Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 1

Exercise 1

Given the function

$f(x,y) = -5\,x^3 - 2\,y^3$ defined over the domain $D \equiv 15\,x^2 + 15\,y^2 \leqslant 435$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.8****

3) The value of the maximum is ****.6****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.0****

Exercise 2

Compute the volume of $D = \left\{ x^2 + y^2 + z^2 \leq 100,\ z \geq 8\,\sqrt{x^2 + y^2} \right\}$

1) -6.46927

2) 1.61732

3) -14.5559

4) 16.1732

5) 19.4078

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (6,6)$.

1) $H(6,6) = -7.10238$

2) $H(6,6) = 0.5$

3) $H(6,6) = 5.77208$

4) $H(6,6) = -0.40802$

5) $H(6,6) = -6.60287$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 2

Exercise 1

Given the function

$f(x,y)=3x^3+5y^3$ defined over the domain $D\equiv 9x^2+15y^2\leqslant 96$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.1****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.5****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 144,\ z\geq 13\left(x^2+y^2\right)\right\}$

1) 17.3439

2) 20.8127

3) 41.6255

4) 19.0783

5) 5.20318

Exercise 3

Compute the mean curvature for $X(u,v)=\{3u,\ 2u,\ v\}$ at the point $(u,v)=(1,4)$.

1) $H(1,4)=-8.3809$

2) $H(1,4)=-6.39195$

3) $H(1,4)=0$

4) $H(1,4)=4.06274$

5) $H(1,4)=7.53019$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 3

Exercise 1

Given the function

$f(x,y)=3x^3+3y^3$ defined over the domain $D\equiv 18x^2+27y^2\leqslant 1260$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.3****

3) The value of the minimum is ****.5****

4) The value of the minimum is ****.6****

5) The value of the minimum is ****.4****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 25,\ z\geq 10\sqrt{x^2+y^2}\right\}$

1) 3.24815

2) 2.85837

3) -0.909482

4) -1.03941

5) 1.29926

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(5,3)$.

1) H(5,3) = -6.38251

2) H(5,3) = -3.8128

3) H(5,3) = 1.2738

4) H(5,3) = 0.5

5) H(5,3) = 1.37318

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 4

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3\left(x^3+y^3\right)}{6x-12x^2-x^3+12x^4-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 196,\ z\ge 8\left(x^2+y^2\right)\right\}$

1) 80.4578

2) 22.9879

3) 38.3132

4) -19.1566

5) -15.3253

Exercise 3

Solve: Unable to decide whether numeric quantity

$$\frac{e^2\,\text{Cos}[3]^2}{(1+e^4)^{3/2}}+\frac{3e^6\,\text{Cos}[3]^2}{(1+e^4)^{3/2}}+\frac{2e^{10}\,\text{Cos}[3]^2}{(1+e^4)^{3/2}}-\frac{e^2\,\text{Cos}[3]^2}{\sqrt{1+e^4}}-\frac{2e^6\,\text{Cos}[3]^2}{\sqrt{1+e^4}}+\frac{e^2\,\text{Sin}[3]^2}{(1+e^4)^{3/2}}+\frac{3e^6\,\text{Sin}[3]^2}{(1+e^4)^{3/2}}+\frac{2e^{10}\,\text{Sin}[3]^2}{(1+e^4)^{3/2}}-\frac{e^2\,\text{Sin}[3]^2}{\sqrt{1+e^4}}-\frac{2e^6\,\text{Sin}[3]^2}{\sqrt{1+e^4}}$$

is equal to zero. Assuming it is.

Compute the Gauss curvature for $X(u,v)=\{e^v\,\text{Cos}[u],\ e^v\,\text{Sin}[u],\ v\}$ at the point $(u,v)=(3,2)$.

1) $K(3,2) = 4.46719$

2) $K(3,2) = -0.000323504$

3) $K(3,2) = -0.526514$

4) $K(3,2) = 7.72663$

5) $K(3,2) = -2.00621$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 5

Exercise 1

Given the function

$f(x,y)=4x^3-3y^3$ defined over the domain $D\equiv 30x^2+18y^2\leqslant 1038$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.1****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.0****

Exercise 2

Compute the volume of $D=\left\{5\left(x^2+y^2\right)\leq z\leq 196-x^2-y^2\right\}$

1) −4022.91

2) 19108.8

3) 10057.3

4) 8045.83

5) 18103.1

Exercise 3

Solve: Unable to decide whether numeric quantity

$$\frac{e^9\,\text{Cos}[3]^2}{(1+e^{18})^{3/2}}+\frac{3e^{27}\,\text{Cos}[3]^2}{(1+e^{18})^{3/2}}+\frac{2e^{45}\,\text{Cos}[3]^2}{(1+e^{18})^{3/2}}-\frac{e^9\,\text{Cos}[3]^2}{\sqrt{1+e^{18}}}-\frac{2e^{27}\,\text{Cos}[3]^2}{\sqrt{1+e^{18}}}+\frac{e^9\,\text{Sin}[3]^2}{(1+e^{18})^{3/2}}+\frac{3e^{27}\,\text{Sin}[3]^2}{(1+e^{18})^{3/2}}+\frac{2e^{45}\,\text{Sin}[3]^2}{(1+e^{18})^{3/2}}-\frac{e^9\,\text{Sin}[3]^2}{\sqrt{1+e^{18}}}-\frac{2e^{27}\,\text{Sin}[3]^2}{\sqrt{1+e^{18}}}$$

is equal to zero. Assuming it is.

Compute the mean curvature for $X(u,v)=\{e^v\,\text{Cos}[u],\ e^v\,\text{Sin}[u],\ v\}$ at the point $(u,v)=(3,9)$.

1) $H(3,9)=-8.92989$

2) $H(3,9)=2.31134$

3) $H(3,9)=7.41131$

4) $H(3,9)=0$

5) $H(3,9)=6.78322$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 6

Exercise 1

Given the function

$f(x,y)=2x^3-3y^3$ defined over the domain $D\equiv 15x^2+18y^2\leqslant 663$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.5****

Exercise 2

Compute the volume of $D=\left\{3\left(x^2+y^2\right)\leq z\leq 64-x^2-y^2\right\}$

1) 4503.79

2) -1286.8

3) 4664.64

4) 4503.79

5) 1608.5

Exercise 3

Compute the mean curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(3,9)$.

1) H(3,9) = 4.4489

2) H(3,9) = 0.0392837

3) H(3,9) = 5.4916

4) H(3,9) = 6.0251

5) H(3,9) = 1.81496

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 7

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^4 + 2y^4}{-3x - 6x^2 + x^4 + 6x^5 + 3x^6 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 225,\ z \ge 7\sqrt{x^2 + y^2}\right\}$

1) 71.0428

2) 120.773

3) 49.73

4) 191.816

5) 127.877

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{u, u, v\}$ at the point $(u,v)=(3,4)$.

1) $K(3,4) = 5.70855$

2) $K(3,4) = 3.62018$

3) $K(3,4) = -4.74001$

4) $K(3,4) = 2.39803$

5) $K(3,4) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 8

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 - 3y^4}{x^4 + 3(-1+x)^2 x\left(1+x+x^2+x^3\right) - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2 \le 64,\ z \ge 10\left(x^2+y^2\right)\right\}$

1) 9.99053

2) 5.99432

3) 0.999053

4) 0.999053

5) 14.9858

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(2,3)$.

1) $K(2,3) = -6.29612$

2) $K(2,3) = 0$

3) $K(2,3) = -4.53797$

4) $K(2,3) = -8.80323$

5) $K(2,3) = -6.91246$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 9

Exercise 1

Given the function

$f(x,y)=5x^3-y^3$ defined over the domain $D\equiv 45x^2+3y^2\leqslant 1632$, compute its absolute maxima and minima.

1) The value of the minimum is ****.3****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.5****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 4,\ z\geq 13\left(x^2+y^2\right)\right\}$

1) -1.12585

2) 0.474146

3) 0.174146

4) -0.525854

5) 1.67415

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(0,8)$.

1) $H(0,8)=0.5$

2) $H(0,8)=3.04934$

3) $H(0,8)=-1.3149$

4) $H(0,8)=-6.76818$

5) $H(0,8)=5.89341$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 10

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{3x^3+y^3}{x^3-3x\left(1-x+x^3\right)+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 225,\ z\ge 6\sqrt{x^2+y^2}\right\}$

1) 67.323

2) 96.1757

3) 28.8527

4) 0.

5) 221.204

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{u,\ 3u^2,\ v\}$ at the point $(u,v)=(2,6)$.

1) $K(2,6) = 5.95422$

2) $K(2,6) = -8.14205$

3) $K(2,6) = 0$

4) $K(2,6) = -1.59537$

5) $K(2,6) = 5.56029$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 11

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 2y^3}{6x - 6x^2 + x^3 - 6x^4 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 196,\ z \ge 13\left(x^2 + y^2\right)\right\}$

1) 66.1299

2) -18.8943

3) -2.36178

4) -18.8943

5) 23.6178

Exercise 3

Solve: Unable to decide whether numeric quantity

$$\frac{e^{10}\,\text{Cos}[4]^2}{(1+e^{20})^{3/2}} + \frac{3e^{30}\,\text{Cos}[4]^2}{(1+e^{20})^{3/2}} + \frac{2e^{50}\,\text{Cos}[4]^2}{(1+e^{20})^{3/2}} - \frac{e^{10}\,\text{Cos}[4]^2}{\sqrt{1+e^{20}}} - \frac{2e^{30}\,\text{Cos}[4]^2}{\sqrt{1+e^{20}}} + \frac{e^{10}\,\text{Sin}[4]^2}{(1+e^{20})^{3/2}} + \frac{3e^{30}\,\text{Sin}[4]^2}{(1+e^{20})^{3/2}} + \frac{2e^{50}\,\text{Sin}[4]^2}{(1+e^{20})^{3/2}} - \frac{e^{10}\,\text{Sin}[4]^2}{\sqrt{1+e^{20}}} - \frac{2e^{30}\,\text{Sin}[4]^2}{\sqrt{1+e^{20}}}$$ is equal to zero. Assuming it is.

General: Further output of Solve::ztest1 will be suppressed during this calculation.

Compute the Gauss curvature for $X(u,v)=\{e^v\,\text{Cos}[u],\ e^v\,\text{Sin}[u],\ v\}$ at the point $(u,v)=(4,10)$.

1) $K(4,10) = -6.57889$

2) $K(4,10) = -3.14031$

3) $K(4,10) = 0$

4) $K(4,10) = 2.65134$

5) $K(4,10) = 1.22194$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 12

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2\left(x^4 - y^4\right)}{-3x + 6x^2 + x^4 - 6x^5 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 49,\ z \ge 12\left(x^2 + y^2\right)\right\}$

1) 2.55042

2) 11.4769

3) 13.3897

4) 3.82563

5) 6.37606

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(6,7)$.

1) $K(6,7) = 5.23761$

2) $K(6,7) = 1.08237$

3) $K(6,7) = 2.36297$

4) $K(6,7) = 0$

5) $K(6,7) = 7.07061$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 13

Exercise 1

Given the function

$f(x,y) = -4x^3 - 4y^3$ defined over the domain $D \equiv 6x^2 + 6y^2 \leq 12$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.6****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.3****

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \leq 225,\ z \geq 13\sqrt{x^2 + y^2}\right\}$

1) 16.6565

2) 18.7386

3) -4.16412

4) -2.08206

5) 20.8206

Exercise 3

Compute the mean curvature for $X(u,v) = \{3u,\ 2u^2,\ v\}$ at the point $(u,v) = (1,2)$.

1) $H(1,2) = 0.048$

2) $H(1,2) = -7.27849$

3) $H(1,2) = 6.00997$

4) $H(1,2) = 7.04709$

5) $H(1,2) = -5.81896$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 14

Exercise 1

Given the function

$f(x,y) = -3x^3 + y^3$ defined over the domain $D \equiv 9x^2 + 3y^2 \leq 48$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.5****

Exercise 2

Compute the volume of $D = \{x^2 + y^2 \leq z \leq 81 - x^2 - y^2\}$

1) 5153.

2) -515.3

3) 515.3

4) 12 882.5

5) 11 336.6

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \text{Cos}[u], v^2 \text{Sin}[u], v\}$ at the point $(u,v) = (1,1)$.

1) H(1,1) = 8.52933

2) H(1,1) = 1.49121

3) H(1,1) = -7.46509

4) H(1,1) = 0.134164

5) H(1,1) = -8.76686

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 15

Exercise 1

Given the function

$f(x,y)=5x^3-2y^3$ defined over the domain $D\equiv 30x^2+9y^2\leqslant 561$, compute its absolute maxima and minima.

1) The value of the maximum is ****.1****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.2****

Exercise 2

Compute the volume of $D=\left\{10\left(x^2+y^2\right)\leq z\leq 36-x^2-y^2\right\}$

1) 351.63

2) 370.137

3) 462.671

4) 185.068

5) -18.5068

Exercise 3

Compute the mean curvature for $X(u,v)=\left\{2u,\,3u^2,\,v\right\}$ at the point $(u,v)=(6,6)$.

1) $H(6,6)=0.000128008$

2) $H(6,6)=-3.66162$

3) $H(6,6)=8.58892$

4) $H(6,6)=-0.507026$

5) $H(6,6)=6.58504$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 16

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 - 3y^3}{x^3 + 6x\left(1 + 2x - 2x^3 + x^4\right) - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 169,\ z \ge 14\left(x^2 + y^2\right)\right\}$

1) -11.3459

2) 24.5827

3) 18.9098

4) -18.9098

5) 43.4924

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v \mathrm{Cos}[u],\ e^v \mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(2,8)$.

1) $K(2,8) = 5.76926$

2) $K(2,8) = -3.0973$

3) $K(2,8) = 3.37955$

4) $K(2,8) = -2.55199$

5) $K(2,8) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 17

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 - y^3}{x^3 - 2x\left(1 + 2x + x^3\right) + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 1,\ z \ge 15\left(x^2 + y^2\right)\right\}$

1) 1.10131

2) 1.90131

3) -0.698694

4) 1.00131

5) 0.101306

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v)=(2,10)$.

1) $K(2,10) = 3.85265$

2) $K(2,10) = 6.56721$

3) $K(2,10) = 2.24888$

4) $K(2,10) = -3.19474$

5) $K(2,10) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 18

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 + 3\,y^4}{x^4 + 3\,\left(x + x^2 - x^5\right) - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 100,\ z \ge 10\ \sqrt{x^2 + y^2}\right\}$

1) 23.9064

2) 12.4729

3) 15.5911

4) 10.3941

5) 8.31527

Exercise 3

Compute the Gauss curvature for $X(u,v)=\left\{u,\ u^2,\ v\right\}$ at the point $(u,v)=(2,3)$.

1) $K(2,3) = 7.72699$

2) $K(2,3) = -1.60816$

3) $K(2,3) = -6.65694$

4) $K(2,3) = 3.32244$

5) $K(2,3) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 19

Exercise 1

Given the function

$f(x,y)=-x^3-2y^3$ defined over the domain $D\equiv 9x^2+3y^2\leqslant 327$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.3****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 4,\ z\geq 5\sqrt{x^2+y^2}\right\}$

1) 0.325374

2) 0.125374

3) -1.07463

4) 1.32537

5) 1.32537

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(4,6)$.

1) $H(4,6)=-5.11011$

2) $H(4,6)=0.5$

3) $H(4,6)=7.11135$

4) $H(4,6)=6.36557$

5) $H(4,6)=-0.738834$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 20

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3+3y^3}{-4x+4x^2+x^3-8x^4-8x^5+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 121,\ z\geq 4\sqrt{x^2+y^2}\right\}$

1) 183.11

2) 233.049

3) -83.232

4) 83.232

5) 149.818

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{2u,\ u,\ v\}$ at the point $(u,v)=(10,2)$.

1) $K(10,2) = 2.93719$

2) $K(10,2) = -1.97454$

3) $K(10,2) = 4.88722$

4) $K(10,2) = -8.92394$

5) $K(10,2) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 21

Exercise 1

Given the function

$f(x,y) = -4x^3 + y^3$ defined over the domain $D \equiv 30x^2 + 6y^2 \leq 846$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.6****

3) The value of the minimum is ****.4****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.0****

Exercise 2

Compute the volume of $D = \left\{8\left(x^2 + y^2\right) \leq z \leq 169 - x^2 - y^2\right\}$

1) 3987.87

2) 12960.6

3) 996.967

4) 4984.83

5) -498.483

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \text{Cos}[u], e^v \text{Sin}[u], v\}$ at the point $(u,v) = (1,4)$.

1) $H(1,4) = 1.0275$

2) $H(1,4) = 5.62393 \times 10^{-8}$

3) $H(1,4) = -5.27768$

4) $H(1,4) = 7.82307$

5) $H(1,4) = -4.81542$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 22

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 - 2y^4}{x^4 - 9\left(x + x^2 - x^5\right) + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 49,\ z \ge 3\sqrt{x^2 + y^2}\right\}$

1) 73.7295

2) 36.8648

3) 44.2377

4) 55.2971

5) 14.7459

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v)=(6,3)$.

1) $K(6,3) = 6.87791$

2) $K(6,3) = 6.54461$

3) $K(6,3) = 1.30319$

4) $K(6,3) = 0$

5) $K(6,3) = -7.46441$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 23

Exercise 1

Given the function

$f(x,y)=-4x^3+5y^3$ defined over the domain $D\equiv 30x^2+45y^2\leqslant 2370$, compute its absolute maxima and minima.

1) The value of the maximum is ****.1****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.8****

Exercise 2

Compute the volume of $D=\left\{13\left(x^2+y^2\right)\leq z\leq 225-x^2-y^2\right\}$

1) 5680.11

2) 11360.2

3) -5112.1

4) 15904.3

5) 17040.3

Exercise 3

Compute the mean curvature for $X(u,v)=\{e^v\,\mathrm{Cos}[u],\ e^v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(6,10)$.

1) $H(6,10)=6.07401$

2) $H(6,10)=0$

3) $H(6,10)=-4.79288$

4) $H(6,10)=1.88195$

5) $H(6,10)=1.13375$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 24

Exercise 1

Given the function

$f(x,y)=-x^3+4y^3$ defined over the domain $D\equiv 6x^2+6y^2\leq 102$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.5****

Exercise 2

Compute the volume of $D=\left\{15\left(x^2+y^2\right)\leq z\leq 4-x^2-y^2\right\}$

1) 3.76991

2) 1.5708

3) 2.67035

4) 0.785398

5) -1.41372

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\text{Sin}[u],v\}$ at the point $(u,v)=(4,3)$.

1) $H(4,3)=-2.07793$

2) $H(4,3)=3.44366$

3) $H(4,3)=8.96097$

4) $H(4,3)=0.5$

5) $H(4,3)=8.7551$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 25

Exercise 1

Given the function

$f(x,y) = -3x^3 - y^3$ defined over the domain $D \equiv 27x^2 + 3y^2 \leq 984$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.3****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.9****

Exercise 2

Compute the volume of $D = \{6(x^2 + y^2) \leq z \leq 81 - x^2 - y^2\}$

1) 1472.28

2) -1472.28

3) 2355.66

4) -1325.06

5) 883.371

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \text{Cos}[u], v^2 \text{Sin}[u], v\}$ at the point $(u,v) = (1,4)$.

1) $H(1,4) = 0.00196786$

2) $H(1,4) = -8.12964$

3) $H(1,4) = -0.85369$

4) $H(1,4) = -0.563691$

5) $H(1,4) = -5.60481$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 26

Exercise 1

Given the function

$f(x,y)=-2x^3+5y^3$ defined over the domain $D\equiv 12x^2+45y^2\leqslant 1812$, compute its absolute maxima and minima.

1) The value of the minimum is ****.4****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.9****

Exercise 2

Compute the volume of $D=\left\{13\left(x^2+y^2\right)\leq z\leq 49-x^2-y^2\right\}$

1) 134.696

2) 457.966

3) 242.452

4) -134.696

5) 269.392

Exercise 3

Compute the mean curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(6,9)$.

1) H(6,9) = 0.0392837

2) H(6,9) = -0.732412

3) H(6,9) = 6.60453

4) H(6,9) = -1.55294

5) H(6,9) = 6.57478

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 27

Exercise 1

Given the function

$f(x,y)=-2x^3-4y^3$ defined over the domain $D\equiv 6x^2+24y^2\leq 408$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.0****

5) The value of the minimum is ****.1****

Exercise 2

Compute the volume of $D=\left\{15\left(x^2+y^2\right)\leq z\leq 36-x^2-y^2\right\}$

1) -50.8938

2) 127.235

3) 50.8938

4) -101.788

5) -76.3407

Exercise 3

Compute the mean curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(4,8)$.

1) $H(4,8)=1.98957$

2) $H(4,8)=0.0441942$

3) $H(4,8)=-3.17451$

4) $H(4,8)=-7.72038$

5) $H(4,8)=2.02683$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 28

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{x^4+3y^4}{x^4-6x\left(1+x+2x^4\right)+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 16,\ z\ge 15\sqrt{x^2+y^2}\right\}$

1) -1.70312

2) -1.20312

3) 0.29688

4) 1.19688

5) 0.59688

Exercise 3

Compute the Gauss curvature for X(u,v)={Cos[u], Sin[u], v} at the point (u,v)=(3,5).

1) K(3,5) = 4.9158

2) K(3,5) = 0

3) K(3,5) = -8.34492

4) K(3,5) = -6.76858

5) K(3,5) = 4.09353

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 29

Exercise 1

Given the function

$f(x,y)=3x^3+4y^3$ defined over the domain $D\equiv 9x^2+12y^2\leqslant 84$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.1****

Exercise 2

Compute the volume of $D=\left\{9\left(x^2+y^2\right)\leq z\leq 100-x^2-y^2\right\}$

1) 4555.31

2) 3455.75

3) 1884.96

4) 1099.56

5) 1570.8

Exercise 3

Compute the mean curvature for $X(u,v)=\left\{v^2\,\text{Cos}[u],\ v^2\,\text{Sin}[u],\ v\right\}$ at the point $(u,v)=(4,1)$.

1) H(4,1) = −2.83013

2) H(4,1) = −7.86884

3) H(4,1) = 0.134164

4) H(4,1) = −2.76686

5) H(4,1) = 4.93854

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 30

Exercise 1

Given the function

$f(x,y)=3x^3-y^3$ defined over the domain $D\equiv 27x^2+6y^2\leq 1068$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.5****

Exercise 2

Compute the volume of $D=\left\{5\left(x^2+y^2\right)\leq z\leq 81-x^2-y^2\right\}$

1) 5153.

2) 3778.86

3) 1717.67

4) -687.066

5) -343.533

Exercise 3

Compute the mean curvature for $X(u,v)=\left\{v^2\,\text{Cos}[u],\ v^2\,\text{Sin}[u],\ v\right\}$ at the point $(u,v)=(1,10)$.

1) $H(1,10)=4.44505$

2) $H(1,10)=5.91046$

3) $H(1,10)=0.000125155$

4) $H(1,10)=8.23747$

5) $H(1,10)=-0.874357$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 31

Exercise 1

Given the function

$f(x,y)=2x^3-4y^3$ defined over the domain $D\equiv 6x^2+30y^2\leqslant 774$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.9****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 9,\ z\geq 15\sqrt{x^2+y^2}\right\}$

1) -1.27475

2) 2.12525

3) 0.125246

4) 1.32525

5) -0.574754

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(4,7)$.

1) $H(4,7)=6.63468$

2) $H(4,7)=0.5$

3) $H(4,7)=-0.845539$

4) $H(4,7)=8.71761$

5) $H(4,7)=2.98842$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 32

Exercise 1

Given the function

$f(x,y)=4x^3-2y^3$ defined over the domain $D\equiv 18x^2+3y^2\leq 165$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.6****

3) The value of the minimum is ****.7****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.5****

Exercise 2

Compute the volume of $D=\left\{12\left(x^2+y^2\right)\leq z\leq 100-x^2-y^2\right\}$

1) 966.644

2) 3141.59

3) 1208.3

4) -241.661

5) -1208.3

Exercise 3

Compute the mean curvature for $X(u,v)=\{e^v\,\mathrm{Cos}[u],\ e^v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(2,8)$.

1) $H(2,8)=-3.7615$

2) $H(2,8)=6.45149$

3) $H(2,8)=0$

4) $H(2,8)=-7.85876$

5) $H(2,8)=7.8952$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 33

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^4+3y^4}{x^4+4x\left(1+2x+x^4\right)-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 144,\ z\ge 12\left(x^2+y^2\right)\right\}$

1) -15.0274

2) 18.7843

3) 0.

4) 41.3254

5) 22.5411

Exercise 3

Compute the Gauss curvature for $X(u,v)=\left\{v^2\,\text{Cos}[u],\ v^2\,\text{Sin}[u],\ v\right\}$ at the point $(u,v)=(4,5)$.

1) $K(4,5) = -7.84237\times10^{-6}$

2) $K(4,5) = -5.88881$

3) $K(4,5) = -3.80499$

4) $K(4,5) = -4.00207$

5) $K(4,5) = -6.56554$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 34

Exercise 1

Given the function

$f(x,y) = -4x^3 + y^3$ defined over the domain $D \equiv 30x^2 + 6y^2 \leq 846$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.6****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.7****

Exercise 2

Compute the volume of $D = \{5(x^2 + y^2) \leq z \leq 169 - x^2 - y^2\}$

1) -6729.53

2) 8972.7

3) 7477.25

4) 747.725

5) 14206.8

Exercise 3

Compute the mean curvature for $X(u,v) = \{e^v \text{Cos}[u], e^v \text{Sin}[u], v\}$ at the point $(u,v) = (1,10)$.

1) $H(1,10) = 5.82309$

2) $H(1,10) = 4.10957$

3) $H(1,10) = -7.67979$

4) $H(1,10) = 0$

5) $H(1,10) = -7.70886$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 35

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 - 2y^4}{x^4 - 6x\left(1 + 2x + x^4\right) + 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 225,\ z \ge 4\left(x^2 + y^2\right)\right\}$

1) 122.675

2) -35.05

3) 148.963

4) 87.6251

5) 157.725

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v \mathrm{Cos}[u],\ e^v \mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(5,7)$.

1) $K(5,7) = -3.74206$

2) $K(5,7) = 7.58742$

3) $K(5,7) = 0$

4) $K(5,7) = 8.32493$

5) $K(5,7) = 5.03265$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 36

Exercise 1

Given the function

$f(x,y) = -3x^3 + 2y^3$ defined over the domain $D \equiv 18x^2 + 15y^2 \leqslant 663$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.2****

Exercise 2

Compute the volume of $D = \left\{12\left(x^2 + y^2\right) \leq z \leq 81 - x^2 - y^2\right\}$

1) 792.769

2) 79.2769

3) -475.661

4) -713.492

5) 2378.31

Exercise 3

Compute the mean curvature for $X(u,v) = \left\{v^2 \text{Cos}[u], v^2 \text{Sin}[u], v\right\}$ at the point $(u,v) = (2,7)$.

1) $H(2,7) = 0.000365351$

2) $H(2,7) = 2.49182$

3) $H(2,7) = -6.28748$

4) $H(2,7) = -2.08674$

5) $H(2,7) = -5.71399$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 37

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^4+y^4}{x^4+6x\left(1-2x+x^4\right)-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 225,\ z\ge 10\left(x^2+y^2\right)\right\}$

1) 28.1803

2) 49.3155

3) 73.9733

4) 17.6127

5) 35.2254

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v\,\mathrm{Cos}[u],\ e^v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(6,3)$.

1) $K(6,3) = 4.90857$

2) $K(6,3) = -6.27075$

3) $K(6,3) = -6.11387\times10^{-6}$

4) $K(6,3) = 6.86528$

5) $K(6,3) = -1.59181$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 38

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 + 3y^4}{-3x - 6x^2 - x^4 + 6x^5 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 49,\ z \ge 6\left(x^2 + y^2\right)\right\}$

1) 12.6767

2) 6.33833

3) -3.803

4) 31.6917

5) -8.87366

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(5,3)$.

1) $K(5,3) = 3.47388$

2) $K(5,3) = -2.40584$

3) $K(5,3) = 0$

4) $K(5,3) = 4.01567$

5) $K(5,3) = 3.42497$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 39

Exercise 1

Given the function

$f(x,y)=-5x^3+5y^3$ defined over the domain $D\equiv 30x^2+30y^2\leqslant 960$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.8****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 81,\ z\geq 12\left(x^2+y^2\right)\right\}$

1) 4.22158

2) 10.5539

3) 29.551

4) 12.6647

5) 16.8863

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(2,3)$.

1) $H(2,3)=-4.96648$

2) $H(2,3)=3.99655$

3) $H(2,3)=8.85026$

4) $H(2,3)=-6.27871$

5) $H(2,3)=0.5$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 40

Exercise 1

Given the function

$f(x,y)=5x^3-3y^3$ defined over the domain $D\equiv 15x^2+9y^2\leqslant 96$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.1****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 25,\ z\geq x^2+y^2\right\}$

1) 39.1544

2) 106.785

3) 46.2734

4) -24.9164

5) 35.5949

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(1,1)$.

1) $H(1,1) = -5.14054$

2) $H(1,1) = -3.69375$

3) $H(1,1) = 2.75031$

4) $H(1,1) = 2.29636$

5) $H(1,1) = 0.5$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 41

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 3y^3}{4x - 8x^2 + x^3 - 4x^4 - 4x^5 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \leq 100,\ z \geq 6\left(x^2 + y^2\right)\right\}$

1) -25.963

2) 62.3112

3) 25.963

4) 0.

5) 20.7704

Exercise 3

Compute the Gauss curvature for $X(u,v)=\left\{v^2\,\mathrm{Cos}[u],\ v^2\,\mathrm{Sin}[u],\ v\right\}$ at the point $(u,v)=(3,4)$.

1) $K(3,4) = -1.14203$

2) $K(3,4) = -5.43801$

3) $K(3,4) = -0.0000295858$

4) $K(3,4) = -8.27165$

5) $K(3,4) = -3.10965$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 42

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4 - 2y^4}{-2x + 4x^2 - x^4 - 2x^5 + 4x^6 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 9,\ z \ge 4\ \left(x^2 + y^2\right)\right\}$

1) -2.71285

2) 3.39106

3) -2.37374

4) 2.71285

5) 9.83406

Exercise 3

Compute the Gauss curvature for X(u,v)={Cos[u], Sin[u], v} at the point (u,v)=(5,8).

1) K(5,8) = -3.33667

2) K(5,8) = 8.52691

3) K(5,8) = -2.83479

4) K(5,8) = 0

5) K(5,8) = 5.62052

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 43

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^4+3y^4}{x^4-9x\left(1+2x+x^4\right)+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 49,\ z\ge 14\sqrt{x^2+y^2}\right\}$

1) 2.19074

2) 2.19074

3) 5.47684

4) 1.82561

5) 0.

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(6,1)$.

1) $K(6,1) = 5.12874$

2) $K(6,1) = 2.98739$

3) $K(6,1) = 0$

4) $K(6,1) = -4.20541$

5) $K(6,1) = 5.10272$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 44

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 2y^3}{x^3 + 2x\left(1 - 2x - 2x^3 + x^4\right) - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 81,\ z \ge 8\sqrt{x^2 + y^2}\right\}$

1) 2.35805

2) 8.25317

3) -11.7902

4) 17.6854

5) 11.7902

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{u, u^2, v\}$ at the point $(u,v)=(8,10)$.

1) $K(8,10) = -6.18286$

2) $K(8,10) = -7.07744$

3) $K(8,10) = 0$

4) $K(8,10) = -7.66165$

5) $K(8,10) = 0.794137$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 45

Exercise 1

Given the function

$f(x,y)=2x^3-3y^3$ defined over the domain $D\equiv 3x^2+18y^2\leqslant 291$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.6****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.0****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 196,\ z\geq 6\left(x^2+y^2\right)\right\}$

1) 91.8152

2) 51.0085

3) 10.2017

4) -5.10085

5) 81.6135

Exercise 3

Compute the mean curvature for $X(u,v)=\left\{3u,\ 2u^2,\ v\right\}$ at the point $(u,v)=(7,9)$.

1) $H(7,9)$ = 6.73239

2) $H(7,9)$ = 0.000268684

3) $H(7,9)$ = 5.44578

4) $H(7,9)$ = 1.23159

5) $H(7,9)$ = 8.48612

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 46

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^3+y^3}{-6x+12x^2-x^3+12x^4+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 196,\ z\geq 2\left(x^2+y^2\right)\right\}$

1) 151.222

2) -45.3665

3) 75.6108

4) 408.298

5) -136.099

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v\,\mathrm{Cos}[u],\ e^v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(3,3)$.

1) $K(3,3) = 6.60683$

2) $K(3,3) = -4.49495$

3) $K(3,3) = -6.11387\times10^{-6}$

4) $K(3,3) = 3.27451$

5) $K(3,3) = 4.81488$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 47

Exercise 1

Given the function

$f(x,y)=-x^3-2y^3$ defined over the domain $D\equiv 9x^2+3y^2\leq 327$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.7****

Exercise 2

Compute the volume of $D=\left\{14\left(x^2+y^2\right)\leq z\leq 1-x^2-y^2\right\}$

1) -1.39528

2) 0.80472

3) 0.10472

4) -0.69528

5) -0.69528

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\text{Sin}[u],v\}$ at the point $(u,v)=(2,9)$.

1) $H(2,9)=-1.86236$

2) $H(2,9)=5.58321$

3) $H(2,9)=0.5$

4) $H(2,9)=-0.134494$

5) $H(2,9)=7.96123$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 48

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^4+2y^4}{x^4+3x\left(1-2x-x^4+x^5\right)-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 64,\ z\geq 8\left(x^2+y^2\right)\right\}$

1) 31.1718

2) 4.98748

3) 9.97496

4) 21.1968

5) 12.4687

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{v\,\mathrm{Cos}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(6,4)$.

1) $K(6,4) = -7.07364$

2) $K(6,4) = 3.09423$

3) $K(6,4) = -0.863972$

4) $K(6,4) = -0.822434$

5) $K(6,4) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 49

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{x^4+y^4}{x^4+6\left(x+x^2+2x^5-2x^6\right)-2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 225,\ z\geq x^2+y^2\right\}$

1) 341.907

2) 512.86

3) -136.763

4) 307.716

5) 68.3813

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v\,\text{Cos}[u],\ e^v\,\text{Sin}[u],\ v\}$ at the point $(u,v)=(5,2)$.

1) $K(5,2) = 7.11015$

2) $K(5,2) = -7.28846$

3) $K(5,2) = 3.54331$

4) $K(5,2) = 6.78355$

5) $K(5,2) = -0.000323504$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 50

Exercise 1

Given the function

$f(x,y) = -3x^3 - 4y^3$ defined over the domain $D \equiv 9x^2 + 12y^2 \leq 84$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.6****

Exercise 2

Compute the volume of $D = \{15\left(x^2 + y^2\right) \leq z \leq 100 - x^2 - y^2\}$

1) 1963.5

2) 687.223

3) 392.699

4) -392.699

5) 981.748

Exercise 3

Compute the mean curvature for $X(u,v) = \{v^2 \text{Cos}[u], v^2 \text{Sin}[u], v\}$ at the point $(u,v) = (4,3)$.

1) $H(4,3) = 0.00469006$

2) $H(4,3) = 5.12392$

3) $H(4,3) = -8.43093$

4) $H(4,3) = -6.9175$

5) $H(4,3) = 3.58012$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 51

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3 - 2y^3}{x^3 + 6x\left(1 + x - x^3 + x^4\right) - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2 + y^2 + z^2 \le 169,\ z \ge 3\left(x^2 + y^2\right)\right\}$

1) 87.3634

2) 96.0997

3) 253.354

4) −78.627

5) 34.9454

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v \mathrm{Cos}[u],\ e^v \mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(1,6)$.

1) $K(1,6) = 0.980175$

2) $K(1,6) = -7.10132$

3) $K(1,6) = 6.99316$

4) $K(1,6) = 6.1937$

5) $K(1,6) = 0$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 52

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3+2y^3}{2x-4x^2+x^3+4x^4-y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\le 4,\ z\ge 4\left(x^2+y^2\right)\right\}$

1) 3.10089

2) -0.590647

3) 2.65791

4) 1.47662

5) 0.

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(3,7)$.

1) $K(3,7) = 0$

2) $K(3,7) = -8.32127$

3) $K(3,7) = -2.57556$

4) $K(3,7) = -8.22468$

5) $K(3,7) = 1.05248$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 53

Exercise 1

Given the function

$f(x,y)=-5x^3-2y^3$ defined over the domain $D\equiv 30x^2+6y^2\leqslant 504$, compute its absolute maxima and minima.

1) The value of the maximum is ****.9****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.0****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 16,\ z\geq 12\left(x^2+y^2\right)\right\}$

1) 1.86546

2) 3.10909

3) 2.90182

4) 0.621819

5) 2.07273

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(0,7)$.

1) $H(0,7)=-4.83869$

2) $H(0,7)=0.5$

3) $H(0,7)=4.79908$

4) $H(0,7)=8.78459$

5) $H(0,7)=6.18145$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 54

Exercise 1

Given the function

$f(x,y)=3x^3+5y^3$ defined over the domain $D\equiv 27x^2+45y^2\leqslant 2592$, compute its absolute maxima and minima.

1) The value of the minimum is ****.8****

2) The value of the minimum is ****.9****

3) The value of the minimum is ****.6****

4) The value of the minimum is ****.7****

5) The value of the minimum is ****.1****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 25,\ z\geq 11\sqrt{x^2+y^2}\right\}$

1) 1.07516

2) 3.01044

3) 0.215031

4) 0.96764

5) 3.01044

Exercise 3

Compute the mean curvature for $X(u,v)=\{3u,\ 3u^2,\ v\}$ at the point $(u,v)=(6,5)$.

1) $H(6,5)=7.71626$

2) $H(6,5)=2.75493$

3) $H(6,5)=2.4909$

4) $H(6,5)=0.000190909$

5) $H(6,5)=-4.69431$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 55

Exercise 1

Given the function

$f(x,y)=5x^3+2y^3$ defined over the domain $D\equiv 15x^2+15y^2\leqslant 435$, compute its absolute maxima and minima.

1) The value of the maximum is ****.1****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.8****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 64,\ z\geq 10\left(x^2+y^2\right)\right\}$

1) 14.9858

2) 14.9858

3) 9.99053

4) -8.99147

5) 28.9725

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(3,8)$.

1) $H(3,8)=1.174$

2) $H(3,8)=6.9036$

3) $H(3,8)=-1.72753$

4) $H(3,8)=5.40393$

5) $H(3,8)=0.5$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 56

Exercise 1

Given the function

$f(x,y)=-4x^3+2y^3$ defined over the domain $D\equiv 30x^2+9y^2\leqslant 831$, compute its absolute maxima and minima.

1) The value of the maximum is ****.7****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.4****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.3****

Exercise 2

Compute the volume of $D=\left\{14\left(x^2+y^2\right)\leq z\leq 169-x^2-y^2\right\}$

1) 6879.07

2) 2990.9

3) -2990.9

4) 4486.35

5) -2093.63

Exercise 3

Compute the mean curvature for $X(u,v)=\{e^v\,\mathrm{Cos}[u],\ e^v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(2,4)$.

1) $H(2,4)=5.62393\times 10^{-8}$

2) $H(2,4)=4.1266$

3) $H(2,4)=-8.95527$

4) $H(2,4)=6.18209$

5) $H(2,4)=6.4197$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 57

Exercise 1

Given the function

$f(x,y)=2x^3+y^3$ defined over the domain $D\equiv 6x^2+9y^2\leq 348$, compute its absolute maxima and minima.

1) The value of the minimum is ****.7****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.1****

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 225,\ z\geq 8\sqrt{x^2+y^2}\right\}$

1) 54.5845

2) 65.5014

3) -38.2091

4) 109.169

5) -43.6676

Exercise 3

Compute the mean curvature for $X(u,v)=\{\text{Cos}[u],\ \text{Sin}[u],\ v\}$ at the point $(u,v)=(4,2)$.

1) $H(4,2)=-8.63707$

2) $H(4,2)=0.5$

3) $H(4,2)=-2.09392$

4) $H(4,2)=1.11184$

5) $H(4,2)=-1.9182$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 58

Exercise 1

Given the function

$f(x,y) = -5\,x^3 + 2\,y^3$ defined over the domain $D \equiv 15\,x^2 + 15\,y^2 \leqslant 435$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.7****

4) The value of the minimum is ****.0****

5) The value of the minimum is ****.9****

Exercise 2

Compute the volume of $D = \left\{x^2 + y^2 + z^2 \leq 121,\ z \geq 5\,\left(x^2 + y^2\right)\right\}$

1) 90.4075

2) 37.6698

3) 33.9028

4) -15.0679

5) 26.3688

Exercise 3

Compute the mean curvature for $X(u,v) = \{\text{Cos}[u], \text{Sin}[u], v\}$ at the point $(u,v) = (2,10)$.

1) $H(2,10) = 5.25308$

2) $H(2,10) = 7.66941$

3) $H(2,10) = 0.5$

4) $H(2,10) = -4.8887$

5) $H(2,10) = -1.25666$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 59

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{3x^4+y^4}{6x+12x^2+x^4-12x^5-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 16,\ z\geq 8\sqrt{x^2+y^2}\right\}$

1) -0.103508

2) -0.931575

3) 1.03508

4) -1.03508

5) 2.79473

Exercise 3

Compute the Gauss curvature for X(u,v)={Cos[u], Sin[u], v} at the point (u,v)=(3,8).

1) K(3,8) = 1.31747

2) K(3,8) = 0

3) K(3,8) = -4.57757

4) K(3,8) = 0.809091

5) K(3,8) = -1.02454

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by hand) - training for serial number: 60

Exercise 1

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{-6x+6x^2-x^3+6x^4+2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 2

Compute the volume of $D=\left\{x^2+y^2+z^2\leq 196,\ z\geq 15\left(x^2+y^2\right)\right\}$

1) 20.4763

2) 10.2381

3) 6.14288

4) 18.4287

5) 12.2858

Exercise 3

Compute the Gauss curvature for $X(u,v)=\{e^v\,\mathrm{Cos}[u],\ e^v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v)=(4,1)$.

1) $K(4,1) = 8.89999$

2) $K(4,1) = -0.0142093$

3) $K(4,1) = -6.23159$

4) $K(4,1) = 1.61183$

5) $K(4,1) = -8.17022$