Surfaces in Riemannian and Lorentzian 3-manifolds admitting a Killing vector field

Ana M. Lerma and José M. Manzano

Departamento de Didáctica de las Ciencias, Campus Las Lagunillas, Universidad de Jaén, 23071 Jaén (Spain) [alerma@ujaen.es]

Department of Mathematics, King's College London, Strand, WC2R 2LS London (United Kingdom) [manzanoprego@gmail.com]

A Killing submersion is a Riemannian submersion from an orientable connected 3-manifold onto a Riemannian surface, such that the fibres of the submersion are the integral curves of a Killing vector field without zeroes. A Killing submersion can be Riemannian or Lorentzian, depending on whether the Killing vector field is assumed spacelike or timelike. The geometry of the total space is characterized by two geometric functions in the base surface, namely the *bundle curvature* τ and the *Killing length* μ. These two functions also yield restrictions to the topology of the 3-manifold. If the base is simply connected, τ and μ can be prescribed arbitrarily giving rise to a unique Killing submersion structure, but uniqueness fails if simple connectedness is dropped.

We will also show the existence of a constant $C(M,\mu)$ such that the total space of any Lorentzian Killing submersion over M with Killing length μ and bundle curvature τ satisfying $\inf_M |\tau| > C(M,\mu)$ does not admit complete spacelike surfaces. In other words, if the space is twisted enough, then it is not distinguishing from the point of view of causality. We will see that the constant $C(M,\mu)$ can be understood as the Cheeger constant of M with density μ, and it is sharp in some specific examples. This second part is based in a joint work of the second author and H. Lee [1].

[1] Lee, H., Manzano, J.M., Generalized Calabi's correspondence and complete spacelike surfaces (2013, arXiv:1301.7241).

[2] Lerma, A.M., Manzano, J.M., Compact stable surfaces with constant mean curvature in Killing submersions. Available at arXiv:1604.00542.