ANDALUSGEOID2002: THE NEW GRAVIMETRIC GEOID MODEL OF ANDALUSIA (SOUTHERN SPAIN)

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ABSTRACT

In 1991 the first determination of a gravimetric geoid in a test area in central Spain was computed by using least square collocation. In 1995 a gravimetric geoid in the Iberian Peninsula, Ibergeo95, was calculated by FFT. Nowadays an improved geoid of Andalusia, ANDALUSGeoid2002, has been computed by fast collocation procedure and remove-restore technique in the GRS80 Reference System. The computations have been done from 16562 free-air gravity anomaly data set, obtained from IGN (Instituto Geográfico Nacional) and BGI (International Gravity Bureau), the Earth Gravity Model EGM96 and detailed (100 m $\times$ 100 m), coarse (5 km $\times$ 5 km) and reference (20 km $\times$ 20 km) digital terrain models. Relative carrier-phase GPS measurements at 69 benchmarks of the Spanish Levelling Network in Andalusia have been done. The standard deviations of differences between ANDALUSGeoid2002 and GPS/levelling undulations after fitting the tilt have been $\pm$11 cm, $\pm$39 cm and $\pm$38 cm in western, eastern and whole Andalusia, respectively. The ANDALUSGeoid2002 shows an improvement of Ibergeo95 in this territory.

Keywords: Geoid, fast collocation, GPS, levelling

1. INTRODUCTION

The Iberian Peninsula is located in the southwest of Europe (36º < φ < 44º, – 10º < λ < – 3º), most of which is occupied by the Meseta, a large plateau that is almost completely surrounded by mountain ranges. Andalusia, the chosen area to compute a gravimetric geoid, is a region located to the south of the Iberian Peninsula (36º < φ < 38.4º, – 7.0º < λ < – 1.5º), it takes up 87268 km^2 area of land approximately. Three main geographical features in the area can be seen: the Sierra Morena is located to the north, the Guadalquivir River Depression to the central part and finally the Betic Mountain Range to the southeast, in which the highest place, Mulhacén, 3.481 m, can be found.

In Andalusia there is an increasing number of users of Global Positioning System (GPS) who demand a detailed geoid model, therefore, a computation of a high resolution regional geoid has been computed in southern Spain.

In 1991 the first determination of a preliminary geoid in a small zone in central Spain was made using the least squares collocation (LSC) method (*Sevilla et al., 1991*). In 1993 the geoid of central Spain was refined taking into account terrain effects (*Gil et al., 1993b*). In 1994 the first determination of a gravimetric geoid in the Iberian Peninsula was set up. Afterwards, the Ibergeo95 geoid was computed (*Sevilla, 1995*).

The new gravimetric geoid of Andalusia has been computed using the fast collocation method and the remove-restore technique from a gravity data set covering the area $36° < \varphi < 38.4°$ and $-7.0° < \lambda < -1.5°$, the EGM96 geopotential coefficient set, a 100 m × 100 m digital terrain model and the ETOPO5U model.

In section 2 the data compilation employed in this work, such as, terrestrial gravity data, the digital elevation models, spirit-levelled heights and GPS data is presented. In section 3 the gravimetric ANDALUSGeoid2002 computation is introduced. In section 4 a comparison with GPS-levelling data is displayed. Finally, in section 5 the conclusions are shown.

Table 1. Source data bank from BGI.

BGI Source number	Country	Points	Owner
35280014	SPAIN	53	Instituto Nacional de Geofísica
25000010	EUROPE	95	IGCS
35280020	SPAIN	7101	IGC
35280003	SPAIN	256	University of Wisconsin
35280021	SPAIN	377	Princeton University
25000002	EUROPE	109	IGC
25000001	EUROPE	666	University of Wisconsin
25000025	EUROPE	132	IGC Western Europe
25000019	EUROPE	123	IGC
35280002	SPAIN	9	DMATC
35280004	SPAIN	133	IUGG
35280016	SPAIN	137	Instituto Geográfico y Catastral
35280001	SPAIN	211	Hawaii Institute of Geophysics
35280009	SPAIN	846	IGC
35280013	SPAIN	559	Instituto Geográfico y Catastral

IGN: Instituto Geográfico Nacional (Spain),
IGC: International Gravity Commission;
DMATC: Defence Mapping Agency Topographic Centre

2. DATA COMPILATION

2.1. Terrestrial gravity data

27691 free-air gravity anomalies for the Iberian Peninsula have been supplied by Instituto Geográfico Nacional and 51610 marine and 9208 terrestrial gravity anomalies by International Gravity Bureau (Table 1) which have been calculated with International Gravity formula 1967 and thus gravity anomalies in the Geodetic Reference System 1980 have been computed according to the following transformation (*National Geodetic Survey, 1986*):

$$\Delta g_{1980} = \Delta g_{1967} - \left(0.8316 + 0.0782 \sin^2 \varphi - 0.0007 \sin^4 \varphi\right) \tag{1}$$

The next step was to limit all these data to the study area, this one comprises a limit of $36° < \varphi < 38.4°$ in latitude and $-7.5° < \lambda < -1.5°$ in longitude. Furthermore, a detection of duplicated values in latitude and longitude has been done. Finally a total of 16562 gravity anomalies were obtained.

2.2. The Andalusian digital elevation model

The Andalusian digital elevation model has been constructed from approximately 21 million point heights held by the Servicio Geográfico del Ejército and from ETOPO5U model for the bathymetric data. This model consists of a 100 m × 100 m grid in UTM (Universal Transverse Mercator) projection referred to Hayford ellipsoid, with an overlapping side of 200 m. In addition, the topography was represented by a coarse (5 km × 5 km) and a reference (20 km × 20 km) digital terrain models which have been computed from the detailed DEM (100 m × 100 m).

2.3. Levelling networks

Two different networks can be distinguished in Spain: the Precision Levelling Network and the High Precision Levelling Network. The Precision levelling network started on 12th August 1871, the first traverse was Alicante – Madrid. The network consisted of 92 traverses (46 on road and 46 on railway), a total of 16611 km long and 18025 (bronze and metallic) benchmarks. The levelling was double and was carried out by different people and instruments as well. For this levelling network the tolerance was $\pm 5\ \text{mm}\ \sqrt{k}$. After surveys and computations works the results were published in the *"Catálogo de altitudes de las señales metálicas de la red" (1925)*. However, the height of this catalogue was geometric height, which was obtained directly of surveys, without any correction and without any adjustment. In 1912 the International Association of Geodesy set the acceptable maximum error in levelling works and therefore a new levelling network was established, the High Precision Levelling Network, with a precision higher than $\pm 1.5\ \text{mm}\sqrt{k}$. The new network consisted of 11000 km (7500 km by railway and 3500 km by road), 19 closed and 8 opened polygons.

2.4. GPS and spirit-levelled height data

The combined use of Global Positioning System (GPS), levelling and geoid has been a key procedure in various geodetic applications. Although these three types of height information are considerably different in terms of physical meaning, reference surface definition/realization, observational methods, accuracy, etc., they should achieve from a well known simple geometrical relationship, $N = h - H$ (*Kotsakis et al. 1999*).

2.4.1. GPS Surveys

The Spanish height system is based on orthometric heights thus the levelling and GPS data have been employed to compute geoid undulations. The benchmarks of the High Precision Levelling Network have been used in which the orthometric height is known. The GPS traverses are composed of 77 GPS control points, of which 69 are benchmarks of the Spanish first-order levelling network, 4 are benchmarks belonging to the Spanish first order geodetic network and 2 are permanent GPS sites, San Fernando and Jaén. For geodetic establishment, the GPS constellation was tracked for 2 – 4 hours over baseline lengths under 25 km. The equipment used throughout the surveys consisted of eight Leica GPS dual frequency carrier phase receivers from University of Jaén and University of Extremadura.

2.4.2. GPS *computation*

GPS data were processed using IGS precise orbits, 15º cut-off angle, Hopfield tropospheric model and iono free solution. A total of 135 baselines have been computed. The processing was split into 6 campaigns from west to east: Sevilla-Huelva-Cádiz, Córdoba, Málaga, Jaén, Granada and Almería. The redundancy of the measurements allowed to check the baselines and eliminate outliers, i.e. NGS826-NGS842 baseline (Málaga Campaign).

3. GRAVIMETRIC GEOID COMPUTATION IN ANDALUSIA

The fast collocation method and the remove-restore technique have been used to compute the Andalusian quasigeoid estimation. The main advantages of this procedure are: all the elements associated with the gravity field (e.g., free-air anomaly, geoidal heights, gravity gradients, etc.) can be used to calculate any element of the gravity field, provided that covariance functions are known; the input data accuracy may be taken into account in the calculation of the gravity field and associated variances; the estimated variances of the calculated elements may be used to identify areas where the model has higher or lower confidence because of the data distribution and/or original data quality (*Barzaghi, 1996*). The limitation of this method is due basically to the fact that covariance functions between all the elements involved in the calculation have to be known. The computation of these covariance functions is not an easy task in most cases due mainly to heterogeneous distribution of data.

The gravimetric geoid computation process is summarized in the following steps:

1. In order to obtain a smoothed gravity field as input for the collocation procedure, gravity anomalies must be reduced for the geopotential model and residual terrain effect to get the residual gravity anomalies Δg_r:

   $$\Delta g_r = \Delta g - \Delta g_{EGM96} - \Delta g_{rtm} \tag{2}$$

   where Δg_{EGM96} is computed, in a spherical approximation, from the geopotential coefficient EGM96 complete up to degree and order 360 set by:

   $$\Delta g_{EGM96} = \frac{GM}{r^2} \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n (n-1) \sum_{m=0}^{n} [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda] \bar{P}_{nm} (\cos\theta) \tag{3}$$

   being θ, λ the geocentric colatitude and longitude of the point; $\bar{C}_{nm}, \bar{S}_{nm}$ the fully normalized spherical geopotential coefficients; $\bar{P}_{nm}$ the fully normalized associated Legendre polynomials; n_{max} the maximum degree of the geopotential model, and GM/r^2 the mean gravity; and Δg_{rtm} is computed by gravitational effect formulas of a homogeneous rectangular prism with the available DTM and with respect to a $10' \times 10'$ reference grid by using TC program (*Tscherning et al., 1992*). Only residual topography out to a distance of 38 km was taken into account.
2. *Outlier detection*. In gravity field approximation it is necessary to be sure that the data do not contain gross errors, therefore, an important task is to search for outliers. Several methods have been proposed in the geodetic literature. For large numerical databases semiautomatic methods which identify gross errors are used, followed by a detailed analysis of each value. A comparison of the difference between the observed and the predicted values with the error estimate can then be used to identify a possible gross error (*Tscherning, 1991*; *Gil et al. 1993a*).
   The gravity anomaly at point *P*, is predicted from a set of values observed in the neighbourhood as regularly as possible in all direction, taken into account the covariance between the observation and predicted value, the covariance of the observations and the covariance of the observation errors. A measurement is rejected or considered suspect if

   $$\left| \Delta g_r - \Delta g_r^{pred} \right| > 3\sigma \tag{4}$$

   where Δg_r is the residual anomaly observed, Δg_r^{pred} is the residual anomaly predicted and σ^2 is the combined error variance of the observation and prediction. By using this technique 1.88% of gravity anomalies were detected as outliers and removed from the data bank prior to other computation. The gridding of Δg_r to produce a $3' \times 3'$ grid of residual anomalies was performed with the GEOGRID program of the GRAVSOFT package (*Tscherning et al., 1992*).
3. Computation of the empirical and model covariance functions of the gravity anomalies which are required to estimate the residual quasigeoid via fast collocation. The covariance function model used in this work was:

$$C(P,Q) = a\sum_{n=2}^{n_{\max}} c_n \left(\frac{R^2}{rr'}\right)^{n+2} P_n(\cos\psi) +$$

$$+\sum_{n=n_{\max}+1}^{\infty} \frac{A(n-1)}{(n-2)(n+24)} \left(\frac{R_B^2}{rr'}\right)^{n+2} P_n(\cos\psi) \tag{5}$$

where ψ is the spherical distance between points P and Q with radial distances r and r', $P_n(\cos\psi)$ are the Legendre polynomials, R is the mean radius of the Earth, R_B is the radius of the Bjerhammar sphere, c_n, are the error anomaly degree variances associated with the model coefficients. For the analytical approximation of the empirical covariance function COVFIT program has been used (*Knudsen, 1987*). The covariance function is adjusted to fit the empirical values using an iterative least square inversion procedure adjusting the scale factor of the degree variances, A, the depth of the Bjerhammar sphere, $R - R_B$ and the factor to scale the error degree variances, a. Unfortunately, the iteration may diverge (e.g. result in a Bjerhammar-sphere radius larger than mean radius of the Earth). This will normally occur, if the data has a very inhomogeneous statistical character. Therefore, simple histograms are always produced together with the covariances in order to check that the data distribution is reasonably symmetric, if not normal.

4. Estimation of the residual quasigeoid height anomaly,ζ_r, via fast collocation using the FASTCOLB program (*Barzaghi, 1996*). *Bottoni and Barzaghi (1993)* proposed some modifications to the original form of the least square collocation method which permit to compute the classical collocation solution in a fast and reliable way, the Fast Collocation. The basic assumption is that the input data are gridded and homogeneous. This implies that the autocovariance matrix entering in the collocation formula is of Toeplitz type. In particular, if observations are placed on a two dimensional planar grid, the autocovariance matrix is a symmetric block Toeplitz matrix and each block is itself a symmetric Toeplitz matrix. The analysis can be extended to a regular geographical grid taking into account the distorsions on the Toeplitz/Toeplitz structure induced by the convergence of the meridians.
5. Restore of the geopotential model and the residual terrain model effects to get the complete height anomaly.

$$\zeta = \zeta_r + \zeta_{EGM96} + \zeta_{rtm} \tag{6}$$

The conversion of the quasigeoid model to the geoid model has been done using the expression (*Heiskanen and Moritz, 1967*):

$$(\zeta - N) \approx 0.1 H^{av} H \tag{7}$$

where H is the orthometric height of the station and H^{av} the average height of the area considered both in kilometres. Table 2 shows the statistical summary of remove-restore procedure and Figure 1 presents the ANDALUSGeoid2002 contour plot.

Table 2. Statistical summary of remove-restore procedure.

	Δg (mGal)	$\Delta g - \Delta g_{EGM96}$ (mGal)	Δg_r (mGal)	Δg_{r*} (mGal)	Δg_r^G (mGal)	ζ_r (m)	ζ (m)	N (m)
Points	16562	16562	16562	16250	5439	5439	5439	5439
Mean	-9.351	-8.212	-5.108	-4.938	-2.57	-.203	49.812	49.691
Std. dev.	48.763	30.348	27.859	25.538	16.73	.161	3.273	3.265
Max.	244.630	198.993	300.163	288.520	-82.53	.157	56.358	56.332
Min.	-170.470	-103.231	-96.905	-96.910	173.31	-.909	41.800	41.818

where $\Delta g,$ $\Delta g_{EGM96},$ $\Delta g_r,$ $\Delta g_{r*},$ Δg_r^G are free-air, model, residual, residual without outliers and residual gridded gravity anomalies, respectively; $\zeta_r,$ ζ are residual and complete height anomalies and N is geoid undulations.

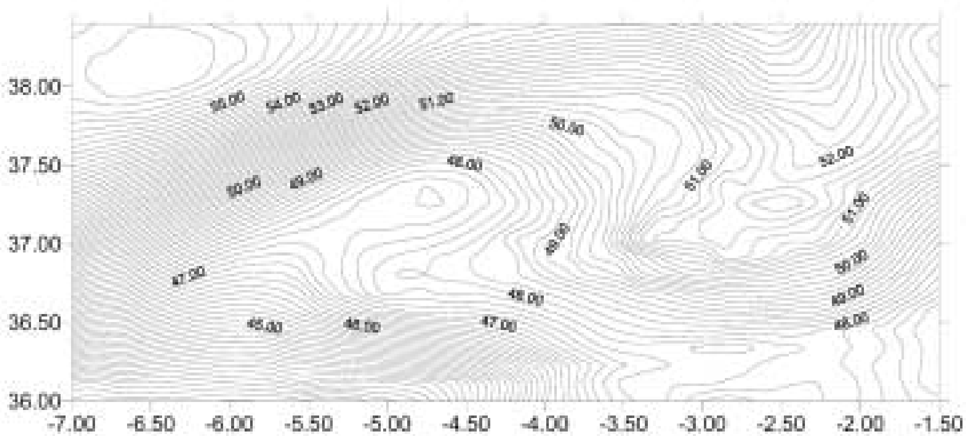

Fig. 1. ANDALUSGeoid2002. Contour interval is 20 cm.

4. COMPARISONS OF ANDALUSGEOID2002 AND GPS-LEVELLING DATA

Geoid undulations (N_{GPS}) were obtained at 69 GPS/levelling stations by simply taking the difference between the WGS84 elipsoidal heights (h) and orthometric heights (H),

$$N_{GPS} = h - H \tag{8}$$

neglecting the term e, quantity due to the deflection of the vertical and the curvature of the plumb line (*Torge, 2001*) which is usually small and below the combined GPS and levelling measurement error level (*Banerjee et al., 1999*).

It is interesting to determine if there is any trend in the differences which could also be attributed to geodetic levelling errors (*Featherstone, 2001*). The linear regression coefficient of the absolute differences ($N - N_{GPS}$) at the 69 controls points as a function of the longitude and latitude are 0.10 and 0.56, respectively, showing an evident trend with latitude.

Only the standard deviation of the differences between the ANDALUSGeoid2002 undulation N and N_{GPS} is used to give an indication of the precision of the gravimetric solution because any gravimetric determination of the geoid is deficient in the zero and first-degree terms (*Featherstone et al., 1996*). The systematic datum difference between the gravimetric geoid and GPS/levelling values was removed by a four-parameter transformation. The use of the datum shift eliminates the possible tilt of the gravimetric geoid (*Lacy M.C et al., 2001*). Table 3 shows the statistics of the differences between the GPS/levelling and the ANDALUSGeoid2002 undulations. The standard deviations of the discrepancies are ± 10.8 cm, ± 38.8 cm and ± 38.4 cm for western, eastern and whole Andalusia, respectively. As the highest standard deviation occurs in eastern Andalusia, a further analysis is done province by province (Table 4). The standard deviations are ± 31 cm, ± 8 cm and ± 5 cm in Granada, Jaén and Almería, respectively. The highest value reached in Granada could be due to errors in levelling. Finally, comparisons between ANDALUSGeoid2002 and Ibergeo95 with respect to GPS/levelling can be seen in Table 5. The standard deviation of the discrepancies at 69 controls points is ± 38 cm for AndalusGeoid2002 versus ± 58 cm for Ibergeo95.

Table 3. Statistics of ANDALUSGeoid2002 (N) and GPS/levelling (N_{GPS}) geoid undulations differences in western, eastern and whole Andalusia, respectively.

	Western area		Eastern area		Whole Andalusia	
	Before fitting	After fitting	Before fitting	After fitting	Before fitting	After fitting
Differences	$N - N_{GPS}$	$N - N_{GPS}$	$N - N_{GPS}$	$N - N_{GPS}$	$N - N_{GPS}$	$N - N_{GPS}$
Mean	-0.235	0	-0.282	0	.252	0
Std. dev.	0.481	0.108	0.695	0.388	.640	.384
Max.	0.398	0.188	0.841	0.934	.923	.979
Min.	-1.199	-0.185	-1.425	-0.986	-1.577	-1.232

Table 4. Statistics of ANDALUSGeoid2002 (N) and GPS/levelling (N_{GPS}) geoid undulations differences in Jaén, Granada and Almería provinces of eastern Andalusia.

Province	Fitting	Mean	Std. Dev.	Max.	Min.
Jaén	Before	-.963	0.310	-0.389	-1.424
	After	0	0.079	0.200	-0.171
Granada	Before	0.328	0.443	0.841	-0.458
	After	0	0.311	0.540	-0.592
Almería	Before	0.217	0.106	0.402	-0.009
	After	0	0.049	0.091	-0.068

Table 5. Statistics of ANDALUSGeoid2002 (N), Ibergeo95 (N_{IBER}) and GPS/levelling (N_{GPS}) geoid undulations differences in western, eastern and whole Andalusia, respectively.

	Western area		Eastern area		Whole Andalusia	
Differences	$N - N_{GPS}$	$N_{IBER} - N_{GPS}$	$N - N_{GPS}$	$N_{IBER} - N_{GPS}$	$N - N_{GPS}$	$N_{IBER} - N_{GPS}$
Mean	0	0	0	0	0	0
Std. dev.	0.108	0.172	0.388	0.540	.384	.584
Max.	0.188	0.252	0.934	0.963	.979	1.068
Min.	-0.185	-0.318	-0.986	-1.233	-1.232	-1.416

5. CONCLUSIONS

This paper deals with the computation of a new high resolution geoid in Andalusia and the study of its accuracy as compared with GPS/levelling traverses which have been done in whole territory.

The results of this work show the fast collocation method and the remove-restore procedure with the residual terrain modelling have been used successfully in geoid modelling. Furthermore, the ANDALUSGeoid2002 shows an improvement of the Ibergeo95 in the Andalusia territory as deduced from the comparison with GPS/levelling data in the region. This is probably due to the use of a better geopotential model and a more accurate residual terrain modelling.

As future work the influence of the DTM resolution in the geoid estimation will be studied employing several digital terrain models and different mathematical models for the terrain correction. In addition, a repetition of the levelling survey in the province of Granada will be planned to check the previous data.

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