# Heat equation

- $$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = c^2 \dfrac{\partial^2 u}{\partial x^2}(t,x), & 0 \le x \le L,\ 0 \le t, \\ u(0,t) = u(L,t) = 0, & 0 \le t, \\ u(x,0) = f(x), & 0 \le x \le L. \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} u(x,t) = \displaystyle\sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi x}{L}\right) e^{-\left(\frac{ck\pi}{L}\right)^2 t}, \\ \displaystyle\sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi x}{L}\right) = f(x). \end{array}}$$

- $$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = c^2 \dfrac{\partial^2 u}{\partial x^2}(t,x), & 0 \le x \le L,\ 0 \le t, \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(L,t) = 0, & 0 \le t, \\ u(x,0) = f(x), & 0 \le x \le L. \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} u(x,t) = \dfrac{a_0}{2} + \displaystyle\sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi x}{L}\right) e^{-\left(\frac{ck\pi}{L}\right)^2 t}, \\ \dfrac{a_0}{2} + \displaystyle\sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi x}{L}\right) = f(x). \end{array}}$$

# Wave equation

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}(t,x) = c^2 \dfrac{\partial^2 u}{\partial x^2}(t,x), & 0 \le x \le L,\ 0 \le t, \\ u(0,t) = u(L,t) = 0, & 0 \le t, \\ u(x,0) = f(x), & 0 \le x \le L, \\ \dfrac{\partial u}{\partial t}(x,t) = g(x), & 0 \le x \le L. \end{cases}$$

$$\Rightarrow \boxed{\begin{array}{l} u(x,t) = \displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{k\pi x}{L}\right) \left( b_k(f) \cos\left(\frac{ck\pi t}{L}\right) + \frac{L}{ck\pi} b_k(g) \sin\left(\frac{ck\pi t}{L}\right)\right), \\ \displaystyle\sum_{k=1}^{\infty} b_k(f) \sin\left(\frac{k\pi x}{L}\right) = f(x), \\ \displaystyle\sum_{k=1}^{\infty} b_k(g) \sin\left(\frac{k\pi x}{L}\right) = g(x). \end{array}}$$