_2 + 3u_2^2 + 3u_3 + 3u_1u_3 - u_4 + u_4^2, -2u_3 + 3u_4 - 3u_1u_4 - 3u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-2, 2)$.

- 1) -11776.3
- 2) -3268.54
- 3) -19522.
- 4) -35613.4
- 5) -5745.25

Exercise 2

Given the system

$$-2xyu_1 - 2yu_2 - u_2^3 = -6$$

$$2xu_1^2 - 2yu_4^2 - u_2u_5^2 = 24$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 around the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (3, -1, -1, 0, -1, 3, 3)$. Compute if possible $\frac{\partial y}{\partial u_3}(-1, 0, -1, 3, 3)$.

- 1) $\frac{\partial y}{\partial u_3}(-1, 0, -1, 3, 3) = 2$
- 2) $\frac{\partial y}{\partial u_3}(-1, 0, -1, 3, 3) = 4$
- 3) $\frac{\partial y}{\partial u_3}(-1, 0, -1, 3, 3) = 1$
- 4) $\frac{\partial y}{\partial u_3}(-1, 0, -1, 3, 3) = 0$
- 5) $\frac{\partial y}{\partial u_3}(-1, 0, -1, 3, 3) = 3$

Exercise 3

Given the function

$f(x, y, z) = -16 + 6x - x^2 + 4y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.484209, -4.14681, ?\}$
- 2) We have a minimum at $\{-0.0842086, -3.84681, ?\}$
- 3) We have a minimum at $\{-0.584209, -3.74681, ?\}$
- 4) We have a minimum at $\{-0.0842086, ?, -0.0791279\}$
- 5) We have a minimum at $\{3, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 24

Exercise 1

Given the functions

$$f(x, y, z) = (2 - 3z + z^2, -2x + x^2 - xy - y^2 - 3z + 3xz - 3z^2, -3 - 2y - 3xy + 2z^2)$$

and

$$g(u, v, w) = (3 - 3v + 2uw + 3vw + 2w^2, 3u + v - v^2 - w, -2w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 2, -1)$.

- 1) -22819.9
- 2) -57380.2
- 3) -17081.1
- 4) -36000.
- 5) -49789.9

Exercise 2

Given the system

$$x^2 z = 4$$

$$y u_2 + 2 x u_4^2 = 4$$

$$-x z^2 = 16$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4$

$) = (-1, -3, 4, 0, -2, 3, 1)$. Compute if possible $\frac{\partial y}{\partial u_2} (0, -2, 3, 1)$.

- 1) $\frac{\partial y}{\partial u_2} (0, -2, 3, 1) = -\frac{1}{2}$
- 2) $\frac{\partial y}{\partial u_2} (0, -2, 3, 1) = -1$
- 3) $\frac{\partial y}{\partial u_2} (0, -2, 3, 1) = -\frac{3}{2}$
- 4) $\frac{\partial y}{\partial u_2} (0, -2, 3, 1) = \frac{1}{2}$
- 5) $\frac{\partial y}{\partial u_2} (0, -2, 3, 1) = 0$

Exercise 3

Given the function

$f(x, y, z) = -31 + 4x - x^2 + 6y - y^2 + 6z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.563523, -4.70459, ?\}$
- 2) We have a minimum at $\{?, -4.20459, -1.04529\}$
- 3) We have a minimum at $\{?, -4.90459, -0.345285\}$
- 4) We have a minimum at $\{?, -4.60459, -0.545285\}$
- 5) We have a minimum at $\{2, 3, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 25

Exercise 1

Given the functions

$$f(x, y, z) = (-3y + 2y^2 - 2z + z^2, 1 - x - 3x^2 + 3yz)$$

and

$$g(u, v) = (2 + u - 3u^2 + v - 3uv, 1 - u - 3v - 2uv - v^2, -3 - u + 3v - 2uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, -2, 1)$.

- 1) 0.840211
- 2) 0.
- 3) -0.581992
- 4) 0.515225
- 5) 0.613692

Exercise 2

Given the system

$$-u^3 - x + ux - 3y^3 + 3uyz = 197$$

$$2y^2 + 2xy^2 - 2y^3 + 2xz = -20$$

$$2uy^2 - 3yz + uyz + 2yz^2 = -8$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (-5, -4, 2, 0)$. Compute if possible $\frac{\partial x}{\partial u}(0)$.

- 1) $\frac{\partial x}{\partial u}(0) = \frac{258}{1739}$
- 2) $\frac{\partial x}{\partial u}(0) = \frac{257}{1739}$
- 3) $\frac{\partial x}{\partial u}(0) = \frac{260}{1739}$
- 4) $\frac{\partial x}{\partial u}(0) = \frac{7}{47}$
- 5) $\frac{\partial x}{\partial u}(0) = \frac{261}{1739}$

Exercise 3

Given the function

$f(x, y, z) = 31 - 6x + x^2 - 6y + y^2 - 4z + z^2$ defined over the domain $D =$

$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{3, 3, ?\}$
- 2) We have a maximum at $\{?, -0.788442, -0.525628\}$
- 3) We have a maximum at $\{-2.24191, ?, -0.625628\}$
- 4) We have a maximum at $\{-2.44191, -1.28844, ?\}$
- 5) We have a maximum at $\{-2.14191, -1.08844, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 26

Exercise 1

Given the functions

$$f(x, y, z) = (3y, x^2 - z, -z + 3xz + 2yz)$$

and

$$g(u, v, w) = (2 - 3u - v + 3uw, -3u - 2u^2 - 2v - 3vw, 2uv + v^2 + 2uw - 3vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, -2, -2)$.

- 1) -293746.
- 2) -382008.
- 3) -344790.
- 4) -231516.
- 5) -77570.1

Exercise 2

Given the system

$$-u^2v - 2vy + 3uxz = -467$$

$$v^2 - uv^2 - x^2 + 3y^2 + 2z - 3x^2z = -291$$

$$3xy - uy^2 = -10$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (5, -1, 5, -5, 4)$. Compute if possible $\frac{\partial z}{\partial v}(-5, 4)$.

- 1) $\frac{\partial z}{\partial v}(-5, 4) = -\frac{37963}{32223}$
- 2) $\frac{\partial z}{\partial v}(-5, 4) = -\frac{12654}{10741}$
- 3) $\frac{\partial z}{\partial v}(-5, 4) = -\frac{12655}{10741}$
- 4) $\frac{\partial z}{\partial v}(-5, 4) = -\frac{37966}{32223}$
- 5) $\frac{\partial z}{\partial v}(-5, 4) = -\frac{37964}{32223}$

Exercise 3

Given the function

$f(x, y, z) = 13 + x^2 - 6y + y^2 - 2z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0, 3, ?\}$
- 2) We have a maximum at $\{-0.4, ?, -0.46091\}$
- 3) We have a maximum at $\{-0.1, ?, -0.56091\}$
- 4) We have a maximum at $\{0., ?, -0.66091\}$
- 5) We have a maximum at $\{0.4, -5.13128, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 27

Exercise 1

Given the functions

$$f(x, y, z) = (-2x^2, -y - xy + xz - z^2, 1 + x - 2z)$$

and

$$g(u, v, w) = (u^2 + 2w - 2uw + vw, 2v + 3v^2, -uv),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 1, -3)$.

- 1) -1.42297×10^6
- 2) -4.39296×10^6
- 3) -3.83357×10^6
- 4) -3.59448×10^6
- 5) -5.42315×10^6

Exercise 2

Given the system

$$-u + v^2 - 3ux + 2xy^2 = -14$$

$$x + 2uxy + uy^2 + 2v^2z - 3uz^2 + 2vz^2 = 190$$

$$-u^2y + 2vxy = 99$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (-3, -3, 4, 3, 4)$. Compute if possible $\frac{\partial x}{\partial v}(3, 4)$.

- 1) $\frac{\partial x}{\partial v}(3, 4) = \frac{44}{27}$
- 2) $\frac{\partial x}{\partial v}(3, 4) = \frac{307}{189}$
- 3) $\frac{\partial x}{\partial v}(3, 4) = \frac{34}{21}$
- 4) $\frac{\partial x}{\partial v}(3, 4) = \frac{305}{189}$
- 5) $\frac{\partial x}{\partial v}(3, 4) = \frac{304}{189}$

Exercise 3

Given the function

$f(x, y, z) = -9 - x^2 + 6y - y^2 - z^2$ defined over the domain $D = \left\{ \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-0.6, ?, -0.3\}$
- 2) We have a maximum at $\{?, 4.5, 1.2\}$
- 3) We have a maximum at $\{0., 3., ?\}$
- 4) We have a maximum at $\{1.2, ?, 0.9\}$
- 5) We have a maximum at $\{?, 2.7, -0.6\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 28

Exercise 1

Given the functions

$$f(x, y) = (-1 - x + 2x^2 + y + xy + 2y^2, -x + 3x^2 + y + 3xy, 1 - x + 2x^2 - 2y - 2xy - 2y^2, -2 + 3x - 2y + 2xy + 2y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (u_2 u_3, 2u_1 u_3 - 2u_2 u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, -1).$$

- 1) -6153.07
- 2) -3049.22
- 3) -3608.
- 4) -3020.52
- 5) -6174.38

Exercise 2

Given the system

$$-v^2 + u^2 x + 2v x^2 + 3v^2 y + x y^2 = -136$$

$$3u v x - 2v^2 y - v y^2 - 3x y^2 = 99$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-1, -4, -3, 3)$. Compute if possible $\frac{\partial x}{\partial v}(-3, 3)$.

- 1) $\frac{\partial x}{\partial v}(-3, 3) = \frac{70}{2391}$
- 2) $\frac{\partial x}{\partial v}(-3, 3) = \frac{23}{797}$
- 3) $\frac{\partial x}{\partial v}(-3, 3) = \frac{67}{2391}$
- 4) $\frac{\partial x}{\partial v}(-3, 3) = \frac{71}{2391}$
- 5) $\frac{\partial x}{\partial v}(-3, 3) = \frac{68}{2391}$

Exercise 3

Given the function

$f(x, y, z) = 1 - 2x + x^2 - 2y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 1, 0\}$
- 2) We have a minimum at $\{0.7, ?, -0.1\}$
- 3) We have a minimum at $\{0.6, ?, -0.3\}$
- 4) We have a minimum at $\{?, 1.4, -0.3\}$
- 5) We have a minimum at $\{0.7, 0.7, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 29

Exercise 1

Given the functions

$$f(x, y) = (-2x + x^2 - 2y + 2xy + 3y^2, -2 + x - 3x^2 - 3y - xy - 3y^2)$$

and

$$g(u, v) = (-3 - 3u + uv + v^2, 2 - 2u + 3u^2 + 3v - 2uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 0)$.

- 1) -0.893766
- 2) -0.804749
- 3) 0.791515
- 4) 0 .
- 5) -0.575111

Exercise 2

Given the system

$$-3u + 2u^2 - 3uy - 3u^2y - xy = -25$$

$$3u - u^3 + 2ux + 3u^2x - 2x^2 + 3ux^2 + 3uxy - y^2 + uy^2 = 34$$

determine if it is possible to solve for variables x, y in terms of variable

u around the point $p = (x, y, u) = (2, 3, 1)$. Compute if possible $\frac{\partial x}{\partial u}(1)$.

- 1) $\frac{\partial x}{\partial u}(1) = -\frac{139}{63}$
- 2) $\frac{\partial x}{\partial u}(1) = -\frac{47}{21}$
- 3) $\frac{\partial x}{\partial u}(1) = -\frac{142}{63}$
- 4) $\frac{\partial x}{\partial u}(1) = -\frac{20}{9}$
- 5) $\frac{\partial x}{\partial u}(1) = -\frac{46}{21}$

Exercise 3

Given the function

$f(x, y, z) = -21 + 4x - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.653107, -0.476554, ?\}$
- 2) We have a minimum at $\{-0.953107, ?, -2.55299\}$
- 3) We have a minimum at $\{?, -0.376554, -2.95299\}$
- 4) We have a minimum at $\{2, 1, ?\}$
- 5) We have a minimum at $\{-0.553107, -0.276554, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 30

Exercise 1

Given the functions

$$f(x, y) = (-2 + 3y - 3y^2, 1 - x - y + 2xy + 3y^2, -1 + 3x - 2x^2 + 2y + 2xy + y^2, 3 - 3x - 2x^2 - 2y - 2xy + 2y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (2u_1^2 - 2u_1u_3 + u_2u_3 - u_2u_4 + 2u_3u_4, -u_3^2 + 3u_4 + u_3u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, -3).$$

- 1) 69411.1
- 2) 310308.
- 3) 416532.
- 4) 194318.
- 5) 122535.

Exercise 2

Given the system

$$-xu_2u_4 - yu_1u_5 = -67$$

$$-2xyu_1 - 2y^2u_3 + 2xu_3u_5 = -104$$

determine if it is possible to solve for variables x, y in terms of variables

$$u_1, u_2, u_3, u_4, u_5 \text{ around the point } p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-1,$$

$$4, -4, 3, 3, -1, -4). \text{ Compute if possible } \frac{\partial x}{\partial u_5}(-4, 3, 3, -1, -4).$$

- 1) $\frac{\partial x}{\partial u_5}(-4, 3, 3, -1, -4) = -\frac{124}{5}$
- 2) $\frac{\partial x}{\partial u_5}(-4, 3, 3, -1, -4) = -\frac{123}{5}$
- 3) $\frac{\partial x}{\partial u_5}(-4, 3, 3, -1, -4) = -\frac{121}{5}$
- 4) $\frac{\partial x}{\partial u_5}(-4, 3, 3, -1, -4) = -24$
- 5) $\frac{\partial x}{\partial u_5}(-4, 3, 3, -1, -4) = -\frac{122}{5}$

Exercise 3

Given the function

$f(x, y, z) = -17 + 6x - x^2 + 4y - y^2 - z^2$ defined over the domain $D =$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 2.07779, -0.541948\}$
- 2) We have a maximum at $\{?, 2, 0\}$
- 3) We have a maximum at $\{?, 1.71649, 0.\}$
- 4) We have a maximum at $\{1.0839, ?, -0.722597\}$
- 5) We have a maximum at $\{1.98714, 0.813243, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 31

Exercise 1

Given the functions

$$f(x, y) = (3 - x + y - xy + 2y^2, -1 + 2x - 3x^2 - 3y + 2xy - 2y^2)$$

and

$$g(u, v) = (-3 - 3u + u^2 - 2v - 3uv + v^2, 3 - 3u + 2u^2 + 2v - 2uv + v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-2, -1)$.

- 1) 3272.39
- 2) 2112.
- 3) 889.978
- 4) 953.741
- 5) 1148.52

Exercise 2

Given the system

$$-2u^2v + vx + 3vxy + 3y^2 = -137$$

$$1 - 3v^2 + 2x - 3x^2 - v^2y + 3y^2 + 2y^3 = -27$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-5, 3, -4, 2)$. Compute if possible $\frac{\partial y}{\partial u}(-4, 2)$.

- 1) $\frac{\partial y}{\partial u}(-4, 2) = \frac{68}{109}$
- 2) $\frac{\partial y}{\partial u}(-4, 2) = \frac{67}{109}$
- 3) $\frac{\partial y}{\partial u}(-4, 2) = \frac{65}{109}$
- 4) $\frac{\partial y}{\partial u}(-4, 2) = \frac{64}{109}$
- 5) $\frac{\partial y}{\partial u}(-4, 2) = \frac{66}{109}$

Exercise 3

Given the function

$f(x, y, z) = 15 - 2x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{0., -1., ?\}$
- 2) We have a minimum at $\{?, 0.8, 1.4\}$
- 3) We have a minimum at $\{?, -0.2, 2.8\}$
- 4) We have a minimum at $\{?, 0.2, 2.4\}$
- 5) We have a minimum at $\{1, ?, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 32

Exercise 1

Given the functions

$$f(x, y, z) = (-2xz + yz - z^2, x + z^2)$$

and

$$g(u, v) = (-3 + 2u + u^2 + uv - v^2, -3u^2 + v - 3uv + 2v^2, 2 - 2u + u^2 + 2v + 2uv + 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, 2, -2)$.

- 1) 0.
- 2) 0.157477
- 3) -0.737009
- 4) -0.684957
- 5) -0.644871

Exercise 2

Given the system

$$3uy^2 - 3xy^2 - 3uyz + 2uz^2 = 334$$

$$3x^2 - u^2y - 2xy - 3uy^2 = -138$$

$$2 + 3u^2y - 2xyz + 2z^2 - uz^2 = -78$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (-2, -5, 1, 2)$. Compute if possible $\frac{\partial z}{\partial u}(2)$.

- 1) $\frac{\partial z}{\partial u}(2) = -\frac{58753}{17756}$
- 2) $\frac{\partial z}{\partial u}(2) = -\frac{58755}{17756}$
- 3) $\frac{\partial z}{\partial u}(2) = -\frac{58757}{17756}$
- 4) $\frac{\partial z}{\partial u}(2) = -\frac{14689}{4439}$
- 5) $\frac{\partial z}{\partial u}(2) = -\frac{29377}{8878}$

Exercise 3

Given the function

$f(x, y, z) = -2 + 2x - x^2 - y^2 + 4z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, -0.4, 2.4\}$
- 2) We have a maximum at $\{0.8, -0.2, ?\}$
- 3) We have a maximum at $\{1, 0, ?\}$
- 4) We have a maximum at $\{?, 0.6, 2.4\}$
- 5) We have a maximum at $\{0.8, -1., ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 33

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (3 - x_1 x_2 + x_2 x_3 + 2 x_3^2 - 2 x_4^2, \\ 1 - 2 x_2 + x_3 - 3 x_2 x_3 - x_3 x_4 - 2 x_4^2, -3 x_1 - 2 x_2^2 - x_4^2, 2 + x_3^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-3 u_1 u_4 + u_4^2, 1 - 3 u_2 - 2 u_3^2, -2 u_1 - 2 u_1^2 + u_2 u_3 - u_1 u_4, -2 u_1 - 3 u_2 u_4 - 2 u_3 u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (3, 0, 0, -1).$$

- 1) 0.599457
- 2) 0.407771
- 3) 0.
- 4) -0.537133
- 5) 0.765744

Exercise 2

Given the system

$$\begin{aligned} -x_3 x_4 &= -6 \\ -3 x_1 x_4 &= -9 \\ 3 v w + v w x_2 + x_2 x_4 &= 7 \\ 2 w x_1 x_3 &= -8 \end{aligned}$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u, v$

, $w) = (-1, -5, -2, -3, 5, -2, -2)$. Compute if possible $\frac{\partial x_3}{\partial w}(5, -2, -2)$.

- 1) $\frac{\partial x_3}{\partial w}(5, -2, -2) = 1$
- 2) $\frac{\partial x_3}{\partial w}(5, -2, -2) = 0$
- 3) $\frac{\partial x_3}{\partial w}(5, -2, -2) = \frac{1}{2}$
- 4) $\frac{\partial x_3}{\partial w}(5, -2, -2) = -\frac{1}{2}$
- 5) $\frac{\partial x_3}{\partial w}(5, -2, -2) = \frac{3}{2}$

Exercise 3

Given the function

$f(x, y, z) = 10 - 2x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{1.6, ?, 1.\}$
- 2) We have a minimum at $\{1.4, -0.8, ?\}$
- 3) We have a minimum at $\{?, 0.2, 2.8\}$
- 4) We have a minimum at $\{0.2, ?, 3.\}$
- 5) We have a minimum at $\{1, ?, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 34

Exercise 1

Given the functions

$$f(x, y) = (-3 + x + x^2 + 2y + xy + 3y^2, -1 + 3x - 3x^2 + 2y - 3xy + 3y^2)$$

and

$$g(u, v) = (3 - 2u + 2u^2 - 2v + 3uv - v^2, 2 + 3u - 3u^2 + 2v - uv - 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, 0)$.

- 1) 683 286.
- 2) 481 096.
- 3) 819 502.
- 4) 920 599.
- 5) 1.20535×10^6

Exercise 2

Given the system

$$u^2 - 3u^3 - 2ux - 2x^3 - y + 3uy + 2u^2y + 3xy + 2uxy - 3uy^2 = -26$$

$$-3 + ux^2 + x^3 - 2x^2y + 3y^2 + 2uy^2 = 22$$

determine if it is possible to solve for variables x, y in terms of variable

u around the point $p = (x, y, u) = (-1, 2, 2)$. Compute if possible $\frac{\partial x}{\partial u}(2)$.

- 1) $\frac{\partial x}{\partial u}(2) = \frac{231}{115}$
- 2) $\frac{\partial x}{\partial u}(2) = \frac{233}{115}$
- 3) $\frac{\partial x}{\partial u}(2) = \frac{47}{23}$
- 4) $\frac{\partial x}{\partial u}(2) = \frac{232}{115}$
- 5) $\frac{\partial x}{\partial u}(2) = \frac{234}{115}$

Exercise 3

Given the function

$f(x, y, z) = 10 - 2x + x^2 - 2y + y^2 - 6z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 1, 3\}$
- 2) We have a maximum at $\{?, -0.703023, -2.20907\}$
- 3) We have a maximum at $\{?, -0.103023, -1.40907\}$
- 4) We have a maximum at $\{-0.603023, ?, -1.80907\}$
- 5) We have a maximum at $\{-0.503023, -0.403023, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 35

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-2x_1^2 + 3x_1x_4, -3x_1x_3 - 3x_2x_4 + x_4^2, 2x_1x_2 - x_2^2 + 3x_3 - 2x_1x_4)$$

and

$$g(u, v, w) = (2u^2 - 3v^2 - 3vw, v + 2uv - 3w - uw + 2vw, 2u^2 - uw + 2w^2, -1 - u - 3w + 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (1, 2, -2, -3).$$

- 1) 0.124178
- 2) 0.634291
- 3) -0.498302
- 4) 0.
- 5) -0.207715

Exercise 2

Given the system

$$u + 3v^2x_3 + 2x_2x_4^2 = -16$$

$$-2ux_3x_4 = 20$$

$$-3x_1^2x_3 + 3x_3^2x_4 = -390$$

$$2uv + uvx_3 + x_2x_4^2 = -23$$

determine if it is possible to solve for variables x_1, x_2, x_3

, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4$

, $u, v) = (4, -4, 5, -2, 1, -1)$. Compute if possible $\frac{\partial x_2}{\partial v}(1, -1)$.

- 1) $\frac{\partial x_2}{\partial v}(1, -1) = -\frac{1361}{100}$
- 2) $\frac{\partial x_2}{\partial v}(1, -1) = -\frac{68}{5}$
- 3) $\frac{\partial x_2}{\partial v}(1, -1) = -\frac{681}{50}$
- 4) $\frac{\partial x_2}{\partial v}(1, -1) = -\frac{1363}{100}$
- 5) $\frac{\partial x_2}{\partial v}(1, -1) = -\frac{1359}{100}$

Exercise 3

Given the function

$f(x, y, z) = -5 + 2x - x^2 - y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -0.4, -2.34434\}$
- 2) We have a minimum at $\{1, 0, ?\}$
- 3) We have a minimum at $\{?, -0.1, -1.64434\}$
- 4) We have a minimum at $\{-4.12082, ?, -1.64434\}$
- 5) We have a minimum at $\{-4.22082, 0., ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 36

Exercise 1

Given the functions

$$f(x, y) = (2 + x + 3x^2 + 3y - y^2, -2 + x - x^2 + y - xy - 3y^2, 1 - 3x + 3x^2 + 2y - 2xy - 3y^2, 2 - x + 3x^2 + y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (2u_2u_3, -1 - u_1 + 3u_1^2 - u_2 - 3u_1u_3 + 3u_2u_3 + 3u_1u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, 2)$.

- 1) -375 840.
- 2) -201 945.
- 3) -653 496.
- 4) -297 289.
- 5) -123 477.

Exercise 2

Given the system

$$3xyu_1 = 24$$

$$3y^2 + xu_1 + 2u_2^3 + xu_3^2 - 3yu_3^2 = 86$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4)$

$= (-4, 1, -2, 5, -5, 0)$. Compute if possible $\frac{\partial y}{\partial u_4}(-2, 5, -5, 0)$.

- 1) $\frac{\partial y}{\partial u_4}(-2, 5, -5, 0) = 0$
- 2) $\frac{\partial y}{\partial u_4}(-2, 5, -5, 0) = 3$
- 3) $\frac{\partial y}{\partial u_4}(-2, 5, -5, 0) = 1$
- 4) $\frac{\partial y}{\partial u_4}(-2, 5, -5, 0) = 2$
- 5) $\frac{\partial y}{\partial u_4}(-2, 5, -5, 0) = 4$

Exercise 3

Given the function

$f(x, y, z) = 19 - 6x + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{2.11162, ?, 1.55406\}$
- 2) We have a minimum at $\{?, 2.05231, 0.498249\}$
- 3) We have a minimum at $\{2.63952, ?, 1.55406\}$
- 4) We have a minimum at $\{1.75968, ?, 0.850186\}$
- 5) We have a minimum at $\{3, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 37

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (1 + 2x_1x_2 + x_2^2 + x_3 + x_1x_3 - x_2x_3 - x_2x_4, \\ -3x_3x_4, -3 - 3x_1x_3 - x_3^2 - 2x_2x_4, -3x_2 - 3x_2x_3 - 2x_3^2 + x_4)$$

and

$$g(u_1, u_2, u_3, u_4) = (u_1 + u_1u_2 + 3u_2^2 + u_2u_3 + u_4 + 2u_1u_4, \\ u_2 - 2u_1u_2 - u_2u_3 + 2u_4 + u_1u_4 + u_2u_4, 2u_1 + u_1u_2 - u_3u_4, -2u_1 + u_2u_3 - 2u_1u_4 + u_3u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 2, 3, 3)$.

- 1) 2.75122×10^{10}
- 2) 5.64834×10^{10}
- 3) 1.03528×10^{11}
- 4) 4.1007×10^{10}
- 5) 8.63976×10^{10}

Exercise 2

Given the system

$$2x_1^2x_2 - 2x_3^2 - 2vx_1x_4 = -56$$

$$3x_1x_3^2 = 36$$

$$3x_1x_2x_3 = -36$$

$$-3wx_2^2 - wx_4 = -14$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u,$

$v, w) = (3, -2, 2, 2, 0, 1, 1)$. Compute if possible $\frac{\partial x_3}{\partial v}(\theta, 1, 1)$.

- 1) $\frac{\partial x_3}{\partial v}(\theta, 1, 1) = \frac{9}{65}$
- 2) $\frac{\partial x_3}{\partial v}(\theta, 1, 1) = \frac{2}{13}$
- 3) $\frac{\partial x_3}{\partial v}(\theta, 1, 1) = \frac{8}{65}$
- 4) $\frac{\partial x_3}{\partial v}(\theta, 1, 1) = \frac{7}{65}$
- 5) $\frac{\partial x_3}{\partial v}(\theta, 1, 1) = \frac{6}{65}$

Exercise 3

Given the function

$f(x, y, z) = 9 - 2x + x^2 - 2y + y^2 - 6z + z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -0.5, 3.3\}$
- 2) We have a minimum at $\{1.6, 1.6, ?\}$
- 3) We have a minimum at $\{-0.2, 0.7, ?\}$
- 4) We have a minimum at $\{?, 1.6, 3.9\}$
- 5) We have a minimum at $\{1, 1, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 38

Exercise 1

Given the functions

$$f(x, y) = (-2x^2 + y + 2y^2, 1 - 2x - x^2 + 3y - 3xy - 3y^2, 1 + x + x^2 + 3y - 2xy, 1 + x - 2x^2 + y + xy + y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-3 + 3u_1 + 3u_2^2, u_1 - 2u_1u_2 + 3u_2^2 + 2u_3^2 + u_4 + 2u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 2)$.

- 1) 7.09162×10^6
- 2) 2.05809×10^6
- 3) 1.17026×10^6
- 4) 7.55102×10^6
- 5) 4.73754×10^6

Exercise 2

Given the system

$$2xu_3^2 + yu_1u_4 = -116$$

$$3y^2u_3 + 3xu_2u_5 + 2u_1u_4u_5 = -105$$

determine if it is possible to solve for variables x, y in terms of variables

$$u_1, u_2, u_3, u_4, u_5 \text{ around the point } p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-3,$$

$$1, 5, -1, -4, -4, 3). \text{ Compute if possible } \frac{\partial x}{\partial u_4} (5, -1, -4, -4, 3).$$

- 1) $\frac{\partial x}{\partial u_4} (5, -1, -4, -4, 3) = \frac{42}{79}$
- 2) $\frac{\partial x}{\partial u_4} (5, -1, -4, -4, 3) = \frac{43}{79}$
- 3) $\frac{\partial x}{\partial u_4} (5, -1, -4, -4, 3) = \frac{41}{79}$
- 4) $\frac{\partial x}{\partial u_4} (5, -1, -4, -4, 3) = \frac{40}{79}$
- 5) $\frac{\partial x}{\partial u_4} (5, -1, -4, -4, 3) = \frac{44}{79}$

Exercise 3

Given the function

$f(x, y, z) = -17 + 6x - x^2 - y^2 + 4z - z^2$ defined over the domain $D =$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.5, -0.294872, ?\}$
- 2) We have a minimum at $\{?, -0.294872, -2.67143\}$
- 3) We have a minimum at $\{-1., -0.494872, ?\}$
- 4) We have a minimum at $\{-1.3, ?, -2.17143\}$
- 5) We have a minimum at $\{?, 0, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 39

Exercise 1

Given the functions

$$f(x, y) = (2x - 3x^2 - 2y - 3xy + y^2, -1 + 2x + 2x^2 + 3y - 3xy + 2y^2)$$

and

$$g(u, v) = (2 - 2u - v + 3v^2, 1 + 2u + 2u^2 - v - uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-2, -3)$.

- 1) 157752.
- 2) 91473.5
- 3) 248373.
- 4) 68361.
- 5) 99466.4

Exercise 2

Given the system

$$3y + xu_2u_3 + yu_2u_4 = 0$$

$$-y^2 - 3xyu_1 + 2xyu_3 + 2yu_2u_4 + 2yu_3u_4 + 3u_2u_3u_4 = 76$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4) = (-2, -1, -5, -3, -1, 3)$. Compute if possible $\frac{\partial y}{\partial u_1}(-5, -3, -1, 3)$.

- 1) $\frac{\partial y}{\partial u_1}(-5, -3, -1, 3) = 0$
- 2) $\frac{\partial y}{\partial u_1}(-5, -3, -1, 3) = -\frac{2}{37}$
- 3) $\frac{\partial y}{\partial u_1}(-5, -3, -1, 3) = -\frac{1}{37}$
- 4) $\frac{\partial y}{\partial u_1}(-5, -3, -1, 3) = \frac{1}{37}$
- 5) $\frac{\partial y}{\partial u_1}(-5, -3, -1, 3) = -\frac{3}{37}$

Exercise 3

Given the function

$f(x, y, z) = -25 + 6x - x^2 + 6y - y^2 + 6z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 3.66177, 0.990083\}$
- 2) We have a maximum at $\{0.0136128, 1.70882, ?\}$
- 3) We have a maximum at $\{3, 3, ?\}$
- 4) We have a maximum at $\{1.2342, 2.44118, ?\}$
- 5) We have a maximum at $\{1.72244, ?, 0.501848\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 40

Exercise 1

Given the functions

$$f(x, y) = (2x - 3x^2 - 3y - xy + 3y^2, -1 + 3x - 3x^2 - 3xy)$$

and

$$g(u, v) = (-2u - 3u^2 + v - 2uv, -3 + 3u + u^2 + 3uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 0)$.

- 1) -101714.
- 2) -82218.1
- 3) -62916.
- 4) -46986.1
- 5) -22679.9

Exercise 2

Given the system

$$-xy^2 - 3y^2u_2 = -128$$

$$-2xy^2 = 32$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 around the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-1, -4, 2, 3, 0, -1, 0)$. Compute if possible $\frac{\partial x}{\partial u_2}(2, 3, 0, -1, 0)$.

- 1) $\frac{\partial x}{\partial u_2}(2, 3, 0, -1, 0) = -\frac{1}{3}$
- 2) $\frac{\partial x}{\partial u_2}(2, 3, 0, -1, 0) = \frac{2}{3}$
- 3) $\frac{\partial x}{\partial u_2}(2, 3, 0, -1, 0) = \frac{1}{3}$
- 4) $\frac{\partial x}{\partial u_2}(2, 3, 0, -1, 0) = 1$
- 5) $\frac{\partial x}{\partial u_2}(2, 3, 0, -1, 0) = 0$

Exercise 3

Given the function

$f(x, y, z) = 1 + x^2 - 2y + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 1, 2\}$
- 2) We have a minimum at $\{0.2, 1.8, ?\}$
- 3) We have a minimum at $\{-0.8, 1.8, ?\}$
- 4) We have a minimum at $\{0.8, ?, 1.6\}$
- 5) We have a minimum at $\{?, 0.8, 1.4\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 41

Exercise 1

Given the functions

$$f(x, y, z) = (-3z, -x + 3x^2 - z)$$

and

$$g(u, v) = (3 + u + u^2 - v - uv - 2v^2, 1 + 2u + 2u^2 - 2v - 2uv - 3v^2, 1 - 3u - 3u^2 + v - uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 1, 1)$.

- 1) 0.314638
- 2) -0.270382
- 3) 0.
- 4) -0.23471
- 5) -0.61457

Exercise 2

Given the system

$$-2x^2y - 3uy^2 + uz + u^2z - 3xz = -66$$

$$3u - 2uxz - 2uyz = 21$$

$$-3u + 2u^2 - u^2x + x^3 + u^2y + uxz - uyz = 15$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (3, -4, 2, 3)$. Compute if possible $\frac{\partial y}{\partial u}(3)$.

- 1) $\frac{\partial y}{\partial u}(3) = \frac{9}{40}$
- 2) $\frac{\partial y}{\partial u}(3) = \frac{47}{200}$
- 3) $\frac{\partial y}{\partial u}(3) = \frac{23}{100}$
- 4) $\frac{\partial y}{\partial u}(3) = \frac{11}{50}$
- 5) $\frac{\partial y}{\partial u}(3) = \frac{43}{200}$

Exercise 3

Given the function

$f(x, y, z) = -1 + 4x - x^2 + 4y - y^2 - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-2.4095, -1.20133, ?\}$
- 2) We have a minimum at $\{-2.6095, ?, 0.2\}$
- 3) We have a minimum at $\{?, -0.801334, -0.4\}$
- 4) We have a minimum at $\{-2.8095, -0.701334, ?\}$
- 5) We have a minimum at $\{?, 2, 0\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 42

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (1 - 2x_1^2 - 2x_1x_2 + 3x_2x_3 + x_1x_4 + x_2x_4, -2x_1 - 2x_4 - 3x_2x_4)$$

and

$$g(u, v) = (-3 + 3u^2 + 3v + 2uv - 2v^2, 3u - u^2 + 3v + 3uv + 2v^2, -1 - 3u + v - uv - v^2, 2 - 3u - 2u^2 - 2v + 2uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, -1, 1, -3).$$

- 1) 0.608628
- 2) -0.654774
- 3) 0.
- 4) 0.404806
- 5) 0.407934

Exercise 2

Given the system

$$-u x_2 x_4 + 3u x_4^2 = 24$$

$$2u x_1 x_2 - x_1 x_4 - x_1^2 x_4 - 2x_1 x_2 x_4 + 3x_2 x_3 x_4 - 3x_1 x_4^2 + 3x_2 x_4^2 = 151$$

$$-x_2 - 2x_2^2 - 3x_2^3 + x_1 x_3 + u x_2 x_3 + 3u x_4 = -9$$

$$3u^2 - 3x_2^2 - 2x_3^3 = -128$$

determine if it is possible to solve for variables $x_1,$

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (-4, 1, 4, 3, 1)$. Compute if possible $\frac{\partial x_2}{\partial u}(1)$.

- 1) $\frac{\partial x_2}{\partial u}(1) = \frac{22549}{13151}$
- 2) $\frac{\partial x_2}{\partial u}(1) = \frac{67645}{39453}$
- 3) $\frac{\partial x_2}{\partial u}(1) = \frac{67648}{39453}$
- 4) $\frac{\partial x_2}{\partial u}(1) = \frac{67649}{39453}$
- 5) $\frac{\partial x_2}{\partial u}(1) = \frac{67646}{39453}$

Exercise 3

Given the function

$f(x, y, z) = -13 + 6x - x^2 - y^2 + 4z - z^2$ defined over the domain $D =$

$\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-4.73368, 0., ?\}$
- 2) We have a minimum at $\{-5.23368, 0.1, ?\}$
- 3) We have a minimum at $\{?, 0.4, -1.68804\}$
- 4) We have a minimum at $\{?, 0.3, -0.888042\}$
- 5) We have a minimum at $\{3, ?, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 43

Exercise 1

Given the functions

$$f(x, y, z) = (-2y + 2xy, 2y, 3x + 2x^2)$$

and

$$g(u, v, w) = (1 + v^2 + 2w^2, -uv - w + 3uw, v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 1, 2)$.

- 1) -0.671579
- 2) 0 .
- 3) -0.484398
- 4) -0.626653
- 5) -0.527676

Exercise 2

Given the system

$$-u^3 - 3vx + 2vxy + vz + xz^2 + 2z^3 = 42$$

$$x^2z = 12$$

$$2 - 2v^3 - 2yz = -44$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (2, 5, 3, 4, 2)$. Compute if possible $\frac{\partial x}{\partial u}(4, 2)$.

- 1) $\frac{\partial x}{\partial u}(4, 2) = -\frac{14}{47}$
- 2) $\frac{\partial x}{\partial u}(4, 2) = -\frac{12}{47}$
- 3) $\frac{\partial x}{\partial u}(4, 2) = -\frac{15}{47}$
- 4) $\frac{\partial x}{\partial u}(4, 2) = -\frac{13}{47}$
- 5) $\frac{\partial x}{\partial u}(4, 2) = -\frac{16}{47}$

Exercise 3

Given the function

$f(x, y, z) = -12 - x^2 + 6y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.1, -2.0875, ?\}$
- 2) We have a minimum at $\{0.4, ?, -3.93399\}$
- 3) We have a minimum at $\{0., ?, -4.13399\}$
- 4) We have a minimum at $\{-0.1, -1.9875, ?\}$
- 5) We have a minimum at $\{0, 3, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 44

Exercise 1

Given the functions

$$f(x, y) = (-2 - y + 2xy - 3y^2, -2 + 3x + 2y + 2xy + y^2, -1 - 2xy + y^2)$$

and

$$g(u, v, w) = (-u - 3u^2 + 2uv + 2v^2 + uw + 2vw - w^2, 3 + 2w + 3vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 0)$.

- 1) -74.3267
- 2) -54.
- 3) -33.1729
- 4) -81.1377
- 5) -42.4694

Exercise 2

Given the system

$$2x^2y - 3u_2u_3 = 169$$

$$-3y + xu_4 = -19$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4)$

$= (-4, 5, 3, -1, 3, 1)$. Compute if possible $\frac{\partial x}{\partial u_1}(3, -1, 3, 1)$.

- 1) $\frac{\partial x}{\partial u_1}(3, -1, 3, 1) = 3$
- 2) $\frac{\partial x}{\partial u_1}(3, -1, 3, 1) = 4$
- 3) $\frac{\partial x}{\partial u_1}(3, -1, 3, 1) = 1$
- 4) $\frac{\partial x}{\partial u_1}(3, -1, 3, 1) = 2$
- 5) $\frac{\partial x}{\partial u_1}(3, -1, 3, 1) = 0$

Exercise 3

Given the function

$f(x, y, z) = 10 + x^2 - 2y + y^2 - 6z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-1.2, 0.4, ?\}$
- 2) We have a minimum at $\{-0.6, ?, 3.6\}$
- 3) We have a minimum at $\{1.5, ?, 3.9\}$
- 4) We have a minimum at $\{0.6, 0.1, ?\}$
- 5) We have a minimum at $\{0, ?, 3\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 45

Exercise 1

Given the functions

$$f(x, y) = (-1 - 2x^2 - 2y + xy - 2y^2, -3 - x + 3x^2 - y + xy - y^2, -3 - 3x + y - xy)$$

and

$$g(u, v, w) = (1 - 2w^2, 2u^2 - 3v^2 + 3w + 2w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 1)$.

- 1) -9216.
- 2) -15033.5
- 3) -13113.4
- 4) -1126.89
- 5) -4401.33

Exercise 2

Given the system

$$-3 + uv - 2w - 3vwx + vwy + 2xy = 11$$

$$2u^2x - 3uvy = -112$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u, v$

, $w) = (4, 4, -4, -5, -1)$. Compute if possible $\frac{\partial y}{\partial w}(-4, -5, -1)$.

- 1) $\frac{\partial y}{\partial w}(-4, -5, -1) = 304$
- 2) $\frac{\partial y}{\partial w}(-4, -5, -1) = 307$
- 3) $\frac{\partial y}{\partial w}(-4, -5, -1) = 305$
- 4) $\frac{\partial y}{\partial w}(-4, -5, -1) = 306$
- 5) $\frac{\partial y}{\partial w}(-4, -5, -1) = 308$

Exercise 3

Given the function

$f(x, y, z) = 19 - 2x + x^2 - 6y + y^2 + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0.216152, -3.46963, ?\}$
- 2) We have a maximum at $\{?, -2.96963, 0.\}$
- 3) We have a maximum at $\{0.216152, ?, -0.1\}$
- 4) We have a maximum at $\{1, 3, ?\}$
- 5) We have a maximum at $\{-0.383848, -2.66963, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 46

Exercise 1

Given the functions

$$f(x, y, z) = (z, x^2, x^2 - 3y^2 + 3z)$$

and

$$g(u, v, w) = (2v^2 + 3vw, -2u - 3u^2 - 3v + 3w + w^2, 3 - uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, -2, -2)$.

- 1) 0.866882
- 2) -0.147264
- 3) -0.41283
- 4) 0.
- 5) 0.412278

Exercise 2

Given the system

$$2yzu_2 + yu_4^2 = 17$$

$$xyz - u_1u_3u_4 = -48$$

$$-2x^2u_3 + z^2u_4 = 148$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4$

$) = (-4, 1, 2, -2, -2, -4, 5)$. Compute if possible $\frac{\partial z}{\partial u_2}(-2, -2, -4, 5)$.

- 1) $\frac{\partial z}{\partial u_2}(-2, -2, -4, 5) = \frac{259}{715}$
- 2) $\frac{\partial z}{\partial u_2}(-2, -2, -4, 5) = \frac{256}{715}$
- 3) $\frac{\partial z}{\partial u_2}(-2, -2, -4, 5) = \frac{258}{715}$
- 4) $\frac{\partial z}{\partial u_2}(-2, -2, -4, 5) = \frac{257}{715}$
- 5) $\frac{\partial z}{\partial u_2}(-2, -2, -4, 5) = \frac{4}{11}$

Exercise 3

Given the function

$f(x, y, z) = -7 - x^2 - y^2 + 2z - z^2$ defined over the domain $D = \left\{ \frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{0.491132, ?, 0.419765\}$
- 2) We have a minimum at $\{?, 0, 1\}$
- 3) We have a minimum at $\{0., 4.91132, ?\}$
- 4) We have a minimum at $\{?, 2.45566, -1.05363\}$
- 5) We have a minimum at $\{0.491132, ?, -1.05363\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 47

Exercise 1

Given the functions

$$f(x, y) = (1 - 2x - 2x^2 + 3y - 3y^2, -3x - 3x^2 + y - 3xy + 3y^2, 3 + 3x - y + xy + y^2, 1 - 3x + 2x^2 - y + 2xy + y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-u_1^2 + u_3 - 2u_1u_4 + 2u_4^2, u_1 + 2u_1u_2 - u_2u_4 + 3u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, 2)$.

- 1) -132918 .
- 2) -1.12061×10^6
- 3) -721513 .
- 4) -548125 .
- 5) -1.16454×10^6

Exercise 2

Given the system

$$uv + 3u^2v - 2uv^2 + 2ux - 3x^3 - 2xy = 50$$

$$-3 - 3u^2 + 3uv^2 + u^2x - 2vx^2 - xy^2 = -33$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-2, -4, -3, 1)$. Compute if possible $\frac{\partial y}{\partial u}(-3, 1)$.

- 1) $\frac{\partial y}{\partial u}(-3, 1) = \frac{367}{180}$
- 2) $\frac{\partial y}{\partial u}(-3, 1) = \frac{551}{270}$
- 3) $\frac{\partial y}{\partial u}(-3, 1) = \frac{55}{27}$
- 4) $\frac{\partial y}{\partial u}(-3, 1) = \frac{1099}{540}$
- 5) $\frac{\partial y}{\partial u}(-3, 1) = \frac{1103}{540}$

Exercise 3

Given the function

$f(x, y, z) = -21 + 4x - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 0.4, 2.7\}$
- 2) We have a maximum at $\{0.8, 1.6, ?\}$
- 3) We have a maximum at $\{2.3, ?, 3.6\}$
- 4) We have a maximum at $\{2.6, 0.7, ?\}$
- 5) We have a maximum at $\{2, ?, 3\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 48

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-3x_1x_2 + 3x_3 - x_2x_3, 2 + 3x_2 - 3x_1x_2)$$

and

$$g(u, v) = (-3u - u^2 + v - v^2, -2u - 2u^2 - v - 2uv + 3v^2, 2 + 2u + u^2 + 2v - 3v^2, -3 + 3u - u^2 + 3v - 2uv - 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-3, 1, -2, 2).$$

- 1) -0.787323
- 2) $0.$
- 3) -0.868412
- 4) -0.268413
- 5) 0.715378

Exercise 2

Given the system

$$3x_1 + x_2^2x_4 = -11$$

$$x_2x_3^2 = 25$$

$$3x_1^3 - 3x_1x_2 + u^2x_4 - x_4^2 = -340$$

$$3x_1^2x_2 + x_1x_2^2 + 3x_2^2x_3 - 2x_3^2x_4 = -115$$

determine if it is possible to solve for variables $x_1,$

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (-5, 1, 5, 4, 3)$. Compute if possible $\frac{\partial x_4}{\partial u}(3)$.

- 1) $\frac{\partial x_4}{\partial u}(3) = \frac{4780}{27221}$
- 2) $\frac{\partial x_4}{\partial u}(3) = \frac{4776}{27221}$
- 3) $\frac{\partial x_4}{\partial u}(3) = \frac{4779}{27221}$
- 4) $\frac{\partial x_4}{\partial u}(3) = \frac{4778}{27221}$
- 5) $\frac{\partial x_4}{\partial u}(3) = \frac{4777}{27221}$

Exercise 3

Given the function

$f(x, y, z) = 4 - 4x + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 2.67054, 0.206098\}$
- 2) We have a minimum at $\{1.72708, 1.13535, ?\}$
- 3) We have a minimum at $\{2, 2, ?\}$
- 4) We have a minimum at $\{?, 1.71105, 0.973691\}$
- 5) We have a minimum at $\{2.68658, ?, 0.589894\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for

serial number: 49

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (1 - 2x_1 - 3x_1^2 + 3x_2x_4 + 2x_3x_4 - 3x_4^2, 3 + 3x_1^2 + 3x_4 + 2x_2x_4 - 2x_3x_4, -x_3 - 2x_3x_4)$$

and

$$g(u, v, w) = (2v^2, 3v + 3v^2, -3u - u^2 - 2uv + 2v^2 + 3w - 2w^2, -2u - 2v - uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-3, -2, -3, 2).$$

- 1) -0.541302
- 2) 0.499604
- 3) 0 .
- 4) 0.187252
- 5) -0.668377

Exercise 2

Given the system

$$-u^2v - 2uv^2 - 3vx_1x_3 = 236$$

$$3x_2^2x_3 + 3x_3x_4^2 = -102$$

$$-3ux_1^2 + 3ux_4^2 = -75$$

$$2vx_3 + 2x_2x_4 = -22$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$$u, v) = (2, -5, -1, 3, -5, -4). \text{ Compute if possible } \frac{\partial x_4}{\partial u}(-5, -4).$$

- 1) $\frac{\partial x_4}{\partial u}(-5, -4) = -\frac{1633}{554}$
- 2) $\frac{\partial x_4}{\partial u}(-5, -4) = -\frac{1631}{554}$
- 3) $\frac{\partial x_4}{\partial u}(-5, -4) = -\frac{816}{277}$
- 4) $\frac{\partial x_4}{\partial u}(-5, -4) = -\frac{815}{277}$
- 5) $\frac{\partial x_4}{\partial u}(-5, -4) = -\frac{1629}{554}$

Exercise 3

Given the function

$f(x, y, z) = -13 + 2x - x^2 + 4y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2., ?, 2.\}$
- 2) We have a maximum at $\{1, 2, ?\}$
- 3) We have a maximum at $\{0., ?, 0.6\}$
- 4) We have a maximum at $\{?, 2.4, 0.8\}$
- 5) We have a maximum at $\{0.8, ?, 2.\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 50

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-2x_2^2 + 2x_1x_4, 2x_1x_2 + 2x_3 + x_3^2 + 3x_4, 3x_1 + x_3)$$

and

$$g(u, v, w) = (-2v + uv - uw - 3vw, 3 - v^2 - w - 3vw, -2 + 3u^2 + uw, u^2 - v^2 + 3vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 1, 0, 0)$.

- 1) 0.
- 2) 0.62988
- 3) 0.388417
- 4) 0.635628
- 5) 0.102694

Exercise 2

Given the system

$$-u x_2 x_4 - 3 x_3 x_4 = 9$$

$$3 u x_2^2 + 3 x_2 x_3 - 2 x_1^2 x_4 = -81$$

$$3 v + 3 x_1^2 x_2 + u x_3 + x_1 x_3^2 = -96$$

$$-3 v x_1 x_2 + v x_3 x_4 = -72$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$u, v) = (-3, -3, -3, 3, -2, 2)$. Compute if possible $\frac{\partial x_4}{\partial u}(-2, 2)$.

- 1) $\frac{\partial x_4}{\partial u}(-2, 2) = \frac{106}{239}$
- 2) $\frac{\partial x_4}{\partial u}(-2, 2) = \frac{109}{239}$
- 3) $\frac{\partial x_4}{\partial u}(-2, 2) = \frac{107}{239}$
- 4) $\frac{\partial x_4}{\partial u}(-2, 2) = \frac{108}{239}$
- 5) $\frac{\partial x_4}{\partial u}(-2, 2) = \frac{105}{239}$

Exercise 3

Given the function

$f(x, y, z) = -1 - 4x + x^2 - 4y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{2.8, 1.4, ?\}$
- 2) We have a minimum at $\{?, 2, 0\}$
- 3) We have a minimum at $\{1., 2.4, ?\}$
- 4) We have a minimum at $\{1.2, ?, 0.6\}$
- 5) We have a minimum at $\{2.4, ?, 0.8\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 51

Exercise 1

Given the functions

$$f(x, y) = (2 + 2x + x^2 + 3xy, -2 + x + x^2 - 2y + 2xy + 2y^2, 2x + 3x^2 - y + 3xy)$$

and

$$g(u, v, w) = (-1 - 2uw - vw, 2w - 3uw - 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, -3)$.

- 1) -23970.5
- 2) -28417.2
- 3) -65622.6
- 4) -34677.
- 5) -12863.7

Exercise 2

Given the system

$$-3xu_1u_2 + u_2^2 - 2yu_1u_4 = 328$$

$$-2xyu_4 = 120$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4)$

$= (-4, 5, -4, -4, 4, 3)$. Compute if possible $\frac{\partial x}{\partial u_2}(-4, -4, 4, 3)$.

- 1) $\frac{\partial x}{\partial u_2}(-4, -4, 4, 3) = -3$
- 2) $\frac{\partial x}{\partial u_2}(-4, -4, 4, 3) = -\frac{28}{9}$
- 3) $\frac{\partial x}{\partial u_2}(-4, -4, 4, 3) = -\frac{25}{9}$
- 4) $\frac{\partial x}{\partial u_2}(-4, -4, 4, 3) = -\frac{8}{3}$
- 5) $\frac{\partial x}{\partial u_2}(-4, -4, 4, 3) = -\frac{26}{9}$

Exercise 3

Given the function

$f(x, y, z) = 21 - 4x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D = \frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2, 2, ?\}$
- 2) We have a maximum at $\{-5.06535, -0.0522282, ?\}$
- 3) We have a maximum at $\{-4.26535, ?, -0.710581\}$
- 4) We have a maximum at $\{-4.66535, -0.252228, ?\}$
- 5) We have a maximum at $\{-4.36535, ?, -1.51058\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 52

Exercise 1

Given the functions

$$f(x, y) = (-1 + x - 2y + xy - 2y^2, 1 - x + 2y + 2xy - 3y^2, -3x + 3x^2 - y + 3xy - y^2)$$

and

$$g(u, v, w) = (3w - 2vw + 3w^2, -u - 3v + 2uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 1)$.

- 1) 431.
- 2) 662.454
- 3) 680.03
- 4) 358.621
- 5) 613.76

Exercise 2

Given the system

$$2x - 2y = -10$$

$$3u^2v + 2wx + x^2y + wy^2 - y^3 = -333$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u, v$

, $w) = (0, 5, -3, -4, -4)$. Compute if possible $\frac{\partial x}{\partial u}(-3, -4, -4)$.

- 1) $\frac{\partial x}{\partial u}(-3, -4, -4) = \frac{28}{41}$
- 2) $\frac{\partial x}{\partial u}(-3, -4, -4) = \frac{27}{41}$
- 3) $\frac{\partial x}{\partial u}(-3, -4, -4) = \frac{24}{41}$
- 4) $\frac{\partial x}{\partial u}(-3, -4, -4) = \frac{26}{41}$
- 5) $\frac{\partial x}{\partial u}(-3, -4, -4) = \frac{25}{41}$

Exercise 3

Given the function

$f(x, y, z) = -6 + 2x - x^2 - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0.5, 0.2, ?\}$
- 2) We have a maximum at $\{?, 0.1, 0.6\}$
- 3) We have a maximum at $\{1, ?, 1\}$
- 4) We have a maximum at $\{1.4, ?, 1.2\}$
- 5) We have a maximum at $\{1.3, ?, 1.4\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 53

Exercise 1

Given the functions

$$f(x, y, z) = (y^2 - 2z^2, -z, 3y^2 + 3xz)$$

and

$$g(u, v, w) = (3 + 3u - uv, 3 - 3v^2, -3 - 3u - 3v - 2v^2 + 2uw + 2vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 3, 2)$.

- 1) 6421.2
- 2) 2430.4
- 3) 4320.
- 4) 7715.71
- 5) 3398.69

Exercise 2

Given the system

$$2y + uz^2 = -22$$

$$u^2v - 2ux + ux^2 - 2vzx = 39$$

$$-2uz + 2xyz = 72$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (-3, -5, 2, -3, 4)$. Compute if possible $\frac{\partial y}{\partial v}(-3, 4)$.

- 1) $\frac{\partial y}{\partial v}(-3, 4) = -13$
- 2) $\frac{\partial y}{\partial v}(-3, 4) = -\frac{103}{8}$
- 3) $\frac{\partial y}{\partial v}(-3, 4) = -\frac{101}{8}$
- 4) $\frac{\partial y}{\partial v}(-3, 4) = -\frac{105}{8}$
- 5) $\frac{\partial y}{\partial v}(-3, 4) = -\frac{51}{4}$

Exercise 3

Given the function

$f(x, y, z) = 13 - 4x + x^2 - 6y + y^2 - 4z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-4.57863, ?, -0.668641\}$
- 2) We have a maximum at $\{-4.87863, -0.702961, ?\}$
- 3) We have a maximum at $\{2, 3, ?\}$
- 4) We have a maximum at $\{-5.07863, ?, -0.268641\}$
- 5) We have a maximum at $\{-4.07863, ?, -0.968641\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 54

Exercise 1

Given the functions

$$f(x, y) = (2 - x + 3x^2 - 3xy - y^2, -3 - x - 3x^2 + 2y - 3y^2, -3 + x - 2x^2 - 2y - 3xy + y^2)$$

and

$$g(u, v, w) = (-2uv + 2v^2, 3u^2 + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, -2)$.

- 1) 2.45474×10^6
- 2) 1.55427×10^6
- 3) 442 027.
- 4) 236 367.
- 5) 1.31862×10^6

Exercise 2

Given the system

$$xy^2 + yu_2^2 + 2u_2^3 + 3u_1u_3 + 3u_3^3 = 19$$

$$2x^3 - 3yu_1^2 + u_2^2 - u_3^3 + 3u_4^3 = -55$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4)$

$= (-2, 2, -2, -3, 3, 1)$. Compute if possible $\frac{\partial y}{\partial u_2}(-2, -3, 3, 1)$.

- 1) $\frac{\partial y}{\partial u_2}(-2, -3, 3, 1) = -13$
- 2) $\frac{\partial y}{\partial u_2}(-2, -3, 3, 1) = -10$
- 3) $\frac{\partial y}{\partial u_2}(-2, -3, 3, 1) = -14$
- 4) $\frac{\partial y}{\partial u_2}(-2, -3, 3, 1) = -12$
- 5) $\frac{\partial y}{\partial u_2}(-2, -3, 3, 1) = -11$

Exercise 3

Given the function

$f(x, y, z) = 9 - 4x + x^2 - 6y + y^2 - 2z + z^2$ defined over the domain $D =$

$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 2.31334, 0.856925\}$
- 2) We have a minimum at $\{0.967811, 3.23868, ?\}$
- 3) We have a minimum at $\{?, 3, 1\}$
- 4) We have a minimum at $\{1.43048, ?, 0.62559\}$
- 5) We have a minimum at $\{?, 2.77601, 0.162922\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 55

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (2 + 3x_2 - x_1x_2 - x_1x_3 - 3x_3^2 - x_2x_4 + 3x_4^2, -2 - 2x_2 - 3x_1x_3 - 3x_3^2 + 3x_1x_4 + 2x_4^2)$$

and

$$g(u, v) = (-2 + 2u + 3v - uv + 2v^2, 1 - 3u + 3u^2 + uv - 3v^2, 3 + 2u + u^2 + 3v - 3uv + 3v^2, -2 - u + u^2 + 2v + 2uv - v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-3, 3, 3, -3).$$

- 1) 0.
- 2) 0.108197
- 3) 0.520785
- 4) 0.324004
- 5) 0.6204

Exercise 2

Given the system

$$-2u^2x_1 - 3x_2^2 + 3x_3^2 = 73$$

$$-2x_1^2x_2 + 3x_1^2x_3 + 2x_3^2x_4 = -40$$

$$2x_2x_4^2 - 3x_3x_4^2 = 8$$

$$-2x_1 - x_2^2 + 2ux_2^2 = -5$$

determine if it is possible to solve for variables x_1, x_2

, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2, x_3$

, $x_4, u) = (-2, 1, -2, -1, -4)$. Compute if possible $\frac{\partial x_2}{\partial u}(-4)$.

- 1) $\frac{\partial x_2}{\partial u}(-4) = \frac{800}{1677}$
- 2) $\frac{\partial x_2}{\partial u}(-4) = \frac{803}{1677}$
- 3) $\frac{\partial x_2}{\partial u}(-4) = \frac{268}{559}$
- 4) $\frac{\partial x_2}{\partial u}(-4) = \frac{802}{1677}$
- 5) $\frac{\partial x_2}{\partial u}(-4) = \frac{267}{559}$

Exercise 3

Given the function

$f(x, y, z) = -18 + 4x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 3, 1\}$
- 2) We have a maximum at $\{2.9, ?, 2.2\}$
- 3) We have a maximum at $\{0.8, ?, 1.9\}$
- 4) We have a maximum at $\{3.2, ?, 2.2\}$
- 5) We have a maximum at $\{2.6, ?, 1.9\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 56

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (2x_4 - x_1x_4 + 2x_2x_4 + 2x_4^2, \\ -2x_1x_2 - 2x_2^2 + 2x_3^2 - 2x_4 + 2x_2x_4, -3x_1^2 - 3x_2 + 3x_2x_3 - 2x_3^2 + 3x_3x_4)$$

and

$$g(u, v, w) = (-3 + 2u - 3v + 2uv - w + 3vw, -3 + u, 3 - 2uv - w + uw - 3vw, 2v + uw + vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-3, -2, 0, -3).$$

- 1) 0.818768
- 2) 0.34795
- 3) 0.
- 4) 0.155416
- 5) -0.888758

Exercise 2

Given the system

$$3vx_3 - 3ux_2x_3 - 2x_3^2x_4 = 28$$

$$-2ux_2x_3 = 20$$

$$3x_1x_2x_4 + x_1x_4^2 = -500$$

$$-v^3 + x_2^3 - 2x_1x_2x_4 = 189$$

determine if it is possible to solve for variables $x_1, x_2, x_3,$

x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$u, v) = (-5, -5, 1, -5, 2, -4)$. Compute if possible $\frac{\partial x_3}{\partial v}(2, -4)$.

- 1) $\frac{\partial x_3}{\partial v}(2, -4) = \frac{12}{145}$
- 2) $\frac{\partial x_3}{\partial v}(2, -4) = \frac{59}{725}$
- 3) $\frac{\partial x_3}{\partial v}(2, -4) = \frac{121}{1450}$
- 4) $\frac{\partial x_3}{\partial v}(2, -4) = \frac{119}{1450}$
- 5) $\frac{\partial x_3}{\partial v}(2, -4) = \frac{117}{1450}$

Exercise 3

Given the function

$f(x, y, z) = -4 + 4x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D = \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2.4, 2., ?\}$
- 2) We have a maximum at $\{2.6, ?, 0.2\}$
- 3) We have a maximum at $\{?, 1, 1\}$
- 4) We have a maximum at $\{?, 1.8, 2.\}$
- 5) We have a maximum at $\{2.8, ?, 2.\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 57

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (1 - x_1 - 3x_2 - 2x_3 - x_3x_4, 3x_1 + 3x_3 - 2x_2x_4 + 3x_4^2, -3x_2x_3 - 2x_3x_4)$$

and

$$g(u, v, w) = (-u - 2u^2 + 3uv - w + uw, 2 + 2u^2 + 3w - uw + 3vw - 2w^2, -3v + vw, 2u^2 - v - 2uv - 3w),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-3, 1, -3, -1).$$

- 1) 0.44874
- 2) 0.452251
- 3) -0.817619
- 4) -0.162687
- 5) 0.

Exercise 2

Given the system

$$-3vx_1x_4 = 180$$

$$ux_1 + 2vx_1 + 3x_1x_2x_3 - 2vx_4^2 = -43$$

$$-ux_3x_4 = -8$$

$$3x_1^2x_2 + 2x_1^2x_4 = 63$$

determine if it is possible to solve for variables x_1, x_2, x_3

, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4$

, $u, v) = (3, 5, 2, -4, -1, 5)$. Compute if possible $\frac{\partial x_4}{\partial v}(-1, 5)$.

- 1) $\frac{\partial x_4}{\partial v}(-1, 5) = \frac{656}{1975}$
- 2) $\frac{\partial x_4}{\partial v}(-1, 5) = \frac{653}{1975}$
- 3) $\frac{\partial x_4}{\partial v}(-1, 5) = \frac{654}{1975}$
- 4) $\frac{\partial x_4}{\partial v}(-1, 5) = \frac{131}{395}$
- 5) $\frac{\partial x_4}{\partial v}(-1, 5) = \frac{652}{1975}$

Exercise 3

Given the function

$f(x, y, z) = -14 + 4x - x^2 + 2y - y^2 - z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{2, 1, ?\}$
- 2) We have a minimum at $\{-5.29056, -0.632303, ?\}$
- 3) We have a minimum at $\{-4.89056, -0.832303, ?\}$
- 4) We have a minimum at $\{?, -0.332303, -0.1\}$
- 5) We have a minimum at $\{-4.49056, -0.932303, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 58

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (2x_1^2 + x_2^2 + 2x_3, 3x_1x_2 + x_3 + 3x_3^2 - 2x_1x_4 - 3x_3x_4 - 3x_4^2, -3x_2x_3, 3 - 3x_1 - 2x_1^2 - x_2x_4)$$

and

$$g(u_1, u_2, u_3, u_4) = (3u_1 - 2u_1u_2 + 2u_2^2 + 3u_3u_4, -3 - 2u_1 - u_1^2 - 2u_1u_2 - u_4 + 2u_3u_4, -2u_1 + 2u_1^2 + 2u_1u_2 + u_1u_4 + 2u_2u_4, -u_1 + u_2 + 3u_1u_2 - 2u_3 - u_1u_3 - 2u_4^2)$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, -3, 0, 1).$$

- 1) 5.7587×10^8
- 2) 3.04269×10^8
- 3) 7.50379×10^8
- 4) 3.8213×10^8
- 5) 2.81028×10^8

Exercise 2

Given the system

$$-2u x_1^2 + 3w x_2^2 + v x_2 x_4 = -56$$

$$x_1 x_3^2 = -5$$

$$-2x_1^3 - u x_3 = 252$$

$$-2x_1 x_2 x_4 = -80$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u,$

$v, w) = (-5, -2, -1, 4, 2, 2, 5)$. Compute if possible $\frac{\partial x_1}{\partial v}(2, 2, 5)$.

- 1) $\frac{\partial x_1}{\partial v}(2, 2, 5) = 1$
- 2) $\frac{\partial x_1}{\partial v}(2, 2, 5) = 0$
- 3) $\frac{\partial x_1}{\partial v}(2, 2, 5) = 4$
- 4) $\frac{\partial x_1}{\partial v}(2, 2, 5) = 3$
- 5) $\frac{\partial x_1}{\partial v}(2, 2, 5) = 2$

Exercise 3

Given the function

$f(x, y, z) = -19 + 4x - x^2 + 4y - y^2 + 6z - z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-3.79714, ?, -2.1653\}$
- 2) We have a minimum at $\{-3.49714, ?, -2.0653\}$
- 3) We have a minimum at $\{-4.29714, -1.84353, ?\}$
- 4) We have a minimum at $\{-3.39714, ?, -1.6653\}$
- 5) We have a minimum at $\{?, 2, 3\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 59

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-x_1 + 2x_1x_2 + 2x_4 + x_3x_4, 2x_1 + x_2^2 + 2x_2x_4)$$

and

$$g(u, v) = (1 - 2u + 3v + 2uv, -3 - 2u - 3u^2 + 3v - uv - 2v^2, -3 - 3u - u^2 + v + 2v^2, -u + 3u^2 + 2v - 3uv),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, -2, 3, 0).$$

- 1) -0.744611
- 2) -0.449005
- 3) 0.562369
- 4) -0.336581
- 5) 0 .

Exercise 2

Given the system

$$-u - 2x_4^2 - x_2x_4^2 = -55$$

$$-2ux_2x_3 = -40$$

$$-2x_1x_2 + 2x_1x_3x_4 + 2x_4^2 - 3ux_4^2 - 3x_3x_4^2 = -144$$

$$-ux_3 + x_1x_3^2 - ux_4^2 = -14$$

determine if it is possible to solve for variables $x_1,$

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (0, 4, 5, -3, 1)$. Compute if possible $\frac{\partial x_2}{\partial u}(1)$.

- 1) $\frac{\partial x_2}{\partial u}(1) = -\frac{884}{563}$
- 2) $\frac{\partial x_2}{\partial u}(1) = -\frac{882}{563}$
- 3) $\frac{\partial x_2}{\partial u}(1) = -\frac{883}{563}$
- 4) $\frac{\partial x_2}{\partial u}(1) = -\frac{881}{563}$
- 5) $\frac{\partial x_2}{\partial u}(1) = -\frac{880}{563}$

Exercise 3

Given the function

$f(x, y, z) = -8 + 4x - x^2 - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2.4, -1., ?\}$
- 2) We have a maximum at $\{?, -0.8, 0.\}$
- 3) We have a maximum at $\{1., -1., ?\}$
- 4) We have a maximum at $\{2.6, 1., ?\}$
- 5) We have a maximum at $\{2, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 60

Exercise 1

Given the functions

$$f(x, y, z) = (2 + x - x^2 - xy + z, 2x + 3x^2 - 2y + z, 2 + 2xy + 2y^2 + 2z^2)$$

and

$$g(u, v, w) = (v + v^2 - 2uw - w^2, 3v, 3u - v + v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, -3, 3)$.

- 1) 20592.
- 2) 27094.1
- 3) 34371.9
- 4) 33109.3
- 5) 37884.3

Exercise 2

Given the system

$$-3vx + 3y^2 - vy^2 = -3$$

$$vy^2 + xy^2 - vyz - 2uz^2 = -83$$

$$2xy + xyz - uz^2 = 11$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (-5, -3, -1, 4, -5)$. Compute if possible $\frac{\partial z}{\partial u}(4, -5)$.

- 1) $\frac{\partial z}{\partial u}(4, -5) = \frac{57}{971}$
- 2) $\frac{\partial z}{\partial u}(4, -5) = \frac{58}{971}$
- 3) $\frac{\partial z}{\partial u}(4, -5) = \frac{117}{1942}$
- 4) $\frac{\partial z}{\partial u}(4, -5) = \frac{113}{1942}$
- 5) $\frac{\partial z}{\partial u}(4, -5) = \frac{115}{1942}$

Exercise 3

Given the function

$f(x, y, z) = -19 + 4x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-5.17369, -0.811305, ?\}$
- 2) We have a minimum at $\{-4.67369, -1.0113, ?\}$
- 3) We have a minimum at $\{-4.47369, ?, 0.0628984\}$
- 4) We have a minimum at $\{?, -0.811305, 0.0628984\}$
- 5) We have a minimum at $\{2, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 61

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (2 - 3x_1 + 2x_1x_3 - 2x_2x_3 + 3x_4^2, 2x_3 + x_1x_4, -2x_1 + x_2 - 2x_1x_2 - 2x_3)$$

and

$$g(u, v, w) = (-3 - v + 2uv, u^2 + 3v - 3v^2, -2 + 3v^2 + 3uw + w^2, 2uw - 2vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, 0, 3, 0).$$

- 1) 0.603615
- 2) -0.558376
- 3) -0.222412
- 4) 0.762514
- 5) 0.

Exercise 2

Given the system

$$-2v^3 + 2x_2x_3 = -130$$

$$2x_3x_4^2 = 2$$

$$3x_1^2 - ux_1x_4 = 1$$

$$x_1^3 - 2x_2 - vx_2^2 - 2x_1x_3 + x_4^2 = -2$$

determine if it is possible to solve for variables x_1, x_2, x_3

, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3,$

$x_4, u, v) = (1, -1, 1, 1, 2, 4)$. Compute if possible $\frac{\partial x_3}{\partial u}(2, 4)$.

- 1) $\frac{\partial x_3}{\partial u}(2, 4) = \frac{3}{11}$
- 2) $\frac{\partial x_3}{\partial u}(2, 4) = \frac{1}{11}$
- 3) $\frac{\partial x_3}{\partial u}(2, 4) = \frac{2}{11}$
- 4) $\frac{\partial x_3}{\partial u}(2, 4) = -\frac{1}{11}$
- 5) $\frac{\partial x_3}{\partial u}(2, 4) = 0$

Exercise 3

Given the function

$f(x, y, z) = -6 + 6x - x^2 + 2y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-3.45061, -0.969973, ?\}$
- 2) We have a minimum at $\{?, -0.969973, 0.1\}$
- 3) We have a minimum at $\{?, 1, 0\}$
- 4) We have a minimum at $\{-3.45061, -0.669973, ?\}$
- 5) We have a minimum at $\{-3.95061, ?, 0.\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 62

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-3x_1^2 + 3x_1x_2 + 2x_1x_3 - 3x_2x_3, 2 + 3x_1x_3 + 2x_4 + 2x_3x_4)$$

and

$$g(u, v) = (-1 + 2u + 2u^2 + 3v - 2v^2, 3 + 3u - 3u^2 + 2v + uv, -2 + 2u - 3v + 3uv - 2v^2, -3 + u + 2v - 2uv - 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, -1, 3, -1).$$

- 1) 0.
- 2) 0.79143
- 3) 0.232493
- 4) -0.870632
- 5) -0.115443

Exercise 2

Given the system

$$2x_1^2 + 2x_3^2x_4 = 36$$

$$-2x_1x_3 + 2x_2^2x_4 = 100$$

$$x_2^3 - 3x_3 - 3x_2x_3^2 + x_1x_3x_4 = -1$$

$$x_2^3 - 2x_1x_2x_4 - 3x_2^2x_4 + 3x_2x_4^2 = 35$$

determine if it is possible to solve for variables $x_1,$

x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2,$

$x_3, x_4, u) = (0, 5, -3, 2, 2)$. Compute if possible $\frac{\partial x_1}{\partial u}(2)$.

- 1) $\frac{\partial x_1}{\partial u}(2) = 1$
- 2) $\frac{\partial x_1}{\partial u}(2) = 4$
- 3) $\frac{\partial x_1}{\partial u}(2) = 2$
- 4) $\frac{\partial x_1}{\partial u}(2) = 3$
- 5) $\frac{\partial x_1}{\partial u}(2) = 0$

Exercise 3

Given the function

$f(x, y, z) = -2 + x^2 + y^2 + z^2$ defined over the domain $D \equiv$
 $\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.2, ?, -0.2\}$
- 2) We have a minimum at $\{-0.4, ?, 0.5\}$
- 3) We have a minimum at $\{0.5, -0.4, ?\}$
- 4) We have a minimum at $\{?, -0.2, 0.4\}$
- 5) We have a minimum at $\{0, 0, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 63

Exercise 1

Given the functions

$$f(x, y) = (2 - x - 2y + xy + 3y^2, -3 + 2x - 2x^2 - xy - 3y^2, -2 + 2x + 3x^2 + 2xy, 3 + 3x + x^2 - 2y - 2xy)$$

and

$$g(u_1, u_2, u_3, u_4) = (-u_1^2 - 3u_1u_3 + 2u_2u_3 - u_1u_4 + 2u_3u_4, 3u_1 + 2u_2^2 - 2u_3 - 3u_1u_3 - 2u_4 + 2u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 1)$.

- 1) -10742.6
- 2) -1852.9
- 3) -12238.
- 4) -2589.11
- 5) -2043.14

Exercise 2

Given the system

$$-3u + 3u^2 + uv^2 - 2uvx + 3x^2 - y^2 = 17$$

$$3v^2 - 3uv^2 + uy - 3vy - 2xy + uxy = -332$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-4, -4, 5, -5)$. Compute if possible $\frac{\partial y}{\partial v}(5, -5)$.

- 1) $\frac{\partial y}{\partial v}(5, -5) = -\frac{206}{19}$
- 2) $\frac{\partial y}{\partial v}(5, -5) = -\frac{207}{19}$
- 3) $\frac{\partial y}{\partial v}(5, -5) = -\frac{203}{19}$
- 4) $\frac{\partial y}{\partial v}(5, -5) = -\frac{204}{19}$
- 5) $\frac{\partial y}{\partial v}(5, -5) = -\frac{205}{19}$

Exercise 3

Given the function

$f(x, y, z) = 17 - 4x + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 2.71488, 2.13312\}$
- 2) We have a minimum at $\{?, 1.55136, 1.55136\}$
- 3) We have a minimum at $\{1.67231, 1.9392, ?\}$
- 4) We have a minimum at $\{2.25408, 0.969602, ?\}$
- 5) We have a minimum at $\{?, 2, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 64

Exercise 1

Given the functions

$$f(x, y) = (-1 + 3x - 3x^2 - 3y + xy - 3y^2, 3 + 2x - 3x^2 + y + 3xy - 2y^2)$$

and

$$g(u, v) = (-3u - 3u^2 + v + 3v^2, 1 + 2u + 2u^2 - v - 2uv),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, 3)$.

- 1) 99987.7
- 2) 78353.4
- 3) 158675.
- 4) 248210.
- 5) 241128.

Exercise 2

Given the system

$$-3x - 3vy - 3vxy = -33$$

$$-2vx - x^2 + vx^2 - v^2y + xy - 2y^2 - xy^2 = 83$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-4, -5, -4, 1)$. Compute if possible $\frac{\partial y}{\partial v}(-4, 1)$.

- 1) $\frac{\partial y}{\partial v}(-4, 1) = -84$
- 2) $\frac{\partial y}{\partial v}(-4, 1) = -83$
- 3) $\frac{\partial y}{\partial v}(-4, 1) = -86$
- 4) $\frac{\partial y}{\partial v}(-4, 1) = -82$
- 5) $\frac{\partial y}{\partial v}(-4, 1) = -85$

Exercise 3

Given the function

$f(x, y, z) = -6 - 2x + x^2 + y^2 + z^2$ defined over the domain $D = \left\{ \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-0.390476, -4.87727, ?\}$
- 2) We have a maximum at $\{-0.690476, -4.87727, ?\}$
- 3) We have a maximum at $\{-0.690476, -4.87727, ?\}$
- 4) We have a maximum at $\{?, -4.97727, 0.\}$
- 5) We have a maximum at $\{?, 0, 0\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 65

Exercise 1

Given the functions

$$f(x, y, z) = (3 + 3x - 2yz, 3x + 2y + 2y^2 - 2yz)$$

and

$$g(u, v) = (1 - 3u + 3v + 2uv - 3v^2, -2 - u - 2u^2 + v - 2uv - 3v^2, 2 - 3u - 2u^2 + v + v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, -2, -3)$.

- 1) -0.788658
- 2) 0 .
- 3) -0.414784
- 4) 0.356025
- 5) -0.552427

Exercise 2

Given the system

$$3x^2 - 2ux^2 + u^2y - 2uxy + 3uz + 2yz - y^2z = 18$$

$$x^2 - u^2y - 2u^2z + uxz + 2x^2z + xz^2 = 32$$

$$-2ux - 3x^3 - 3uy^2 + xy^2 + y^3 + 2u^2z - 3x^2z = 52$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (-3, 3, 2, -1)$. Compute if possible $\frac{\partial z}{\partial u}(-1)$.

- 1) $\frac{\partial z}{\partial u}(-1) = -\frac{1857}{2087}$
- 2) $\frac{\partial z}{\partial u}(-1) = -\frac{1858}{2087}$
- 3) $\frac{\partial z}{\partial u}(-1) = -\frac{1856}{2087}$
- 4) $\frac{\partial z}{\partial u}(-1) = -\frac{1855}{2087}$
- 5) $\frac{\partial z}{\partial u}(-1) = -\frac{1854}{2087}$

Exercise 3

Given the function

$f(x, y, z) = 4 - 2x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{1, 0, ?\}$
- 2) We have a minimum at $\{1.4, ?, 2.6\}$
- 3) We have a minimum at $\{?, -0.8, 1.8\}$
- 4) We have a minimum at $\{0.6, ?, 1.\}$
- 5) We have a minimum at $\{0.4, ?, 1.8\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 66

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (3x_1 + 2x_3 + 2x_1x_3, 2x_3 + 3x_1x_3 + 2x_2x_4, 3 - 3x_1 - 2x_1^2 + 3x_4)$$

and

$$g(u, v, w) = (-3w, 2 - 2u - 3u^2 - w, -2uv - 3w, 2v + 3uv + 2v^2 + 2w + uw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (0, 3, -3, 2).$$

- 1) 0.534668
- 2) 0.412235
- 3) 0.
- 4) 0.198178
- 5) 0.736504

Exercise 2

Given the system

$$2x_2x_3 - x_3^2 - 2x_1x_2x_4 = -12$$

$$2u^2x_3 - ux_2x_4 - 2x_3^2x_4 = -110$$

$$-x_2x_3 - x_1x_2x_3 + 3vx_2x_4 = -86$$

$$uvx_3 + 3x_2x_4 = 0$$

determine if it is possible to solve for variables x_1, x_2, x_3

, x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4$

, $u, v) = (0, 2, -2, -5, -5, 3)$. Compute if possible $\frac{\partial x_3}{\partial u}(-5, 3)$.

- 1) $\frac{\partial x_3}{\partial u}(-5, 3) = -\frac{84}{145}$
- 2) $\frac{\partial x_3}{\partial u}(-5, 3) = -\frac{421}{725}$
- 3) $\frac{\partial x_3}{\partial u}(-5, 3) = -\frac{1262}{2175}$
- 4) $\frac{\partial x_3}{\partial u}(-5, 3) = -\frac{1261}{2175}$
- 5) $\frac{\partial x_3}{\partial u}(-5, 3) = -\frac{1264}{2175}$

Exercise 3

Given the function

$f(x, y, z) = 7 - 2x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-1.20808, -0.889145, ?\}$
- 2) We have a maximum at $\{?, -0.389145, -4.02423\}$
- 3) We have a maximum at $\{-1.10808, -0.489145, ?\}$
- 4) We have a maximum at $\{-1.00808, ?, -3.42423\}$
- 5) We have a maximum at $\{?, 2, 3\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 67

Exercise 1

Given the functions

$$f(x, y, z) = (-3x - 3yz, -2x + 2y - 3xy - z, z + 2z^2)$$

and

$$g(u, v, w) = (-1 + 2v + uv + v^2, -2 - 3uv + 3v^2 + vw, -3u - v - 2uv + 2v^2 + vw + 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, -1, -3)$.

- 1) -122 211.
- 2) -514 404.
- 3) -968 527.
- 4) -945 095.
- 5) -794 800.

Exercise 2

Given the system

$$uyz - uz^2 = 20$$

$$uvy + 3xy^2 = 96$$

$$-2uy + 3uxy - 2vz = -22$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (1, 4, -1, -4, -3)$. Compute if possible $\frac{\partial z}{\partial u}(-4, -3)$.

$u, v) = (1, 4, -1, -4, -3)$. Compute if possible $\frac{\partial z}{\partial u}(-4, -3)$.

- 1) $\frac{\partial z}{\partial u}(-4, -3) = -\frac{14}{99}$
- 2) $\frac{\partial z}{\partial u}(-4, -3) = -\frac{13}{99}$
- 3) $\frac{\partial z}{\partial u}(-4, -3) = -\frac{16}{99}$
- 4) $\frac{\partial z}{\partial u}(-4, -3) = -\frac{5}{33}$
- 5) $\frac{\partial z}{\partial u}(-4, -3) = -\frac{4}{33}$

Exercise 3

Given the function

$f(x, y, z) = 1 + x^2 - 4y + y^2 + z^2$ defined over the domain $D = \left\{ \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.4, 1.4, ?\}$
- 2) We have a minimum at $\{-0.6, ?, -0.2\}$
- 3) We have a minimum at $\{?, 2.6, 0.6\}$
- 4) We have a minimum at $\{-0.4, 1.4, ?\}$
- 5) We have a minimum at $\{?, 2, 0\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 68

Exercise 1

Given the functions

$$f(x, y, z) = (xy + z^2, -3x^2 - y + 3z - xz + yz, 3x - 2xz + z^2)$$

and

$$g(u, v, w) = (2u + v + 3v^2, -1 + 3uv - 2vw, 1 + u^2 + 2w + 3vw + 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 1, 3)$.

- 1) -3.79644×10^6
- 2) -2.49389×10^6
- 3) -2.35305×10^7
- 4) -1.43458×10^7
- 5) -1.35752×10^7

Exercise 2

Given the system

$$3u^2v - 2v^2 - u^2x + ux^2 - 3vy^2 + 2xy^2 + 2vzx = 184$$

$$2u^2x + vy + 3vyz = -36$$

$$2uvy + 2uxz = -2$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (2, -5, -3, 1, -1)$. Compute if possible $\frac{\partial x}{\partial v}(1, -1)$.

- 1) $\frac{\partial x}{\partial v}(1, -1) = -\frac{8366}{5171}$
- 2) $\frac{\partial x}{\partial v}(1, -1) = -\frac{8370}{5171}$
- 3) $\frac{\partial x}{\partial v}(1, -1) = -\frac{8369}{5171}$
- 4) $\frac{\partial x}{\partial v}(1, -1) = -\frac{8368}{5171}$
- 5) $\frac{\partial x}{\partial v}(1, -1) = -\frac{8367}{5171}$

Exercise 3

Given the function

$f(x, y, z) = 2 + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0, 2, ?\}$
- 2) We have a maximum at $\{0.5, -2.8509, ?\}$
- 3) We have a maximum at $\{0., -2.9509, ?\}$
- 4) We have a maximum at $\{0.2, ?, 0.139634\}$
- 5) We have a maximum at $\{0.3, ?, 0.0396343\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 69

Exercise 1

Given the functions

$$f(x, y, z) = (2y + 3y^2 + yz, 2x^2 - y, -2x^2, -2xz + 2yz)$$

and

$$g(u_1, u_2, u_3, u_4) = (-3u_2 - 3u_1u_2 + 3u_3 + u_2u_3, -3u_2 - u_1u_2 + 3u_3, 3 + 3u_1^2 + 2u_3 - 2u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, 1, -2)$.

- 1) 4713.86
- 2) 3624.52
- 3) 417.181
- 4) 4692.42
- 5) 3024.

Exercise 2

Given the system

$$w^2x + 2uvy = 152$$

$$-2xy = -20$$

$$3w^3 + 3uxy + vyz - 2yz^2 = 93$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v, w around the point $p = (x, y, z, u, v, w) = (2, 5, 0, 3, 5, 1)$. Compute if possible $\frac{\partial x}{\partial w}(3, 5, 1)$.

- 1) $\frac{\partial x}{\partial w}(3, 5, 1) = \frac{4}{37}$
- 2) $\frac{\partial x}{\partial w}(3, 5, 1) = \frac{2}{37}$
- 3) $\frac{\partial x}{\partial w}(3, 5, 1) = \frac{5}{37}$
- 4) $\frac{\partial x}{\partial w}(3, 5, 1) = \frac{3}{37}$
- 5) $\frac{\partial x}{\partial w}(3, 5, 1) = \frac{6}{37}$

Exercise 3

Given the function

$f(x, y, z) = -1 + 4x - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2, 1, ?\}$
- 2) We have a maximum at $\{1., 1.2, ?\}$
- 3) We have a maximum at $\{?, 1.6, 2.8\}$
- 4) We have a maximum at $\{1.6, 0., ?\}$
- 5) We have a maximum at $\{?, 1.6, 2.2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 70

Exercise 1

Given the functions

$$f(x, y) = (2 + x - 3y - 3xy, 2 - x + 3x^2 - 3y + 2xy - y^2)$$

and

$$g(u, v) = (-1 - u - 2u^2 + uv - v^2, 1 - u - u^2 + v - 2uv - v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, -2)$.

- 1) 5572.
- 2) 4487.41
- 3) 8336.19
- 4) 4681.66
- 5) 3842.36

Exercise 2

Given the system

$$-3u^2v + v^3 + 3vxy + 3uy^2 = -608$$

$$-2u - v + 3x^2 - 3y + 3xy + 3x^2y = -134$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (3, -5, -4, 4)$. Compute if possible $\frac{\partial y}{\partial v}(-4, 4)$.

- 1) $\frac{\partial y}{\partial v}(-4, 4) = \frac{1}{3}$
- 2) $\frac{\partial y}{\partial v}(-4, 4) = \frac{429}{1288}$
- 3) $\frac{\partial y}{\partial v}(-4, 4) = \frac{1289}{3864}$
- 4) $\frac{\partial y}{\partial v}(-4, 4) = \frac{643}{1932}$
- 5) $\frac{\partial y}{\partial v}(-4, 4) = \frac{1285}{3864}$

Exercise 3

Given the function

$f(x, y, z) = 15 - 6x + x^2 - 6y + y^2 + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 1.41421, 0.\}$
- 2) We have a minimum at $\{1.27279, ?, -0.424264\}$
- 3) We have a minimum at $\{1.55563, ?, -0.565685\}$
- 4) We have a minimum at $\{?, 3, 0\}$
- 5) We have a minimum at $\{0.707107, 0.707107, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 71

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (2 + x_1^2 - 2x_2 - x_2^2 - 2x_3 + 2x_4 - x_3x_4, -2 - 2x_1 - x_3 - 3x_3^2 + 3x_4 - 3x_1x_4 + x_2x_4, 1 - 3x_2^2 + 3x_1x_4)$$

and

$$g(u, v, w) = (-u^2, 1 + u^2 - 3v - uv + 3uw, 3u^2 - 3v - uv - v^2 + uw, -v + uv + 3v^2 - w + 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-3, -1, -2, -1).$$

- 1) -0.654162
- 2) 0.578378
- 3) 0.32783
- 4) 0.264902
- 5) 0 .

Exercise 2

Given the system

$$-v x_2 x_3 = 50$$

$$-3u x_1 + 3x_2^2 x_3 = 363$$

$$3v x_1^2 + 3u x_2 x_3 + 3u x_2 x_4 = 84$$

$$-3u x_2^2 - v x_4 - v^2 x_4 = 168$$

determine if it is possible to solve for variables $x_1, x_2, x_3,$

x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$u, v) = (-2, -5, 5, -3, -2, 2)$. Compute if possible $\frac{\partial x_2}{\partial u}(-2, 2)$.

- 1) $\frac{\partial x_2}{\partial u}(-2, 2) = -\frac{62}{97}$
- 2) $\frac{\partial x_2}{\partial u}(-2, 2) = -\frac{123}{194}$
- 3) $\frac{\partial x_2}{\partial u}(-2, 2) = -\frac{125}{194}$
- 4) $\frac{\partial x_2}{\partial u}(-2, 2) = -\frac{127}{194}$
- 5) $\frac{\partial x_2}{\partial u}(-2, 2) = -\frac{63}{97}$

Exercise 3

Given the function

$f(x, y, z) = 23 - 2x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.5, 2.3, ?\}$
- 2) We have a minimum at $\{0.1, ?, 4.2\}$
- 3) We have a minimum at $\{?, 2.9, 4.5\}$
- 4) We have a minimum at $\{1, ?, 3\}$
- 5) We have a minimum at $\{-0.5, 0.8, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 72

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (x_1^2 + 3x_2 + 2x_4 + 2x_1x_4, \\ 2x_1^2 - 2x_1x_2 - 3x_3^2 + 3x_4 - x_3x_4, -3x_2x_3 - 2x_3^2 + 3x_4 + x_3x_4, -2x_1^2 + 2x_2^2 - x_1x_3 + 3x_1x_4)$$

and

$$g(u_1, u_2, u_3, u_4) = (2u_4 + 2u_1u_4, -2u_1 + u_1^2 + 2u_3 - 2u_1u_3 + 2u_3^2 + u_1u_4 + 2u_4^2, \\ -3u_1 - 2u_2 + 3u_2u_3 - u_4, 1 + 2u_2 - 3u_1u_2 + u_3^2 + u_1u_4)$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 1, -2, 1)$.

- 1) -342347.
- 2) -291663.
- 3) -252962.
- 4) -410400.
- 5) -283380.

Exercise 2

Given the system

$$-3v x_2 - 3x_3^3 = -330$$

$$2x_2 x_4^2 = 40$$

$$-2u w x_4 + x_1 x_4^2 = 44$$

$$x_2 x_3 x_4 = -50$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u, v$

, $w) = (5, 5, 5, -2, -2, -3, -3)$. Compute if possible $\frac{\partial x_4}{\partial w}(-2, -3, -3)$.

- 1) $\frac{\partial x_4}{\partial w}(-2, -3, -3) = 3$
- 2) $\frac{\partial x_4}{\partial w}(-2, -3, -3) = 1$
- 3) $\frac{\partial x_4}{\partial w}(-2, -3, -3) = 4$
- 4) $\frac{\partial x_4}{\partial w}(-2, -3, -3) = 0$
- 5) $\frac{\partial x_4}{\partial w}(-2, -3, -3) = 2$

Exercise 3

Given the function

$f(x, y, z) = 26 - 6x + x^2 - 6y + y^2 + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-2.62843, -3.32843, ?\}$
- 2) We have a maximum at $\{?, -2.82843, 0.\}$
- 3) We have a maximum at $\{-2.92843, -2.72843, ?\}$
- 4) We have a maximum at $\{3, 3, ?\}$
- 5) We have a maximum at $\{-2.72843, ?, 0.4\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 73

Exercise 1

Given the functions

$$f(x, y) = (2 + 3x - 2x^2 - y + 3xy, -1 + x^2 + 2xy + 3y^2, 3 - x - 3x^2 + 3y - xy + 3y^2)$$

and

$$g(u, v, w) = (-2u^2, -3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-2, 2)$.

- 1) -392017 .
- 2) -139158 .
- 3) -594048 .
- 4) -1.11356×10^6
- 5) -332370 .

Exercise 2

Given the system

$$-x^2 + 3y^3 = -19$$

$$3x^2y = -48$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4$

$) = (-4, -1, 1, 4, 0, -3)$. Compute if possible $\frac{\partial y}{\partial u_3} (1, 4, 0, -3)$.

- 1) $\frac{\partial y}{\partial u_3} (1, 4, 0, -3) = 4$
- 2) $\frac{\partial y}{\partial u_3} (1, 4, 0, -3) = 0$
- 3) $\frac{\partial y}{\partial u_3} (1, 4, 0, -3) = 2$
- 4) $\frac{\partial y}{\partial u_3} (1, 4, 0, -3) = 1$
- 5) $\frac{\partial y}{\partial u_3} (1, 4, 0, -3) = 3$

Exercise 3

Given the function

$f(x, y, z) = 14 - 4x + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, -1.92172, -3.71351\}$
- 2) We have a maximum at $\{-0.375378, -1.42172, ?\}$
- 3) We have a maximum at $\{-0.475378, -1.52172, ?\}$
- 4) We have a maximum at $\{-0.0753781, ?, -3.51351\}$
- 5) We have a maximum at $\{?, 2, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 74

Exercise 1

Given the functions

$$f(x, y) = (-3 + 2x - 3x^2 + 3y - xy + 3y^2, \\ 2 - x - 3x^2 + 2y + 2xy - y^2, -2 + x + 3y + 2y^2, -1 - x^2 + 2y - 2xy - 3y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-u_1^2 - 2u_1u_2 - 3u_3^2 - 3u_3u_4 + 2u_4^2, -3u_1 - 2u_1^2 + 3u_2 - 3u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, 0)$.

- 1) 81030.2
- 2) 261039.
- 3) 181331.
- 4) 266927.
- 5) 150298.

Exercise 2

Given the system

$$-3x^2y = 24$$

$$-3xyu_4 = 60$$

determine if it is possible to solve for variables x, y in terms of variables

$$u_1, u_2, u_3, u_4, u_5 \text{ around the point } p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-2,$$

$$-2, 0, 4, 3, -5, -4). \text{ Compute if possible } \frac{\partial x}{\partial u_1} (0, 4, 3, -5, -4).$$

- 1) $\frac{\partial x}{\partial u_1} (0, 4, 3, -5, -4) = 4$
- 2) $\frac{\partial x}{\partial u_1} (0, 4, 3, -5, -4) = 0$
- 3) $\frac{\partial x}{\partial u_1} (0, 4, 3, -5, -4) = 3$
- 4) $\frac{\partial x}{\partial u_1} (0, 4, 3, -5, -4) = 2$
- 5) $\frac{\partial x}{\partial u_1} (0, 4, 3, -5, -4) = 1$

Exercise 3

Given the function

$f(x, y, z) = 12 - 2x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D =$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{1, 2, ?\}$
- 2) We have a maximum at $\{-0.395944, ?, -0.087832\}$
- 3) We have a maximum at $\{?, -4.00319, -0.987832\}$
- 4) We have a maximum at $\{?, -3.80319, -0.587832\}$
- 5) We have a maximum at $\{?, -3.90319, -0.287832\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 75

Exercise 1

Given the functions

$$f(x, y) = (-3 + 2y - 2xy + y^2, -3 + 3x - 3x^2 + y + 2xy + y^2, 2 + 3x - 2x^2 + y - 3xy + 3y^2, -3 + 2x + x^2 + 2y)$$

and

$$g(u_1, u_2, u_3, u_4) = (3u_1^2 - 2u_1u_2 - 3u_2^2 + u_4 + 2u_1u_4, -2u_1^2 - 3u_1u_2 + u_1u_4 + u_2u_4 - u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, 0)$.

- 1) 4281.99
- 2) 3000.
- 3) 4605.5
- 4) 3567.67
- 5) 4382.7

Exercise 2

Given the system

$$2xyu_3 = 200$$

$$-2x^2u_1 + xyu_5 = -228$$

determine if it is possible to solve for variables x, y in terms of variables

$$u_1, u_2, u_3, u_4, u_5 \text{ around the point } p = (x, y, u_1, u_2, u_3, u_4, u_5) = (4$$

$$, 5, 4, -5, 5, -1, -5). \text{ Compute if possible } \frac{\partial x}{\partial u_3} (4, -5, 5, -1, -5).$$

- 1) $\frac{\partial x}{\partial u_3} (4, -5, 5, -1, -5) = \frac{5}{16}$
- 2) $\frac{\partial x}{\partial u_3} (4, -5, 5, -1, -5) = \frac{9}{16}$
- 3) $\frac{\partial x}{\partial u_3} (4, -5, 5, -1, -5) = \frac{7}{16}$
- 4) $\frac{\partial x}{\partial u_3} (4, -5, 5, -1, -5) = \frac{3}{8}$
- 5) $\frac{\partial x}{\partial u_3} (4, -5, 5, -1, -5) = \frac{1}{2}$

Exercise 3

Given the function

$f(x, y, z) = 9 + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{6.68047, ?, -0.887048\}$
- 2) We have a maximum at $\{?, 1, 1\}$
- 3) We have a maximum at $\{?, -2.4173, -1.33241\}$
- 4) We have a maximum at $\{?, -0.190476, -1.77778\}$
- 5) We have a maximum at $\{?, 1.59098, -1.33241\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 76

Exercise 1

Given the functions

$$f(x, y) = (2 - 3x + 3x^2 + 3xy + 2y^2, 1 - 2x - y - 2xy - 3y^2, 3x^2 + xy - y^2, -2 + x - 2x^2 - 3xy - 3y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-2u_1^2 + 3u_1u_3 - u_3^2, -u_1^2 + 2u_3^2 + u_4 - u_1u_4 + 3u_2u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, -3)$.

- 1) -28130.3
- 2) -292003.
- 3) -322366.
- 4) -172854.
- 5) -289486.

Exercise 2

Given the system

$$2xy - yu_2^2 + 3u_2u_3u_4 = 24$$

$$-3x^2u_3 - 3xyu_3 = 0$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4$

$) = (2, -2, -4, 4, -5, 0)$. Compute if possible $\frac{\partial y}{\partial u_4}(-4, 4, -5, 0)$.

- 1) $\frac{\partial y}{\partial u_4}(-4, 4, -5, 0) = -\frac{13}{2}$
- 2) $\frac{\partial y}{\partial u_4}(-4, 4, -5, 0) = -\frac{15}{2}$
- 3) $\frac{\partial y}{\partial u_4}(-4, 4, -5, 0) = -\frac{11}{2}$
- 4) $\frac{\partial y}{\partial u_4}(-4, 4, -5, 0) = -6$
- 5) $\frac{\partial y}{\partial u_4}(-4, 4, -5, 0) = -7$

Exercise 3

Given the function

$f(x, y, z) = -23 + 6x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-0.679444, ?, -1.62803\}$
- 2) We have a minimum at $\{?, -4.08408, -0.928028\}$
- 3) We have a minimum at $\{?, -3.38408, -1.12803\}$
- 4) We have a minimum at $\{?, -3.68408, -1.22803\}$
- 5) We have a minimum at $\{3, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 77

Exercise 1

Given the functions

$$f(x, y) = (2 - x + 3x^2 + y - 2xy + 3y^2, -3 - x + x^2 + xy - 2y^2)$$

and

$$g(u, v) = (-u + u^2 - v - uv, 1 + 3u - 3u^2 - 3uv - 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, -1)$.

- 1) 101115.
- 2) 23379.1
- 3) 175835.
- 4) 187772.
- 5) 141538.

Exercise 2

Given the system

$$-xy^2 = 36$$

$$3yu_2u_3 + xu_1u_5 = 16$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 around the point $p = (x, y, u_1, u_2, u_3, u_4, u_5)$

$= (-4, 3, -1, 1, 4, 2, -5)$. Compute if possible $\frac{\partial x}{\partial u_2}(-1, 1, 4, 2, -5)$.

- 1) $\frac{\partial x}{\partial u_2}(-1, 1, 4, 2, -5) = -\frac{70}{19}$
- 2) $\frac{\partial x}{\partial u_2}(-1, 1, 4, 2, -5) = -\frac{69}{19}$
- 3) $\frac{\partial x}{\partial u_2}(-1, 1, 4, 2, -5) = -\frac{72}{19}$
- 4) $\frac{\partial x}{\partial u_2}(-1, 1, 4, 2, -5) = -\frac{71}{19}$
- 5) $\frac{\partial x}{\partial u_2}(-1, 1, 4, 2, -5) = -\frac{68}{19}$

Exercise 3

Given the function

$f(x, y, z) = 9 + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-0.3, ?, -4.06025\}$
- 2) We have a maximum at $\{0.5, ?, -4.56025\}$
- 3) We have a maximum at $\{?, 2, 3\}$
- 4) We have a maximum at $\{0., -2.7735, ?\}$
- 5) We have a maximum at $\{?, -2.9735, -3.86025\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 78

Exercise 1

Given the functions

$$f(x, y, z) = (3x^2 - 2xz, -2yz + 3z^2)$$

and

$$g(u, v) = (-3 + 3u + u^2 - v - 3uv, -3 - 2u + 2u^2 - v - 3uv, 1 + 3u + u^2 + 3v - 3uv + 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, -2, -3)$.

- 1) 0.231278
- 2) 0.535542
- 3) 0.709021
- 4) 0.
- 5) -0.594464

Exercise 2

Given the system

$$xu_3u_4 + yu_4^2 = 18$$

$$3x^2u_1 - 2z^2u_1 - 2yu_1^2 - 2zu_2^2 = -7$$

$$y^2z - 2yzu_2 = 33$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4$

$) = (-3, -3, 1, -5, 4, -5, 3)$. Compute if possible $\frac{\partial z}{\partial u_4}(-5, 4, -5, 3)$.

- 1) $\frac{\partial z}{\partial u_4}(-5, 4, -5, 3) = -5$
- 2) $\frac{\partial z}{\partial u_4}(-5, 4, -5, 3) = -3$
- 3) $\frac{\partial z}{\partial u_4}(-5, 4, -5, 3) = -4$
- 4) $\frac{\partial z}{\partial u_4}(-5, 4, -5, 3) = -6$
- 5) $\frac{\partial z}{\partial u_4}(-5, 4, -5, 3) = -7$

Exercise 3

Given the function

$f(x, y, z) = -17 + 4x - x^2 - y^2 + 6z - z^2$ defined over the domain $D =$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{1.33464, ?, 2.42079\}$
- 2) We have a maximum at $\{?, -0.864567, 1.38331\}$
- 3) We have a maximum at $\{2, 0, ?\}$
- 4) We have a maximum at $\{?, 0.345827, 1.21039\}$
- 5) We have a maximum at $\{1.50755, ?, 1.72913\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 79

Exercise 1

Given the functions

$$f(x, y, z) = (-1 - y^2, 1 + 2x^2, 3z + 3yz)$$

and

$$g(u, v, w) = (u + 2w + 2uw + w^2, -uv - w + 3uw - 2vw, 2uv - 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, -2, -3)$.

- 1) -1.31754×10^6
- 2) $-846912.$
- 3) $-382352.$
- 4) $-390822.$
- 5) -91322.2

Exercise 2

Given the system

$$-u x^2 + 3 y^2 z = -112$$

$$-2 v^2 x + x^2 + u y - v z + 3 x z^2 = 162$$

$$3 u x z = 24$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (-4, 4, -2, 1, -5)$. Compute if possible $\frac{\partial z}{\partial u}(1, -5)$.

- 1) $\frac{\partial z}{\partial u}(1, -5) = \frac{539}{437}$
- 2) $\frac{\partial z}{\partial u}(1, -5) = \frac{542}{437}$
- 3) $\frac{\partial z}{\partial u}(1, -5) = \frac{543}{437}$
- 4) $\frac{\partial z}{\partial u}(1, -5) = \frac{541}{437}$
- 5) $\frac{\partial z}{\partial u}(1, -5) = \frac{540}{437}$

Exercise 3

Given the function

$f(x, y, z) = -20 + 2x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{0.904534, 2.7136, ?\}$
- 2) We have a maximum at $\{1, 3, ?\}$
- 3) We have a maximum at $\{?, 1.3568, 1.71861\}$
- 4) We have a maximum at $\{?, 2.44224, 0.361814\}$
- 5) We have a maximum at $\{0.633174, ?, -0.452267\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 80

Exercise 1

Given the functions

$$f(x, y, z) = (-2x + xy - xz, -1 + 3x - y^2 - xz, z)$$

and

$$g(u, v, w) = (u + u^2 + v^2 - w + 2w^2, v^2 + 2w, -2u^2 - uv + 3w),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 0, 3)$.

- 1) 0.823258
- 2) 0.
- 3) 0.369412
- 4) -0.722986
- 5) 0.824788

Exercise 2

Given the system

$$2u^2 + 2v^2x - 2u^2y = 56$$

$$-1 + x - xy + 3yz^2 = -98$$

$$2u^2x - 3y - 2u^2y - 2v^2z - uxz + 2vxz - 2yz^2 = 61$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (-4, -3, -3, -4, -3)$. Compute if possible $\frac{\partial x}{\partial v}(-4, -3)$.

- 1) $\frac{\partial x}{\partial v}(-4, -3) = \frac{11211}{5713}$
- 2) $\frac{\partial x}{\partial v}(-4, -3) = \frac{11208}{5713}$
- 3) $\frac{\partial x}{\partial v}(-4, -3) = \frac{11212}{5713}$
- 4) $\frac{\partial x}{\partial v}(-4, -3) = \frac{11209}{5713}$
- 5) $\frac{\partial x}{\partial v}(-4, -3) = \frac{11210}{5713}$

Exercise 3

Given the function

$f(x, y, z) = 18 - 4x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 0.6, 2.4\}$
- 2) We have a minimum at $\{?, -0.8, 1.\}$
- 3) We have a minimum at $\{2, ?, 2\}$
- 4) We have a minimum at $\{1.2, ?, 1.8\}$
- 5) We have a minimum at $\{1.2, ?, 1.2\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 81

Exercise 1

Given the functions

$$f(x, y) = (2 - 3x - 2x^2 - 2y - xy, 1 + x - x^2 - 3xy - 2y^2, -2 + 3x - 3x^2 - y + 3xy + y^2, -3 - 2x + x^2 + 3y - y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (2u_2^2 - u_1u_3 - 2u_3^2 + 2u_3u_4, u_3 + 2u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 2)$.

- 1) -1371.88
- 2) -2778.97
- 3) -5856.
- 4) -2238.
- 5) -1111.05

Exercise 2

Given the system

$$-uv^2 + 2w^3 + 2wx + uv y = -164$$

$$2v^2 + 2vx^2 - y^3 = 2$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u, v$

, $w) = (2, 2, 4, -5, -2)$. Compute if possible $\frac{\partial y}{\partial v}(4, -5, -2)$.

- 1) $\frac{\partial y}{\partial v}(4, -5, -2) = \frac{124}{47}$
- 2) $\frac{\partial y}{\partial v}(4, -5, -2) = \frac{126}{47}$
- 3) $\frac{\partial y}{\partial v}(4, -5, -2) = \frac{127}{47}$
- 4) $\frac{\partial y}{\partial v}(4, -5, -2) = \frac{125}{47}$
- 5) $\frac{\partial y}{\partial v}(4, -5, -2) = \frac{123}{47}$

Exercise 3

Given the function

$f(x, y, z) = -2 - 4x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 1.92121, 0.233564\}$
- 2) We have a minimum at $\{1.87516, 0.983629, ?\}$
- 3) We have a minimum at $\{2.43771, 1.35866, ?\}$
- 4) We have a minimum at $\{2, 1, ?\}$
- 5) We have a minimum at $\{?, 1.92121, 1.17115\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 82

Exercise 1

Given the functions

$$f(x, y) = (3 + 3x^2 - y - xy + 2y^2, -3 + 2x - x^2 - y - 3xy + 3y^2, -2 + y + 2y^2, 1 - 2x - 3x^2 + 3y + 3xy + 3y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-1 + 2u_1^2 + 3u_2 - 2u_2^2 - 2u_1u_3 - 2u_2u_3 + u_4^2, u_1u_2 - 2u_4 + 2u_2u_4 - 2u_3u_4 - 3u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 1)$.

- 1) -57715.4
- 2) -32504.
- 3) -58510.2
- 4) -53263.4
- 5) -4378.93

Exercise 2

Given the system

$$3xu_4^2 - 2yu_1u_5 = 67$$

$$2xyu_4 = 6$$

determine if it is possible to solve for variables x, y in terms of variables

$$u_1, u_2, u_3, u_4, u_5 \text{ around the point } p = (x, y, u_1, u_2, u_3, u_4, u_5) = (1,$$

$$-1, -4, -1, 0, -3, -5). \text{ Compute if possible } \frac{\partial x}{\partial u_5}(-4, -1, 0, -3, -5).$$

- 1) $\frac{\partial x}{\partial u_5}(-4, -1, 0, -3, -5) = -\frac{8}{13}$
- 2) $\frac{\partial x}{\partial u_5}(-4, -1, 0, -3, -5) = -\frac{4}{13}$
- 3) $\frac{\partial x}{\partial u_5}(-4, -1, 0, -3, -5) = -\frac{5}{13}$
- 4) $\frac{\partial x}{\partial u_5}(-4, -1, 0, -3, -5) = -\frac{6}{13}$
- 5) $\frac{\partial x}{\partial u_5}(-4, -1, 0, -3, -5) = -\frac{7}{13}$

Exercise 3

Given the function

$f(x, y, z) = -17 - x^2 + 4y - y^2 + 4z - z^2$ defined over the domain $D =$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 1.6, 1.2\}$
- 2) We have a maximum at $\{-0.8, ?, 2.2\}$
- 3) We have a maximum at $\{-0.4, ?, 2.6\}$
- 4) We have a maximum at $\{0, 2, ?\}$
- 5) We have a maximum at $\{-0.8, ?, 1.\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 83

Exercise 1

Given the functions

$$f(x, y) = (-2 + 3x^2 - y + y^2, 3 + 3x + 2x^2 + y - 3xy - y^2)$$

and

$$g(u, v) = (2u + u^2 + 2v + 2uv - 2v^2, 3 + 2u - 3u^2 - v + 3uv + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 1)$.

- 1) -2.32934×10^6
- 2) -4.10759×10^6
- 3) -1.0947×10^6
- 4) -3.53819×10^6
- 5) -315527 .

Exercise 2

Given the system

$$3u^2 - 2x - 2ux^2 + y + 3uy - 3y^2 - uy^2 + 3xy^2 + 3y^3 = -465$$

$$2u - 2u^2 - u^3 + 2x + 2ux - ux^2 - 3x^2y + uy^2 + 3y^3 = -105$$

determine if it is possible to solve for variables x, y in terms of variable

u around the point $p = (x, y, u) = (5, -5, 5)$. Compute if possible $\frac{\partial y}{\partial u}(5)$.

- 1) $\frac{\partial y}{\partial u}(5) = \frac{249}{607}$
- 2) $\frac{\partial y}{\partial u}(5) = \frac{997}{2428}$
- 3) $\frac{\partial y}{\partial u}(5) = \frac{2989}{7284}$
- 4) $\frac{\partial y}{\partial u}(5) = \frac{2987}{7284}$
- 5) $\frac{\partial y}{\partial u}(5) = \frac{1495}{3642}$

Exercise 3

Given the function

$f(x, y, z) = 15 + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, -4.85843, -0.557368\}$
- 2) We have a maximum at $\{0.5, -5.35843, ?\}$
- 3) We have a maximum at $\{?, -4.95843, -0.257368\}$
- 4) We have a maximum at $\{-0.4, ?, -0.657368\}$
- 5) We have a maximum at $\{0, ?, 2\}$

Further Mathematics - Degree in Engineering - 2024/2025
 01-Multivariate Functions-Training computers exam for for
 serial number: 84

Exercise 1

Given the functions

$$f(x, y) = (-2 + x - 2x^2 - 2y + 3xy + y^2, \\ 2 - 3x - 2x^2 + 3y - xy + 2y^2, 3 + x^2 + xy - y^2, 1 + 2x - x^2 - 3y + xy + y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-2 + u_3 + 2u_1u_3 + u_2u_3 + u_1u_4 - 3u_4^2, -u_1^2 + 3u_2 + 3u_1u_2 - 3u_2^2 - 2u_3 + 2u_3^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-1, 2).$$

- 1) -25 099.7
- 2) -46 286.5
- 3) -59 732.3
- 4) -69 742.
- 5) -109 941.

Exercise 2

Given the system

$$-2u - 2v^2 + v^2x + 2vx^2 + wx^2 + 3xy^2 = -60$$

$$-2x + uvv + uxy - x^2y = 30$$

determine if it is possible to solve for variables x, y

in terms of variables u, v, w around the point $p = (x, y, u, v,$

$$w) = (-1, 2, -3, -4, 2). \text{ Compute if possible } \frac{\partial x}{\partial u}(-3, -4, 2).$$

- 1) $\frac{\partial x}{\partial u}(-3, -4, 2) = \frac{19}{64}$
- 2) $\frac{\partial x}{\partial u}(-3, -4, 2) = \frac{39}{128}$
- 3) $\frac{\partial x}{\partial u}(-3, -4, 2) = \frac{5}{16}$
- 4) $\frac{\partial x}{\partial u}(-3, -4, 2) = \frac{37}{128}$
- 5) $\frac{\partial x}{\partial u}(-3, -4, 2) = \frac{41}{128}$

Exercise 3

Given the function

$f(x, y, z) = 16 - 4x + x^2 - 6y + y^2 - 4z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, 2.83405, 1.46416\}$
- 2) We have a minimum at $\{0.472342, ?, 0.897346\}$
- 3) We have a minimum at $\{?, 3, 2\}$
- 4) We have a minimum at $\{3.02299, ?, 0.330536\}$
- 5) We have a minimum at $\{2.17277, 1.98383, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 85

Exercise 1

Given the functions

$$f(x, y, z) = (3 - 3x + yz, -z)$$

and

$$g(u, v) = (-2 + 2u + 2v + uv + v^2, -3 - 2u + v - 2uv + v^2, 3 + u - 2u^2 - 2v + 2uv - v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 3, -3)$.

- 1) 0.494319
- 2) 0.
- 3) -0.228499
- 4) -0.609678
- 5) 0.769484

Exercise 2

Given the system

$$xz = 8$$

$$2y^2z = 128$$

$$-zu_1 - xu_4 = 0$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4)$

$= (2, -4, 4, 1, 4, 5, -2)$. Compute if possible $\frac{\partial z}{\partial u_3} (1, 4, 5, -2)$.

- 1) $\frac{\partial z}{\partial u_3} (1, 4, 5, -2) = 2$
- 2) $\frac{\partial z}{\partial u_3} (1, 4, 5, -2) = 0$
- 3) $\frac{\partial z}{\partial u_3} (1, 4, 5, -2) = 4$
- 4) $\frac{\partial z}{\partial u_3} (1, 4, 5, -2) = 3$
- 5) $\frac{\partial z}{\partial u_3} (1, 4, 5, -2) = 1$

Exercise 3

Given the function

$f(x, y, z) = -4 + 6x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{-1.35445, -4.12621, ?\}$
- 2) We have a minimum at $\{-1.05445, ?, -0.45081\}$
- 3) We have a minimum at $\{-1.25445, ?, -0.15081\}$
- 4) We have a minimum at $\{-0.75445, -4.82621, ?\}$
- 5) We have a minimum at $\{3, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 86

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-x_1^2 - 2x_1x_2 + x_1x_3, \\ -3x_1x_2 + x_1x_3 + 2x_3^2, -1 + 2x_1 - 3x_1^2 - 3x_1x_3, -2 + 2x_1^2 + 3x_1x_2 - 2x_3 + 3x_4)$$

and

$$g(u_1, u_2, u_3, u_4) = (-u_2 + 2u_2^2 + u_3^2 - 2u_4 - 2u_1u_4 - u_3u_4, \\ -2u_1u_3 + 2u_1u_4 - u_2u_4, 3u_2^2 - u_4 + 2u_3u_4 - 2u_4^2, 3u_1 - u_2 + 2u_1u_2 + 3u_3 + 2u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, 2, 1, 0)$.

1) 1.67172×10^7

2) 1.23785×10^7

3) 2.48247×10^7

4) 2.34033×10^7

5) 2.15517×10^7

Exercise 2

Given the system

$$-3v^2x_2 + 3vw x_4 = 63$$

$$3x_1^2x_2 = -108$$

$$3wx_1x_3 + 2x_1x_2x_4 = -114$$

$$-2vx_1x_3 = 36$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u,$

$v, w) = (-3, -4, 2, -1, 1, 3, 5)$. Compute if possible $\frac{\partial x_3}{\partial v}(1, 3, 5)$.

1) $\frac{\partial x_3}{\partial v}(1, 3, 5) = -\frac{19}{29}$

2) $\frac{\partial x_3}{\partial v}(1, 3, 5) = -\frac{35}{58}$

3) $\frac{\partial x_3}{\partial v}(1, 3, 5) = -\frac{37}{58}$

4) $\frac{\partial x_3}{\partial v}(1, 3, 5) = -\frac{18}{29}$

5) $\frac{\partial x_3}{\partial v}(1, 3, 5) = -\frac{39}{58}$

Exercise 3

Given the function

$f(x, y, z) = 30 - 4x + x^2 - 6y + y^2 - 6z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-2.2371, -2.55565, ?\}$
- 2) We have a maximum at $\{-1.9371, ?, -1.38898\}$
- 3) We have a maximum at $\{?, -2.65565, -0.788978\}$
- 4) We have a maximum at $\{-2.0371, ?, -1.18898\}$
- 5) We have a maximum at $\{2, 3, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 87

Exercise 1

Given the functions

$$f(x, y) = (3 + x - x^2 + 3y - 2xy + y^2, -1 + 3x - x^2 + 2y - 2xy + y^2, 3 - 2x + 2x^2 + y - xy + y^2)$$

and

$$g(u, v, w) = (2 + 2u^2, v),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-3, 3)$.

- 1) -6262.72
- 2) -3348.
- 3) -5768.59
- 4) -1636.96
- 5) -4732.39

Exercise 2

Given the system

$$2xu_2u_3 - 3yu_2u_4 = 66$$

$$-u_1 - yu_1 - 3xu_2^2 + u_1u_2^2 = 1$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4) = (-1, -5, -2, -3, -4, -2)$. Compute if possible $\frac{\partial y}{\partial u_1}(-2, -3, -4, -2)$.

- 1) $\frac{\partial y}{\partial u_1}(-2, -3, -4, -2) = \frac{52}{73}$
- 2) $\frac{\partial y}{\partial u_1}(-2, -3, -4, -2) = \frac{54}{73}$
- 3) $\frac{\partial y}{\partial u_1}(-2, -3, -4, -2) = \frac{55}{73}$
- 4) $\frac{\partial y}{\partial u_1}(-2, -3, -4, -2) = \frac{56}{73}$
- 5) $\frac{\partial y}{\partial u_1}(-2, -3, -4, -2) = \frac{53}{73}$

Exercise 3

Given the function

$f(x, y, z) = -28 + 6x - x^2 + 4y - y^2 + 6z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -0.571454, -4.83914\}$
- 2) We have a minimum at $\{0.17122, -0.371454, ?\}$
- 3) We have a minimum at $\{?, -0.0714537, -4.63914\}$
- 4) We have a minimum at $\{-0.52878, ?, -4.53914\}$
- 5) We have a minimum at $\{3, ?, 3\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 88

Exercise 1

Given the functions

$$f(x, y) = (-1 - 3x - x^2 + 2y - 2xy - 3y^2, 1 - 2x - 3x^2 + 2y + 3xy, 3x - 2x^2 + 3xy, x^2 - 2y + xy - y^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-u_1 u_3 - 2u_3^2 + 3u_4, 1 - 3u_1 + 2u_1 u_3 - 2u_3 u_4 - u_4^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, -3).$$

- 1) 333.216
- 2) 1340.79
- 3) 946.
- 4) 418.452
- 5) 1072.16

Exercise 2

Given the system

$$2xyu_1 = 96$$

$$-y^3 - 3x^2u_1 - y^2u_2 + 2u_1u_4 = 12$$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, u_1, u_2, u_3, u_4)$

$$p = (-3, -4, 4, -3, 5, 1). \text{ Compute if possible } \frac{\partial x}{\partial u_2}(4, -3, 5, 1).$$

- 1) $\frac{\partial x}{\partial u_2}(4, -3, 5, 1) = \frac{1}{7}$
- 2) $\frac{\partial x}{\partial u_2}(4, -3, 5, 1) = \frac{2}{7}$
- 3) $\frac{\partial x}{\partial u_2}(4, -3, 5, 1) = \frac{2}{21}$
- 4) $\frac{\partial x}{\partial u_2}(4, -3, 5, 1) = \frac{4}{21}$
- 5) $\frac{\partial x}{\partial u_2}(4, -3, 5, 1) = \frac{5}{21}$

Exercise 3

Given the function

$f(x, y, z) = 23 - 6x + x^2 - 4y + y^2 - 4z + z^2$ defined over the domain $D = \left\{ \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{3, 2, ?\}$
- 2) We have a minimum at $\{1.85066, 0.887407, ?\}$
- 3) We have a minimum at $\{?, 1.06489, 0.781696\}$
- 4) We have a minimum at $\{?, 1.24237, 1.31414\}$
- 5) We have a minimum at $\{1.67318, 1.77481, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025
01-Multivariate Functions-Training computers exam for for
serial number: 89

Exercise 1

Given the functions

$$f(x, y, z) = (-3y - 2y^2, -3 - y + 2z + 3xz + 2yz)$$

and

$$g(u, v) = (3 + 3u - 2u^2 + 3v, 1 - 2u + 3u^2 + 3v - 2uv + 3v^2, 1 + 3u + 2u^2 - v + uv - 3v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (2, -1, 3)$.

- 1) 0.284543
- 2) 0.427965
- 3) 0.
- 4) -0.295672
- 5) 0.6166

Exercise 2

Given the system

$$3x^2 - uy - 2y^3 - 2uz - 2xz + x^2z - 2xyz - xz^2 = 129$$

$$3u^2y + xy^2 - 2z^2 - 2uz^2 = -182$$

$$3u^3 - 3x^3 + u^2y + 3yz - 3z^2 = -130$$

determine if it is possible to solve for variables x, y, z in terms of variable u around the point $p = (x, y, z, u) = (1, -4, 5, 2)$. Compute if possible $\frac{\partial z}{\partial u}(2)$.

- 1) $\frac{\partial z}{\partial u}(2) = -\frac{10977}{56432}$
- 2) $\frac{\partial z}{\partial u}(2) = -\frac{686}{3527}$
- 3) $\frac{\partial z}{\partial u}(2) = -\frac{10975}{56432}$
- 4) $\frac{\partial z}{\partial u}(2) = -\frac{5487}{28216}$
- 5) $\frac{\partial z}{\partial u}(2) = -\frac{10973}{56432}$

Exercise 3

Given the function

$f(x, y, z) = 24 - 6x + x^2 + y^2 - 6z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{-4.93088, ?, -0.331398\}$
- 2) We have a maximum at $\{-4.43088, -0.3, ?\}$
- 3) We have a maximum at $\{?, 0.3, -0.431398\}$
- 4) We have a maximum at $\{3, 0, ?\}$
- 5) We have a maximum at $\{?, -0.2, -0.731398\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 90

Exercise 1

Given the functions

$$f(x, y, z) = (3 + 3x^2 + 3xy, -x^2 + y^2, -2 + 3x^2 - 3y^2 - 2z - 2z^2)$$

and

$$g(u, v, w) = (-3v + 2uv + 3v^2 - 2uw, -2u + 2uv - 3w - 2w^2, -1 + u + 2v - 3uv - 2v^2 + 2vw),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (0, 3, 0).$$

- 1) -1.22618×10^8
- 2) -8.82547×10^7
- 3) -7.56512×10^7
- 4) -8.33358×10^7
- 5) -1.33029×10^8

Exercise 2

Given the system

$$-u^2x - 3ux^2 + 2y^2z - 3z^2 - vz^2 = 162$$

$$-3y + xz^2 = -39$$

$$-2x - 3z + 3x^2z + yz^2 = 114$$

determine if it is possible to solve for variables x, y

$, z$ in terms of variables u, v around the point $p = (x, y, z,$

$u, v) = (-3, 4, 3, -2, -3)$. Compute if possible $\frac{\partial z}{\partial v}(-2, -3)$.

- 1) $\frac{\partial z}{\partial v}(-2, -3) = -\frac{257}{8048}$
- 2) $\frac{\partial z}{\partial v}(-2, -3) = -\frac{65}{2012}$
- 3) $\frac{\partial z}{\partial v}(-2, -3) = -\frac{261}{8048}$
- 4) $\frac{\partial z}{\partial v}(-2, -3) = -\frac{259}{8048}$
- 5) $\frac{\partial z}{\partial v}(-2, -3) = -\frac{129}{4024}$

Exercise 3

Given the function

$f(x, y, z) = -18 + 2x - x^2 + 6y - y^2 - z^2$ defined over the domain $D =$

$\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -1.9875, -4.22628\}$
- 2) We have a minimum at $\{-0.9625, -1.4875, ?\}$
- 3) We have a minimum at $\{?, -1.6875, -4.02628\}$
- 4) We have a minimum at $\{1, 3, ?\}$
- 5) We have a minimum at $\{-0.2625, ?, -4.42628\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 91

Exercise 1

Given the functions

$$f(x, y, z) = (-3y + 2xz + z^2, -3 + x + xy - 2yz, 2x^2 + z - 3yz)$$

and

$$g(u, v, w) = (-3u^2 + v^2 + 3w + 2vw - w^2, 2u + 2u^2 - vw - 3w^2, -u^2 + v - 3uw + 3w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 0, 3)$.

- 1) -680642.
- 2) -300364.
- 3) -710313.
- 4) -615417.
- 5) -860598.

Exercise 2

Given the system

$$-u^2x - 3x^2y = -30$$

$$3u - 3v^3 + 3uy + v^2y - uvz = 13$$

$$-3u^2 - 3u^3 + ux + 3uz^2 = -93$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (2, 1, 1, 3, 1)$. Compute if possible $\frac{\partial x}{\partial v}(3, 1)$.

- 1) $\frac{\partial x}{\partial v}(3, 1) = -\frac{10}{17}$
- 2) $\frac{\partial x}{\partial v}(3, 1) = -\frac{7}{17}$
- 3) $\frac{\partial x}{\partial v}(3, 1) = -\frac{9}{17}$
- 4) $\frac{\partial x}{\partial v}(3, 1) = -\frac{8}{17}$
- 5) $\frac{\partial x}{\partial v}(3, 1) = -\frac{6}{17}$

Exercise 3

Given the function

$f(x, y, z) = -3 + 4x - x^2 + 2y - y^2 - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 1.8, 0.8\}$
- 2) We have a maximum at $\{3., 0.4, ?\}$
- 3) We have a maximum at $\{2.4, ?, -1.\}$
- 4) We have a maximum at $\{1.6, ?, 1.\}$
- 5) We have a maximum at $\{2, 1, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 92

Exercise 1

Given the functions

$$f(x, y, z) = (y^2 - 2z, -2y, -1 + 3y + z - 3z^2, 3xy - 2z - 2xz)$$

and

$$g(u_1, u_2, u_3, u_4) = (2 + 2u_1u_3 + u_4^2, u_2 - 3u_2^2 + 3u_3 - u_3u_4, u_1^2 - 3u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, 3, 0)$.

- 1) -260262.
- 2) -111129.
- 3) -325836.
- 4) -503380.
- 5) -398864.

Exercise 2

Given the system

$$2uvz - wxz = -1$$

$$-2w + 2uyz = -12$$

$$-wy^2 - 2z^2 = -27$$

determine if it is possible to solve for variables x, y, z

in terms of variables u, v, w around the point $p = (x, y, z, u,$

$v, w) = (3, 5, -1, 1, 2, 1)$. Compute if possible $\frac{\partial x}{\partial w}(1, 2, 1)$.

- 1) $\frac{\partial x}{\partial w}(1, 2, 1) = -\frac{123}{46}$
- 2) $\frac{\partial x}{\partial w}(1, 2, 1) = -\frac{60}{23}$
- 3) $\frac{\partial x}{\partial w}(1, 2, 1) = -\frac{61}{23}$
- 4) $\frac{\partial x}{\partial w}(1, 2, 1) = -\frac{121}{46}$
- 5) $\frac{\partial x}{\partial w}(1, 2, 1) = -\frac{119}{46}$

Exercise 3

Given the function

$f(x, y, z) = -6 - x^2 + 4y - y^2 + 2z - z^2$ defined over the domain $D \equiv$

$\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -3.62091, -1.09361\}$
- 2) We have a minimum at $\{?, -3.92091, -0.593614\}$
- 3) We have a minimum at $\{0.1, -3.42091, ?\}$
- 4) We have a minimum at $\{-0.1, -3.72091, ?\}$
- 5) We have a minimum at $\{0, 2, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 93

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 - x_1x_3 - 3x_3^2 + 2x_4^2, -3x_1^2 - 3x_3^2)$$

and

$$g(u, v) = (1 - 3u + 2u^2 - 2v - 3uv + v^2, -1 + 2u - 3u^2 + 3uv - 3v^2, -1 - u^2 + 2uv - v^2, -2 + u^2 + 2v - uv - 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (1, -2, 2, -2)$.

- 1) 0.
- 2) 0.754714
- 3) -0.15255
- 4) -0.505568
- 5) -0.657501

Exercise 2

Given the system

$$-x_2^2 x_3 = 20$$

$$-x_1 x_2 x_3 = 30$$

$$x_1 - x_2^2 + 2x_3^2 x_4 = 143$$

$$-2u^2 x_2 - 2x_1 x_2 x_4 + 3x_4^3 = 109$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variable u around the point $p = (x_1, x_2, x_3, x_4, u) = (-3, -2, -5, 3, -4)$. Compute if possible $\frac{\partial x_4}{\partial u}(-4)$.

- 1) $\frac{\partial x_4}{\partial u}(-4) = \frac{19552}{41759}$
- 2) $\frac{\partial x_4}{\partial u}(-4) = \frac{19554}{41759}$
- 3) $\frac{\partial x_4}{\partial u}(-4) = \frac{19553}{41759}$
- 4) $\frac{\partial x_4}{\partial u}(-4) = \frac{19555}{41759}$
- 5) $\frac{\partial x_4}{\partial u}(-4) = \frac{19556}{41759}$

Exercise 3

Given the function

$f(x, y, z) = -7 - x^2 - y^2 + 6z - z^2$ defined over the domain $D = \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 0.9, 2.4\}$
- 2) We have a maximum at $\{1.2, 0.9, ?\}$
- 3) We have a maximum at $\{?, 0., 3.\}$
- 4) We have a maximum at $\{-1.5, ?, 2.4\}$
- 5) We have a maximum at $\{-1.5, ?, 3.3\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 94

Exercise 1

Given the functions

$$f(x, y, z) = (-x^2, 3x^2 - 3xy - 3yz, -2y + 3z)$$

and

$$g(u, v, w) = (2u - 2u^2 + 3vw, 3v^2 - 2w, v^2 + w^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (-1, -3, 0)$.

- 1) -174645.
- 2) -198997.
- 3) -39863.5
- 4) -109999.
- 5) -147744.

Exercise 2

Given the system

$$-3 + 3xy^2 + uz = 103$$

$$u^2 + 3xy^2 - 2z^3 = 114$$

$$-3uvx - vy^2 + z^2 - 3uz^2 = 61$$

determine if it is possible to solve for variables x, y, z in terms of variables u, v around the point $p = (x, y, z, u, v) = (4, 3, -1, 2, -2)$. Compute if possible $\frac{\partial y}{\partial u}(2, -2)$.

- 1) $\frac{\partial y}{\partial u}(2, -2) = \frac{109}{80}$
- 2) $\frac{\partial y}{\partial u}(2, -2) = \frac{977}{720}$
- 3) $\frac{\partial y}{\partial u}(2, -2) = \frac{163}{120}$
- 4) $\frac{\partial y}{\partial u}(2, -2) = \frac{979}{720}$
- 5) $\frac{\partial y}{\partial u}(2, -2) = \frac{49}{36}$

Exercise 3

Given the function

$f(x, y, z) = 5 + 2x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D =$

$\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{?, -3.50567, -0.389083\}$
- 2) We have a minimum at $\{?, -3.40567, -0.689083\}$
- 3) We have a minimum at $\{?, -3.80567, -0.589083\}$
- 4) We have a minimum at $\{?, -3.40567, 0.0109168\}$
- 5) We have a minimum at $\{1, ?, 1\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 95

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (x_2 - x_1 x_2 - 3x_2^2 - x_1 x_3 + x_2 x_3 - 2x_4 + 2x_1 x_4, \\ 3x_2 - 3x_1 x_3, -3x_1^2 - x_1 x_3 + 3x_2 x_3 - 3x_3^2 + 2x_4 - x_2 x_4 + 2x_3 x_4)$$

and

$$g(u, v, w) = (2 + uw, -3uv + 3w^2, -u^2 - 3uv, 2uv),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (-2, 2, -2, -2).$$

- 1) -0.14865
- 2) 0 .
- 3) -0.689388
- 4) -0.683585
- 5) -0.681209

Exercise 2

Given the system

$$-2v x_2 x_4 + 2u x_3 x_4 = -4$$

$$3x_2 x_4 = -6$$

$$2u^2 x_1 + 2x_3 + 2x_2 x_4 = -76$$

$$-2v^2 x_1 + 2x_1 x_2^2 = -24$$

determine if it is possible to solve for variables $x_1, x_2, x_3,$

x_4 in terms of variables u, v around the point $p = (x_1, x_2, x_3, x_4,$

$u, v) = (-4, -2, 0, 1, -3, -1)$. Compute if possible $\frac{\partial x_2}{\partial u}(-3, -1)$.

- 1) $\frac{\partial x_2}{\partial u}(-3, -1) = \frac{1}{2}$
- 2) $\frac{\partial x_2}{\partial u}(-3, -1) = 2$
- 3) $\frac{\partial x_2}{\partial u}(-3, -1) = \frac{3}{2}$
- 4) $\frac{\partial x_2}{\partial u}(-3, -1) = \frac{5}{2}$
- 5) $\frac{\partial x_2}{\partial u}(-3, -1) = 1$

Exercise 3

Given the function

$f(x, y, z) = 18 - 6x + x^2 + y^2 - 2z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 0.2, -0.169973\}$
- 2) We have a maximum at $\{?, -0.2, -0.569973\}$
- 3) We have a maximum at $\{-4.35061, -0.5, ?\}$
- 4) We have a maximum at $\{?, 0, 1\}$
- 5) We have a maximum at $\{?, 0., -0.469973\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 96

Exercise 1

Given the functions

$$f(x, y, z) = (2x^2 - y + z^2, -x - 2x^2 + xy - 3y^2, -2y^2 + yz, 1 + x)$$

and

$$g(u_1, u_2, u_3, u_4) = (2 + u_1 u_3 + 3u_4 - 2u_2 u_4, 3 - 3u_1 u_4 - u_3 u_4, 2u_1^2 + 3u_3 + 2u_2 u_3 + u_1 u_4 - 3u_2 u_4),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point

$$p = (2, -1, 0).$$

- 1) -352588.
- 2) -53155.7
- 3) -44098.5
- 4) -53367.7
- 5) -221841.

Exercise 2

Given the system

$$2yzu_3 = -16$$

$$3xyu_3 = 36$$

$$-2x^3 + 3yz = -60$$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 around the point $p = (x, y, z, u_1, u_2, u_3, u_4)$

$$p = (3, 1, -2, 4, -5, 4, 3). \text{ Compute if possible } \frac{\partial z}{\partial u_3}(4, -5, 4, 3).$$

- 1) $\frac{\partial z}{\partial u_3}(4, -5, 4, 3) = -\frac{1}{54}$
- 2) $\frac{\partial z}{\partial u_3}(4, -5, 4, 3) = 0$
- 3) $\frac{\partial z}{\partial u_3}(4, -5, 4, 3) = \frac{1}{27}$
- 4) $\frac{\partial z}{\partial u_3}(4, -5, 4, 3) = \frac{1}{54}$
- 5) $\frac{\partial z}{\partial u_3}(4, -5, 4, 3) = \frac{1}{18}$

Exercise 3

Given the function

$f(x, y, z) = 1 - 4x + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain $D =$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, -4.36405, -2.18203\}$
- 2) We have a maximum at $\{2, 2, ?\}$
- 3) We have a maximum at $\{-0.555565, ?, -2.38203\}$
- 4) We have a maximum at $\{-0.255565, -4.86405, ?\}$
- 5) We have a maximum at $\{-0.355565, -4.06405, ?\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for serial number: 97

Exercise 1

Given the functions

$$f(x_1, x_2, x_3, x_4) = (-1 - 3x_1x_2 - 3x_3^2 + 2x_4 - 3x_2x_4, \\ -x_1 - x_1x_2 + x_1x_3 + x_3^2, x_1x_2 - 2x_3^2 - x_4 - x_4^2, 3x_1x_2 + 2x_1x_3 - 3x_2x_3 + 2x_3^2)$$

and

$$g(u_1, u_2, u_3, u_4) = (-3u_1 + 3u_1u_2 - 3u_2^2 + 3u_3^2 + u_4^2, \\ u_2^2 + 3u_2u_4 - 3u_4^2, -2 - u_1u_3 - 2u_2u_3 + 2u_3^2 - 3u_2u_4 + u_4^2, 2 + 3u_2)$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, -1, -2, -3)$.

1) -7.08459×10^8

2) -6.34128×10^8

3) -7.56728×10^7

4) -2.28804×10^8

5) -8.22832×10^8

Exercise 2

Given the system

$$x_1x_3^2 = 125$$

$$2x_2x_3^2 = -200$$

$$-3v x_1x_4 + 2w x_1x_4 = -300$$

$$2x_1x_2^2 = 160$$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4

in terms of variables u, v, w around the point $p = (x_1, x_2, x_3, x_4, u, v,$

$w) = (5, -4, -5, 4, -4, 3, -3)$. Compute if possible $\frac{\partial x_1}{\partial v}(-4, 3, -3)$.

1) $\frac{\partial x_1}{\partial v}(-4, 3, -3) = 1$

2) $\frac{\partial x_1}{\partial v}(-4, 3, -3) = 2$

3) $\frac{\partial x_1}{\partial v}(-4, 3, -3) = 3$

4) $\frac{\partial x_1}{\partial v}(-4, 3, -3) = 4$

5) $\frac{\partial x_1}{\partial v}(-4, 3, -3) = 0$

Exercise 3

Given the function

$f(x, y, z) = 6 - 2x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain $D =$

$\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{0.7, ?, 0.8\}$
- 2) We have a minimum at $\{0.6, ?, 1.3\}$
- 3) We have a minimum at $\{1, 1, ?\}$
- 4) We have a minimum at $\{?, 0.5, 1.1\}$
- 5) We have a minimum at $\{?, 1.4, 1.3\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 98

Exercise 1

Given the functions

$$f(x, y) = (3 - 3x + 2x^2 - y - xy - y^2, 2 - 3x + 2x^2 - 3y - 3xy - y^2)$$

and

$$g(u, v) = (-3 + u + u^2 + 2uv - 3v^2, -1 + u - 2u^2 - 2v + 3uv - v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 3)$.

- 1) -1.33504×10^6
- 2) -594617 .
- 3) -947700 .
- 4) -515763 .
- 5) -802761 .

Exercise 2

Given the system

$$u^2v - 2v^2 + vx + 2u^2y + 2vxy = 302$$

$$u^2 - 3u^3 - 3uv + 2x + u^2x - uvx - 2v^2x + uy^2 = 200$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-5, 5, -5, -2)$. Compute if possible $\frac{\partial x}{\partial u}(-5, -2)$.

- 1) $\frac{\partial x}{\partial u}(-5, -2) = -\frac{1545}{47}$
- 2) $\frac{\partial x}{\partial u}(-5, -2) = -\frac{1548}{47}$
- 3) $\frac{\partial x}{\partial u}(-5, -2) = -\frac{1544}{47}$
- 4) $\frac{\partial x}{\partial u}(-5, -2) = -\frac{1546}{47}$
- 5) $\frac{\partial x}{\partial u}(-5, -2) = -\frac{1547}{47}$

Exercise 3

Given the function

$f(x, y, z) = 11 - 6x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv$

$\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a minimum at $\{3, 0, ?\}$
- 2) We have a minimum at $\{2.89507, 0., ?\}$
- 3) We have a minimum at $\{2.02655, ?, 2.20965\}$
- 4) We have a minimum at $\{2.31606, 0.289507, ?\}$
- 5) We have a minimum at $\{?, 1.15803, 3.07817\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 99

Exercise 1

Given the functions

$$f(x, y) = (1 - x + 3x^2 + y + 2xy - 2y^2, -3 - x - 3x^2 - 3y + 2xy + 2y^2)$$

and

$$g(u, v) = (-3 + 3u + 2u^2 - 2v + 2uv + 2v^2, -3 + 3u + u^2 - 2v + 3uv - v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (3, 3)$.

- 1) 1.15964×10^6
- 2) 2.36656×10^6
- 3) 1.18792×10^6
- 4) 3.62167×10^6
- 5) 1.47601×10^6

Exercise 2

Given the system

$$-v^2 y + 3u x y + 2x^2 y = 10$$

$$u^3 - 2u^2 v + 2y - 3v y - 2x y = 124$$

determine if it is possible to solve for variables x, y in terms of variables u, v

around the point $p = (x, y, u, v) = (-3, 5, -1, -5)$. Compute if possible $\frac{\partial x}{\partial v}(-1, -5)$.

- 1) $\frac{\partial x}{\partial v}(-1, -5) = \frac{108}{155}$
- 2) $\frac{\partial x}{\partial v}(-1, -5) = \frac{237}{341}$
- 3) $\frac{\partial x}{\partial v}(-1, -5) = \frac{1186}{1705}$
- 4) $\frac{\partial x}{\partial v}(-1, -5) = \frac{1184}{1705}$
- 5) $\frac{\partial x}{\partial v}(-1, -5) = \frac{1187}{1705}$

Exercise 3

Given the function

$f(x, y, z) = 4 + 2x - x^2 - y^2 - z^2$ defined over the domain $D = \left\{ (x, y, z) \mid \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1 \right\}$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{?, 0.4, -0.1\}$
- 2) We have a maximum at $\{?, 0.1, -0.2\}$
- 3) We have a maximum at $\{1, ?, 0\}$
- 4) We have a maximum at $\{0.9, -0.2, ?\}$
- 5) We have a maximum at $\{?, 0.1, -0.3\}$

Further Mathematics - Degree in Engineering - 2024/2025

01-Multivariate Functions-Training computers exam for for serial number: 100

Exercise 1

Given the functions

$$f(x, y, z) = (-xy + z^2, y^2 - xz)$$

and

$$g(u, v) = (-2u - u^2 - v + 3uv + 3v^2, -1 + u + 2u^2 + 3v - 3uv - v^2, 1 - 3u + 2u^2 - 2v + 2v^2),$$

compute the determinant of the Jacobian matrix of the composition $g \circ f$ at the point $p = (0, -3, -2)$.

- 1) -0.780512
- 2) -0.898243
- 3) 0.229769
- 4) 0.206079
- 5) 0 .

Exercise 2

Given the system

$$3x + ux + 3y - uy^2 + 2y^3 + u^2z - 3uyz - z^2 = -101$$

$$-2ux + 3u^2x - 2uy^2 + u^2z + 3y^2z + xz^2 = 24$$

$$-x - 2uxy - 3xy^2 - 3uz + 2uxz = 61$$

determine if it is possible to solve for variables x, y, z in terms of variable u

around the point $p = (x, y, z, u) = (-5, -4, 4, 2)$. Compute if possible $\frac{\partial z}{\partial u}(2)$.

- 1) $\frac{\partial z}{\partial u}(2) = \frac{913}{105}$
- 2) $\frac{\partial z}{\partial u}(2) = \frac{1823}{210}$
- 3) $\frac{\partial z}{\partial u}(2) = \frac{87}{10}$
- 4) $\frac{\partial z}{\partial u}(2) = \frac{365}{42}$
- 5) $\frac{\partial z}{\partial u}(2) = \frac{304}{35}$

Exercise 3

Given the function

$f(x, y, z) = -11 + 4x - x^2 - y^2 + 6z - z^2$ defined over the domain $D =$

$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

- 1) We have a maximum at $\{2.02629, ?, 2.07496\}$
- 2) We have a maximum at $\{?, 0.691654, 1.38331\}$
- 3) We have a maximum at $\{2, 0, ?\}$
- 4) We have a maximum at $\{?, 0., 1.72913\}$
- 5) We have a maximum at $\{0.642985, 0.345827, ?\}$