Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 1

Exercise 1

Given the function

$f(x,y,z)=6-2x+x^2-2y+y^2-2z+z^2$ defined over the domain $D\equiv \frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-3.04987, -0.502113, -2.54987\}$

2) We have a maximum at $\{-2.74987, -0.702113, -2.74987\}$

3) We have a maximum at $\{1, 1, 1\}$

4) We have a maximum at $\{-3.04987, -1.20211, -2.44987\}$

5) We have a maximum at $\{-2.84987, -0.402113, -2.94987\}$

Exercise 2

Compute $\int_D (x^2 z)\,dx\,dy\,dz$ for $D=\{3x^4 \leq y^4 z^2 \leq 11x^4,\ 6y^5 z^4 \leq x \leq 14y^5 z^4,\ 5x \leq y^2 z^2 \leq 11x,\ x>0,\ y>0,\ z>0\}$

1) 0.90182

2) 0.30182

3) 0.00182039

4) -1.69818

5) 1.30182

Exercise 3

Consider the vectorial field $F(x,y,z)=\left\{-6+5xz+\text{Cos}\left[y^2+2z^2\right],\ 2xyz+\text{Cos}\left[2x^2-2z^2\right],\ e^{-2x^2-y^2}+9y-7xy\right\}$ and the surface

$S\equiv\left(\frac{3+x}{3}\right)^2+\left(\frac{-1+y}{3}\right)^2+\left(\frac{z}{6}\right)^2=1$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) 0.   2) -0.4   3) 3.7   4) 0.6

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ 21 - 7x & 2 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2 en el instante t=0.4 mediante un desarrollo en serie de Fourier de orden 8.

1) u(2,0.4) = 3.50001

2) u(2,0.4) = 2.02025

3) u(2,0.4) = 0.811132

4) u(2,0.4) = −0.501193

5) u(2,0.4) = −4.61118

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 2

Exercise 1

Given the system

$-x\,u_3^2 + z\,u_3^2 + 3\,z\,u_3\,u_4 == 18$
$2\,x\,z\,u_1 - 2\,u_1^2\,u_2 == 20$
$x\,y\,z == 40$

determine if it is possible to solve for variables x,y,z in terms of variables u_1,u_2,u_3,u_4 arround the point p=(x,y,z,u_1,u_2,u_3,u_4)=(4,-5,-2,-1,-2,3,-4). Compute if possible $\frac{\partial x}{\partial u_3}(-1,-2,3,-4)$.

1) $\frac{\partial x}{\partial u_3}(-1,-2,3,-4) = -\frac{1}{3}$

2) $\frac{\partial x}{\partial u_3}(-1,-2,3,-4) = -\frac{4}{15}$

3) $\frac{\partial x}{\partial u_3}(-1,-2,3,-4) = -\frac{2}{5}$

4) $\frac{\partial x}{\partial u_3}(-1,-2,3,-4) = -\frac{8}{15}$

5) $\frac{\partial x}{\partial u_3}(-1,-2,3,-4) = -\frac{7}{15}$

Exercise 2

Compute the volume of the domain limited by the plane $2\,x + z == 9$ and the paraboloid $z == 8\,x^2 + 8\,y^2$.

1) 78.1923

2) 16.3492

3) 18.6058

4) 14.701

5) 20.8165

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{(3\,t + 2)\,\sin(2\,t)\,(9\cos(17\,t) + 10),\ (2\,t + 9)\,\sin(t)\,(9\cos(17\,t) + 10)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 15 541.   2) 12 432.8   3) 6216.42   4) 18 649.2

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 20x & 0\le x\le \frac{1}{5} \\ 5-5x & \frac{1}{5}\le x\le 1 \end{cases} & 0\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto $x=\frac{3}{10}$ en el instante $t=0.3$ mediante un desarrollo en serie de Fourier de orden 11.

1) $u\left(\frac{3}{10},0.3\right) = -0.523712$

2) $u\left(\frac{3}{10},0.3\right) = -0.54349$

3) $u\left(\frac{3}{10},0.3\right) = 4.03046$

4) $u\left(\frac{3}{10},0.3\right) = 3.81993$

5) $u\left(\frac{3}{10},0.3\right) = 2.$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 3

Exercise 1

Given the functions

$f(x,y) = (-3 - 2x - 3x^2 + 2xy - y^2, 3 + 3x - x^2 + xy - 2y^2, 3 - 2x + 3x^2 - y - 2xy + 3y^2)$

and

$g(u,v,w) = (2v, u^2 + 2uv + 3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2,1)$.

1) -5501.18

2) -14 200.4

3) -15 300.9

4) -18 087.4

5) -9792.

Exercise 2

Compute the volume of the domain limited by the plane $4x + 4z == 5$ and the paraboloid $z == 7x^2 + 7y^2$.

1) 0.291696

2) 0.286991

3) 0.228629

4) 0.370946

5) 1.19181

Exercise 3

Consider the vectorial field $F(x,y,z) = \{-2x^2 z, 3y^2 z^2, -3x^2 yz + 5x^2 yz^2\}$ and the surface

$S \equiv \left(\frac{2+x}{9}\right)^2 + \left(\frac{-7+y}{2}\right)^2 + \left(\frac{3+z}{7}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -3.49977×10^6 2) 1.23521×10^6 3) -2.05869×10^6 4) -4.52911×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-2)\,x^2\,(x-\pi)^2 & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = -2\,(x-3)\,x^2\,(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.8 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.8) = −8.0652

2) u(1,0.8) = 6.79007

3) u(1,0.8) = 6.99236

4) u(1,0.8) = 6.66497

5) u(1,0.8) = −6.8577

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 4

Exercise 1

Given the functions

$f(x,y) = (-3 + 3x + 3x^2 + y + 2xy, -2 + 2x + 2x^2 + 2y - 2xy + 3y^2, 2x - 3y + 2xy + 2y^2)$

and

$g(u,v,w) = (-3v^2 + 3w^2, 3u + 3uv + 2uw + 3vw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1,2)$.

1) 1.11297×10^6

2) 950 104.

3) 679 104.

4) 449 820.

5) 128 215.

Exercise 2

Compute $\int_D (y+z)\,dx\,dy\,dz$ for D=

$\{4y^6 \le x^8 z^4 \le 10y^6,\ 9y^7 \le x^2 z^4 \le 13y^7,\ 9 \le x^4 y^6 z \le 15,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.0183069

2) 1.51831

3) 0.818307

4) 1.11831

5) -1.88169

Exercise 3

Consider the vectorial field $F(x,y,z) = \{e^{-z^2} + 2yz,\ -10xy - \text{Sin}[x^2],\ e^{-2x^2+y^2} + 2z\}$ and the surface

$$S \equiv \left(\frac{-8+x}{4}\right)^2 + \left(\frac{7+y}{8}\right)^2 + \left(\frac{2+z}{6}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -106 644. 2) -62 731.3 3) -94 097.3 4) 94 098.7

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -10x & 0\le x\le \frac{2}{5} \\ \frac{20x}{3}-\frac{20}{3} & \frac{2}{5}\le x\le 1 \end{cases} & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} \frac{30x}{7} & 0\le x\le \frac{7}{10} \\ 10-10x & \frac{7}{10}\le x\le 1 \end{cases} & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{3}{5}$ en el instante t= 0.5 mediante un desarrollo en serie de Fourier de orden 10.

1) $u(\frac{3}{5},0.5) = -7.52021$

2) $u(\frac{3}{5},0.5) = -5.79459$

3) $u(\frac{3}{5},0.5) = 3.83177$

4) $u(\frac{3}{5},0.5) = -4.98195$

5) $u(\frac{3}{5},0.5) = -4.25217$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 5

Exercise 1

Given the functions

$f(x,y)=(-1-x+x^2-2y+2xy+3y^2, 3-x+3x^2-y+2xy-2y^2, 2-3x+2x^2-2y+2xy-3y^2, 2x-3x^2+2y+3xy-2y^2)$

and

$g(u_1,u_2,u_3,u_4)=(-u_2+3u_2^2-2u_1u_3-3u_2u_3-3u_2u_4, 3u_1u_3+2u_4-u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-1,2)$.

1) 83 552.

2) 151 764.

3) 133 977.

4) 149 782.

5) 135 508.

Exercise 2

Compute the volume of the domain limited by the plane $6x+2z==9$ and the paraboloid $z==6x^2+6y^2$.

1) 6.22183

2) 3.92905

3) 14.9659

4) 7.41857

5) 7.30803

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-4xyz^2, -y^2, -4xy-5yz\}$ and the surface

$S\equiv\left(\frac{-2+x}{9}\right)^2+\left(\frac{-5+y}{6}\right)^2+\left(\frac{-1+z}{8}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -2.36365×10^6 2) -1.57576×10^6 3) $-562772.$ 4) -1.85715×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(4,t)=0 & 0\le t \\ u(x,0)=2\,(x-4)^2\,(x-3)\,x^2 & 0\le x\le 4 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2 en el instante t=0.7 mediante un desarrollo en serie de Fourier de orden 9.

1) u(2,0.7) = 1.74709

2) u(2,0.7) = −1.12185

3) u(2,0.7) = −17.0824

4) u(2,0.7) = −2.74807

5) u(2,0.7) = 3.35233

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 6

Exercise 1

Given the functions

$f(x,y)=(-3+3x^2+y-3xy+2y^2,-2x-3x^2+3y+2xy+3y^2)$

and

$g(u,v)=(-3-3u+u^2-3v+3uv+3v^2,-2+3u-2u^2-v-3uv+3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-1,3)$.

1) -5.34066×10^7

2) -2.05896×10^7

3) -2.90122×10^7

4) -5.44377×10^6

5) -3.2789×10^7

Exercise 2

Compute $\int_D (6yz)\,dx\,dy\,dz$ for D=

$\{x^9\le z^2\le 7x^9,\ 9x^6y^9z^8\le 1\le 14x^6y^9z^8,\ 8y^7z^7\le x^7\le 10y^7z^7,\ x>0,\ y>0,\ z>0\}$

1) 0.705414

2) 1.80541

3) -1.59459

4) 0.00541351

5) 1.30541

Exercise 3

Consider the vectorial field F(x,y,z) =

$\{3+e^{-2y^2},\ -8yz+\text{Cos}[x^2+2z^2],\ e^{-2x^2+2y^2}-7x\}$ and the surface

$S\equiv\left(\frac{9+x}{7}\right)^2+\left(\frac{-9+y}{4}\right)^2+\left(\frac{2+z}{1}\right)^2=1$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) 8630.18 2) 1876.58 3) 7692.18 4) 2251.78

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=3\,(x-3)\,x^2\,(x-\pi) & 0\le x\le \pi \\ \frac{\partial}{\partial t}u(x,0)=3\,(x-3)\,(x-1)\,x\,(x-\pi)^2 & 0.\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.4 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.4) = −7.42524

2) u(1,0.4) = 1.25181

3) u(1,0.4) = 4.27964

4) u(1,0.4) = −0.616687

5) u(1,0.4) = −0.724249

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 7

Exercise 1

Given the system

$x^2 - 3y + 2uxy + uy^2 - 2xy^2 - z + 3xz^2 == 213$

$-2yz^2 == -50$

$-3u + 2ux^2 - xy - 3uxy - u^2 z - uz^2 + yz^2 == 36$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point p=(x,y,z,u)=(3,1,5,-1). Compute if possible $\frac{\partial x}{\partial u}(-1)$.

1) $\frac{\partial x}{\partial u}(-1) = -\frac{4650}{7451}$

2) $\frac{\partial x}{\partial u}(-1) = -\frac{4646}{7451}$

3) $\frac{\partial x}{\partial u}(-1) = -\frac{4647}{7451}$

4) $\frac{\partial x}{\partial u}(-1) = -\frac{4648}{7451}$

5) $\frac{\partial x}{\partial u}(-1) = -\frac{4649}{7451}$

Exercise 2

Compute the volume of the domain limited by the plane $10x + 9z == 5$ and the paraboloid $z == 9x^2 + 9y^2$.

1) 0.195347

2) 0.135776

3) 0.0607238

4) 0.0750808

5) 0.29756

Exercise 3

Consider the vectorial field $F(x,y,z) = \{x^2 y^2, -9xy^2 z + 2z^2, 5 + 6y\}$ and the surface

$S \equiv \left(\frac{8+x}{1}\right)^2 + \left(\frac{3+y}{4}\right)^2 + \left(\frac{-7+z}{6}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 453 083. 2) −323 629. 3) 873 802. 4) -1.1327×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -7x & 0 \le x \le 1 \\ \frac{7x}{\pi-1} - \frac{7}{\pi-1} - 7 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.1 mediante un desarrollo en serie de Fourier de orden 9.

1) u(1,0.1) = -0.477644

2) u(1,0.1) = -3.78068

3) u(1,0.1) = 3.30475

4) u(1,0.1) = 4.39334

5) u(1,0.1) = 3.84663

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 8

Exercise 1

Given the system

$$-u^2 + 2\,u\,x + y + 3\,u\,x\,y - 2\,y^3 - 3\,u\,z + 3\,x^2\,z == 12$$
$$u\,x\,y - 3\,u\,x\,z == -90$$
$$x^3 + 2\,u^2\,z + 3\,u\,x\,z + z^2 == 168$$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point p=(x,y,z,u)=(2,-3,4,3). Compute if possible $\frac{\partial z}{\partial u}(3)$.

1) $\frac{\partial z}{\partial u}(3) = -\frac{1166}{911}$

2) $\frac{\partial z}{\partial u}(3) = -\frac{1170}{911}$

3) $\frac{\partial z}{\partial u}(3) = -\frac{1169}{911}$

4) $\frac{\partial z}{\partial u}(3) = -\frac{1168}{911}$

5) $\frac{\partial z}{\partial u}(3) = -\frac{1167}{911}$

Exercise 2

Compute $\int_D (3\,x + 2\,z)\,dx\,dy\,dz$ for D=
$\{9 \le x^8\,y\,z^9 \le 15,\ 3\,x^3\,y^4 \le z^7 \le 12\,x^3\,y^4,\ 9\,y^2\,z^4 \le x^5 \le 16\,y^2\,z^4,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.00415566

2) -1.59584

3) -1.69584

4) -1.49584

5) 2.00416

Exercise 3

Consider the vectorial field $F(x,y,z) = \left\{e^{-2y^2+z^2} - 3y - 4xy,\ -4 + e^{x^2} - 8xyz,\ \mathrm{Sin}[x^2]\right\}$ and the surface

$$S \equiv \left(\frac{x}{6}\right)^2 + \left(\frac{6+y}{6}\right)^2 + \left(\frac{-6+z}{5}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) 1810.07   2) 9048.07   3) 18 095.6   4) 79 618.6

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -3x & 0 \le x \le 2 \\ \frac{6x}{\pi-2} - \frac{12}{\pi-2} - 6 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -\frac{8x}{3} & 0 \le x \le 3 \\ \frac{8x}{\pi-3} - \frac{24}{\pi-3} - 8 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.3 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.3) = -8.14207

2) u(1,0.3) = -0.814428

3) u(1,0.3) = -3.82555

4) u(1,0.3) = -1.3091

5) u(1,0.3) = 4.78577

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 9

Exercise 1

Given the system

$-v\,x\,y == -50$

$2\,y^2\,z == 150$

$-3\,v\,y^2 - x\,z == 135$

determine if it is possible to solve for variables x,y,z in terms of variables u,v,w arround the point $p=(x,y,z,u,v,w)=(5,-5,3,0,-2,1)$. Compute if possible $\frac{\partial z}{\partial u}(0,-2,1)$.

1) $\frac{\partial z}{\partial u}(0,-2,1)=4$

2) $\frac{\partial z}{\partial u}(0,-2,1)=2$

3) $\frac{\partial z}{\partial u}(0,-2,1)=0$

4) $\frac{\partial z}{\partial u}(0,-2,1)=1$

5) $\frac{\partial z}{\partial u}(0,-2,1)=3$

Exercise 2

Compute $\int_D (3\,x)\,dx\,dy\,dz$ for D=

$\{8 \le x^6\,y^9 \le 11,\ 7\,x^6\,y^9\,z^4 \le 1 \le 11\,x^6\,y^9\,z^4,\ 8\,y^2\,z^8 \le x^3 \le 9\,y^2\,z^8,\ x>0,\ y>0,\ z>0\}$

1) -0.699979

2) -0.599979

3) -0.799979

4) 0.0000208697

5) 1.60002

Exercise 3

Consider the vectorial field $F(x,y,z) =$
$\{-7y - 9z + \text{Sin}[y^2 - 2z^2],\ e^{z^2} - 3z,\ -5z + 8yz - \text{Sin}[2y^2]\}$ and the surface

$$S \equiv \left(\frac{5+x}{4}\right)^2 + \left(\frac{2+y}{6}\right)^2 + \left(\frac{-2+z}{7}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) −10 344.4 2) −14 778.1 3) −7388.55 4) 41 382.1

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-3)\,(x-1)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.3 mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,0.3) = −4.7215

2) u(2,0.3) = 0.217078

3) u(2,0.3) = −1.93934

4) u(2,0.3) = −3.39409

5) u(2,0.3) = 1.29319

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 10

Exercise 1

Given the function

$f(x,y,z) = -8 + 6x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-3.27872, -1.15957, -3.35773\}$

2) We have a minimum at $\{-2.87872, -0.959574, -3.25773\}$

3) We have a minimum at $\{-3.27872, -0.859574, -3.75773\}$

4) We have a minimum at $\{-2.67872, -0.559574, -2.85773\}$

5) We have a minimum at $\{3, 1, 1\}$

Exercise 2

Compute $\int_D (3x^3)\,dx\,dy\,dz$ for $D = \{9 \le x^2 y^2 z^8 \le 12,\ 2 \le y^3 z^6 \le 9,\ 4x^6 \le y z^8 \le 7x^6,\ x > 0,\ y > 0,\ z > 0\}$

1) -0.287925

2) 0.0120745

3) -1.88793

4) -0.387925

5) -0.387925

Exercise 3

Consider the vectorial field $F(x,y,z) = \{e^{-y^2+z^2} + 8xy,\ 8y + 4z - \mathrm{Sin}[x^2 - 2z^2],\ e^{x^2} - 5yz\}$ and the surface

$S \equiv \left(\frac{-2+x}{8}\right)^2 + \left(\frac{-8+y}{9}\right)^2 + \left(\frac{7+z}{6}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -69485.2 2) -81066.2 3) 57905.8 4) 214249.

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -\frac{9x}{2} & 0\le x\le 2 \\ x-11 & 2\le x\le 3 \\ \frac{8x}{\pi-3}-\frac{24}{\pi-3}-8 & 3\le x\le \pi \end{cases} & 0\le x\le \pi \\ \frac{\partial}{\partial t}u(x,0)=-2(x-3)(x-2)x(x-\pi)^2 & 0.\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.2 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.2) = 4.23328

2) u(1,0.2) = 7.76745

3) u(1,0.2) = 1.0317

4) u(1,0.2) = 3.18954

5) u(1,0.2) = -7.75628

Further Mathematics - 2023/2024 Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 11

Exercise 1

Given the functions

$f(x,y)=(3x+3x^2-2y-3xy-y^2, 3-2x-2x^2+y-xy+y^2, -3-3x-3x^2-3y-2xy-3y^2)$

and

$g(u,v,w)=(-3-2u, 1+2uv-2v^2-2w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,-2)$.

1) 6629.7

2) 1365.15

3) 10456.

4) 8669.63

5) 13011.8

Exercise 2

Compute the volume of the domain limited by the plane $8x+4z==8$ and the paraboloid $z==3x^2+3y^2$.

1) 2.8507

2) 13.2931

3) 8.58974

4) 6.29326

5) 1.98989

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$
$r(t)=\{(6t+6)\sin(2t)(5\cos(4t)+6), (9t+7)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 1971.36 2) 985.86 3) 394.56 4) 1774.26

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = 3\,(x-3)\,(x-2)\,(x-1)\,x^2 & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=1. mediante un desarrollo en serie de Fourier de orden 8.

1) u(2,1.) = 3.75581

2) u(2,1.) = −1.9891

3) u(2,1.) = −1.36456

4) u(2,1.) = −3.63773

5) u(2,1.) = 0.386338

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 12

Exercise 1

Given the function

$f(x,y,z) = -9 - x^2 + 4y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.8, 3., 0.6}

2) We have a maximum at {-0.8, 1.2, -0.6}

3) We have a maximum at {0, 2, 0}

4) We have a maximum at {-1., 2.2, 0.2}

5) We have a maximum at {1., 1., 0.4}

Exercise 2

Compute $\int_D (2\,x\,z^2)\,dx\,dy\,dz$ for D=

$\{6\,y^3 \le x^9 \le 14\,y^3,\ 5\,y^6\,z^8 \le x^3 \le 13\,y^6\,z^8,\ 5\,x^5 \le y^9 \le 10\,x^5,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.09807

2) 0.00193234

3) 0.501932

4) 0.901932

5) -1.29807

Exercise 3

Consider the vectorial field F(x,y,z) = $\{e^{y^2-2z^2} - 5\,z,\ -4\,x\,y + \mathrm{Sin}[x^2 - z^2],\ -7\,y - \mathrm{Sin}[x^2]\}$ and the surface

$S \equiv \left(\frac{-4+x}{5}\right)^2 + \left(\frac{5+y}{1}\right)^2 + \left(\frac{-4+z}{7}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -2111.12 2) -234.323 3) 2346.28 4) -2345.72

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 4x & 0 \le x \le \frac{1}{2} \\ \frac{61}{3} - \frac{110x}{3} & \frac{1}{2} \le x \le \frac{4}{5} \\ 45x - 45 & \frac{4}{5} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto $x=\frac{1}{2}$ en el instante t=0.8 mediante un desarrollo en serie de Fourier de orden 10.

1) $u(\frac{1}{2},0.8) = 0.63801$

2) $u(\frac{1}{2},0.8) = -2.37458$

3) $u(\frac{1}{2},0.8) = 1.56941$

4) $u(\frac{1}{2},0.8) = -1.45$

5) $u(\frac{1}{2},0.8) = 1.68839$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 13

Exercise 1

Given the system

$-2\,u\,x - u\,y - 3\,x\,y - 3\,z == -36$
$-x^2 + 2\,u\,x^2 - 2\,z + 2\,u^2\,z - 2\,z^2 == 151$
$-2\,u\,x^2 - 2\,u\,y^2 + 2\,u^2\,z - z^2 == 40$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point p=(x,y,z,u)=(3,0,4,4). Compute if possible $\frac{\partial z}{\partial u}(4)$.

1) $\frac{\partial z}{\partial u}(4) = -\frac{122}{35}$

2) $\frac{\partial z}{\partial u}(4) = -\frac{487}{140}$

3) $\frac{\partial z}{\partial u}(4) = -\frac{97}{28}$

4) $\frac{\partial z}{\partial u}(4) = -\frac{243}{70}$

5) $\frac{\partial z}{\partial u}(4) = -\frac{489}{140}$

Exercise 2

Compute the volume of the domain limited by the plane $9\,x + 7\,z == 2$ and the paraboloid $z == 3\,x^2 + 3\,y^2$.

1) 0.14289

2) 0.456844

3) 0.0867401

4) 0.093895

5) 0.0879744

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{(2\,t + 4)\sin(2\,t)\,(9\cos(15\,t) + 10),\ (7\,t + 6)\sin(t)\,(9\cos(15\,t) + 10)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 23 374.9 2) 4675.7 3) 11 687.9 4) 2338.3

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -x & 0\le x\le 1 \\ x-2 & 1\le x\le 2 \end{cases} & 0\le x\le 2 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} 3x & 0\le x\le 1 \\ 6-3x & 1\le x\le 2 \end{cases} & 0.\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{7}{5}$

en el instante t=0.3 mediante un desarrollo en serie de Fourier de orden 9.

1) $u(\frac{7}{5},0.3) = 0.310133$

2) $u(\frac{7}{5},0.3) = -3.1235$

3) $u(\frac{7}{5},0.3) = 8.31709$

4) $u(\frac{7}{5},0.3) = -4.02229$

5) $u(\frac{7}{5},0.3) = -1.18535$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 14

Exercise 1

Given the function

$f(x,y,z) = 16 - 4x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {3.03909, 0.984341, 1.56296}

2) We have a minimum at {1.99712, 2.02631, 1.56296}

3) We have a minimum at {1.47613, 0.723847, 3.64691}

4) We have a minimum at {2, 2, 3}

5) We have a minimum at {1.73662, 1.24483, 2.60494}

Exercise 2

Compute $\int_D (y z^3)\, dx\, dy\, dz$ for $D =$
$\{8 \leq x^8 y z \leq 13,\ 5 x y^7 z^2 \leq 1 \leq 14 x y^7 z^2,\ 2 y^6 \leq x^9 z^5 \leq 11 y^6,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.89975

2) 0.000252895

3) -1.29975

4) -1.49975

5) 0.900253

Exercise 3

Consider the vectorial field $F(x,y,z) =$
$\{9xy + 5xz + \mathrm{Cos}[y^2 + 2z^2],\ 3 + e^{-2x^2+2z^2},\ e^{x^2-2y^2} - 2yz\}$ and the surface

$S \equiv \left(\frac{7+x}{3}\right)^2 + \left(\frac{-8+y}{3}\right)^2 + \left(\frac{-3+z}{1}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -4548.56   2) 1873.84   3) 2676.64   4) -7759.76

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=3\,(x-3)^2\,(x-2)\,x^2 & 0\le x\le 3 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} -2\,x & 0\le x\le 1 \\ x-3 & 1\le x\le 2 \lor 2\le x\le 3 \end{cases} & 0.\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2 en el instante t= 0.4 mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,0.4) = −7.16827

2) u(2,0.4) = 2.40782

3) u(2,0.4) = −1.73786

4) u(2,0.4) = 3.97819

5) u(2,0.4) = −0.251851

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 15

Exercise 1

Given the system

$-v^2 + 3 u x^2 - v y - 2 v^2 y + 2 v z + z^3 == 80$

$-2 u - 2 v x^2 - u^2 y - 3 v y z == 8$

$3 u - 3 u x - 3 v y - z^2 == -40$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point p=(x,y,z,u,v)=(2,-2,2,4,-4). Compute if possible $\frac{\partial x}{\partial v}(4,-4)$.

1) $\frac{\partial x}{\partial v}(4,-4)=\frac{4}{5}$

2) $\frac{\partial x}{\partial v}(4,-4)=\frac{6}{5}$

3) $\frac{\partial x}{\partial v}(4,-4)=\frac{2}{5}$

4) $\frac{\partial x}{\partial v}(4,-4)=1$

5) $\frac{\partial x}{\partial v}(4,-4)=\frac{3}{5}$

Exercise 2

Compute the volume of the domain limited by the plane $7 x + 10 z == 2$ and the paraboloid $z == 10 x^2 + 10 y^2$.

1) 0.00959935

2) 0.0535061

3) 0.031718

4) 0.00707645

5) 0.0465248

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$

$r(t) = \{(8 t + 8) \sin(2 t) (6 \cos(17 t) + 6), (7 t + 4) \sin(t) (6 \cos(17 t) + 6)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 27 827.1 2) 9275.92 3) 23 189.3 4) 20 870.4

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(4,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -3x & 0\le x\le 2 \\ 3x-12 & 2\le x\le 4 \end{cases} & 0\le x\le 4 \\ 0 & \text{True} \end{cases}$$

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.7 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.7) = -3.

2) u(1,0.7) = 4.34603

3) u(1,0.7) = -1.80828

4) u(1,0.7) = -0.224732

5) u(1,0.7) = 2.40757

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 16

Exercise 1

Given the functions

$f(x,y)=(-1-3x^2-3y+2xy, 2-x+3x^2-y-2xy+2y^2, -2+2x-y+2xy+y^2)$

and

$g(u,v,w)=(u^2-2v^2-2vw, -3+3u^2+uw+vw+3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(3,-1).

1) -5.09388×10^6

2) -2.10224×10^6

3) -5.36281×10^6

4) -1.03288×10^6

5) -3.48036×10^6

Exercise 2

Compute the volume of the domain limited by the plane $3x+9z=4$ and the paraboloid $z=8x^2+8y^2$.

1) 0.0493098

2) 0.140536

3) 0.164099

4) 0.0393935

5) 0.0823718

Exercise 3

Consider the vectorial field $F(x,y,z)=\{6yz-8xz^2, -2x^2y^2, 9x^2+3xz\}$ and the surface

$S\equiv\left(\frac{4+x}{6}\right)^2+\left(\frac{9+y}{5}\right)^2+\left(\frac{-2+z}{8}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 692457. 2) 1.03869×10^6 3) -1.86963×10^6 4) 1.45416×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{3x}{2} & 0 \le x \le 2 \\ -\frac{3x}{\pi-2} + \frac{6}{\pi-2} + 3 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = -3(x-3)(x-2)x^2(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.6 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.6) = 7.74948

2) u(1,0.6) = 5.72664

3) u(1,0.6) = -7.70733

4) u(1,0.6) = -3.97827

5) u(1,0.6) = 4.27987

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 17

Exercise 1

Given the function

$f(x,y,z)=11-2x+x^2+y^2-2z+z^2$ defined over the domain D≡ $\frac{x^2}{16}+\frac{y^2}{16}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-3.86875, 0.4, -0.349509}

2) We have a maximum at {-3.96875, 0., -0.249509}

3) We have a maximum at {-3.56875, -0.3, 0.250491}

4) We have a maximum at {1, 0, 1}

5) We have a maximum at {-4.26875, 0.3, -0.349509}

Exercise 2

Compute $\int_D (z^2)\,dx\,dy\,dz$ for D=

$\{6x^5 z \le y^3 \le 7x^5 z,\ 3y^9 \le x^9 z^6 \le 4y^9,\ 6z^4 \le x^6 y^7 \le 11z^4,\ x>0,\ y>0,\ z>0\}$

1) 3.064

2) 6.12801

3) 2.18857

4) 5.25258

5) 0.218857

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,2\pi]\dashrightarrow R^2$

$$r(t)=\left\{\frac{\left(-\frac{1}{2}\sqrt{3}\sin(t)-\frac{1}{2}\right)\cos(t)\,(9\cos(t)+9)}{\sin^2(t)+1},\ \frac{\left(\frac{\sqrt{3}}{2}-\frac{\sin(t)}{2}\right)\cos(t)\,(9\cos(t)+9)}{\sin^2(t)+1}\right\}$$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 135.531 2) 150.531 3) 195.531 4) 225.531

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 3x & 0 \le x \le 1 \\ -\frac{3x}{\pi-1} + \frac{3}{\pi-1} + 3 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = (x-3)\, x\, (x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2
en el instante t=1. mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,1.) = -8.64312

2) u(2,1.) = -8.70281

3) u(2,1.) = 3.77128

4) u(2,1.) = 1.14092

5) u(2,1.) = -1.65354

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 18

Exercise 1

Given the function

$f(x,y,z)=-8+2x-x^2+2y-y^2-z^2$ defined over the domain $D\equiv \frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1, 1, 0}

2) We have a maximum at {0.6, 1.4, 0.5}

3) We have a maximum at {1.4, 1.1, -0.3}

4) We have a maximum at {1.4, 0.9, 0.1}

5) We have a maximum at {1.2, 1.3, -0.2}

Exercise 2

Compute $\int_D (x+2y)\,dx\,dy\,dz$ for $D=$

$\{3x^4y^8z^8\le 1\le 5x^4y^8z^8,\ 5x^4\le y^8z^8\le 13x^4,\ 2z^3\le x^3y\le 6z^3,\ x>0,\ y>0,\ z>0\}$

1) 0.506511

2) 0.606511

3) 1.10651

4) -1.79349

5) 0.00651102

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,2\pi]\dashrightarrow R^2$

$$r(t)=\left\{\frac{\left(\frac{1}{2}-\frac{1}{2}\sqrt{3}\sin(t)\right)\cos(t)\,(6\cos(t)+6)}{\sin^2(t)+1},\ \frac{\left(\frac{\sin(t)}{2}+\frac{\sqrt{3}}{2}\right)\cos(t)\,(6\cos(t)+6)}{\sin^2(t)+1}\right\}$$

Indication: it is necessary to represent
 the curve to check whether it has intersection points.

1) 66.9027 2) 106.503 3) 113.103 4) 7.50266

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(3,t)=0 & 0\leq t \\ u(x,0)=-2\ (x-3)\ (x-1)\ x & 0\leq x\leq 3 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.3 mediante un desarrollo en serie de Fourier de orden 9.

1) u(2,0.3) = 4.97342

2) u(2,0.3) = 1.50032

3) u(2,0.3) = 0.0736357

4) u(2,0.3) = 2.27294

5) u(2,0.3) = -3.76162

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 19

Exercise 1

Given the system

$3\,w\,y^2 + u\,v\,z - w\,z == -30$
$3\,u^2\,w - 2\,v^2\,z - 3\,x\,y\,z == 66$
$-3\,w^2\,x + u\,x\,y == -10$

determine if it is possible to solve for variables x, y,z in terms of variables u,v,w arround the point p=(x,y,z, u,v,w)=(2,-1,-3,2,4,-1). Compute if possible $\frac{\partial x}{\partial v}(2,4,-1)$.

1) $\frac{\partial x}{\partial v}(2,4,-1) = -\frac{182}{211}$

2) $\frac{\partial x}{\partial v}(2,4,-1) = -\frac{183}{211}$

3) $\frac{\partial x}{\partial v}(2,4,-1) = -\frac{184}{211}$

4) $\frac{\partial x}{\partial v}(2,4,-1) = -\frac{181}{211}$

5) $\frac{\partial x}{\partial v}(2,4,-1) = -\frac{180}{211}$

Exercise 2

Compute $\int_D (6\,y^2)\,dx\,dy\,dz$ for D=

$\{4\,x\,y^3\,z^3 \le 1 \le 7\,x\,y^3\,z^3,\ 9\,z^9 \le x^4\,y^7 \le 15\,z^9,\ 5\,y^6\,z^4 \le x^9 \le 14\,y^6\,z^4,\ x > 0,\ y > 0,\ z > 0\}$

1) -0.698611

2) 1.60139

3) 0.00138944

4) 1.60139

5) 0.701389

Exercise 3

Consider the vectorial field $F(x,y,z) = \{4z + \text{Cos}[z^2], -6yz - 4xyz - \text{Sin}[x^2], xz - 2yz + \text{Cos}[2y^2]\}$ and the surface

$$S \equiv \left(\frac{-8+x}{7}\right)^2 + \left(\frac{-9+y}{7}\right)^2 + \left(\frac{-3+z}{1}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) −73 809.9   2) −55 993.5   3) 50 904.9   4) −25 451.1

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-3)\, x\, (x-\pi)^2 & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} x & 0 \le x \le 1 \\ 3 - 2x & 1 \le x \le 2 \\ \frac{x}{\pi-2} - \frac{2}{\pi-2} - 1 & 2 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.7 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.7) = 2.3208

2) u(1,0.7) = −5.19539

3) u(1,0.7) = −0.864329

4) u(1,0.7) = 7.43462

5) u(1,0.7) = −6.25074

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 20

Exercise 1

Given the functions

$f(x,y) = (-3 + 3x - 3x^2 + 3y + 3y^2, -3 + 2x - x^2 + y + 2xy + y^2)$

and

$g(u,v) = (-3 - u - 2u^2 - uv, -1 - u + 2u^2 + 2v - uv - 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,3)$.

1) -1.27648×10^6

2) $-956046.$

3) $-801321.$

4) -1.69908×10^6

5) $-734308.$

Exercise 2

Compute $\int_D (2y^2 z)\,dx\,dy\,dz$ for $D=\{6 \le x^7 y^4 z^3 \le 8,\ 1 \le x^6 y^9 z^2 \le 6,\ 3y^3 z \le x \le 11 y^3 z,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.307221

2) 0.00722146

3) 0.607221

4) -0.592779

5) -1.49278

Exercise 3

Consider the vectorial field $F(x,y,z) = \{-xy + \text{Cos}[2z^2], -6xyz - \text{Sin}[2x^2], 9xyz + \text{Cos}[2x^2 - 2y^2]\}$ and the surface

$S \equiv \left(\frac{9+x}{3}\right)^2 + \left(\frac{-3+y}{6}\right)^2 + \left(\frac{-8+z}{9}\right)^2 = 1$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) 378649. 2) -353404. 3) 126217. 4) 252433.

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ -\frac{7x}{\pi-2} + \frac{14}{\pi-2} + 7 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.2 mediante un desarrollo en serie de Fourier de orden 9.

1) u(2,0.2) = 3.51487

2) u(2,0.2) = −0.283967

3) u(2,0.2) = −4.86629

4) u(2,0.2) = −0.530194

5) u(2,0.2) = 1.81185

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 21

Exercise 1

Given the functions

$f(x,y) = (-2 + 3x - 2y + 2xy + 2y^2, 2 - x^2 - 2y + 3xy - 3y^2)$

and

$g(u,v) = (2 - 2u + 3u^2 + v - 2uv - v^2, 3 - 3u - 2v + 3uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,-3)$.

1) -1.37246×10^6

2) $-261052.$

3) -1.62477×10^6

4) -2.09174×10^6

5) $-539015.$

Exercise 2

Compute the volume of the domain limited by the plane $x + 5z == 5$ and the paraboloid $z == 4x^2 + 4y^2$.

1) 0.859256

2) 0.147588

3) 1.58633

4) 1.69189

5) 0.394665

Exercise 3

Consider the vectorial field $F(x,y,z) = \{8xy - x^2yz, -4, x^2 + 7z^2\}$ and the surface

$S \equiv \left(\frac{9+x}{1}\right)^2 + \left(\frac{-5+y}{3}\right)^2 + \left(\frac{z}{4}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 2010.62 2) −5828.38 3) −4421.38 4) 2211.62

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0 \leq t \\ u(x,0) = \begin{cases} -\frac{50x}{7} & 0 \leq x \leq \frac{7}{10} \\ \frac{50x}{3} - \frac{50}{3} & \frac{7}{10} \leq x \leq 1 \end{cases} & 0 \leq x \leq 1 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -5x & 0 \leq x \leq \frac{4}{5} \\ 20x - 20 & \frac{4}{5} \leq x \leq 1 \end{cases} & 0. \leq x \leq 1 \\ 0 & \text{True} \end{cases}$$

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

General: Further output of NIntegrate::slwcon will be suppressed during this calculation.

Calcular la posición de la cuerda en el punto $x=\frac{2}{5}$ en el instante $t=0.2$ mediante un desarrollo en serie de Fourier de orden 11.

1) $u(\frac{2}{5},0.2) = -3.24703$

2) $u(\frac{2}{5},0.2) = 1.00674$

3) $u(\frac{2}{5},0.2) = -7.09196$

4) $u(\frac{2}{5},0.2) = -8.11949$

5) $u(\frac{2}{5},0.2) = -5.42437$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 22

Exercise 1

Given the function

$f(x,y,z)=-4+6x-x^2-y^2+2z-z^2$ defined over the domain $D\equiv \frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-5.49229, -0.2, -0.31104}

2) We have a minimum at {-4.99229, 0., -0.11104}

3) We have a minimum at {-4.59229, 0.3, -0.51104}

4) We have a minimum at {3, 0, 1}

5) We have a minimum at {-4.59229, -0.2, -0.41104}

Exercise 2

Compute $\int_D (y^3+z^3)\,dx\,dy\,dz$ for $D=\{4z^2\le x^3y^3\le 6z^2,\ 8\le x^3z^9\le 10,\ 3x^3y\le z^8\le 6x^3y,\ x>0,\ y>0,\ z>0\}$

1) -0.683845

2) -0.483845

3) 0.0161551

4) -1.58384

5) -0.983845

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-1+3x+\mathrm{Cos}[2y^2],\ 2xz-\mathrm{Sin}[2x^2-z^2],\ e^{-y^2}\}$ and the surface

$$S\equiv\left(\frac{-6+x}{4}\right)^2+\left(\frac{4+y}{6}\right)^2+\left(\frac{2+z}{8}\right)^2=1$$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 5548.34 2) -4340.86 3) -6994.06 4) 2412.74

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = -(x-2)^2 (x-1)\, x & 0 \le x \le 2 \\ \frac{\partial}{\partial t} u(x,0) = \begin{cases} -9x & 0 \le x \le 1 \\ 9x - 18 & 1 \le x \le 2 \end{cases} & 0. \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.5 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.5) = −8.86542

2) u(1,0.5) = 4.59589

3) u(1,0.5) = −3.18732

4) u(1,0.5) = 2.95041

5) u(1,0.5) = −8.44074

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 23

Exercise 1

Given the function

$f(x,y,z) = -19 + 4x - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.95609, 2.95751, 1.92324}

2) We have a maximum at {2, 3, 2}

3) We have a maximum at {2.54759, 4.14051, 3.10624}

4) We have a maximum at {0.773083, 2.36601, 3.40199}

5) We have a maximum at {3.43484, 3.54901, 0.444488}

Exercise 2

Compute $\int_D (2y + z^3)\, dx\, dy\, dz$ for D=

$\{2y^3 \le x \le 9y^3,\ 3x^2 \le y^5 z^8 \le 7x^2,\ 7x^9 z^2 \le y^2 \le 14x^9 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.0116137

2) 0.811614

3) -0.288386

4) 1.91161

5) 1.51161

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] ----\to R^2$

$$r(t) = \left\{ \frac{\left(-\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} - \frac{1+\sqrt{3}}{2\sqrt{2}}\right)\cos(t)\,(9\cos(t)+10)}{\sin^2(t)+1},\ \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(9\cos(t)+10)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 152.631 2) 101.931 3) 169.531 4) 203.331

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 2x & 0\le x\le 2 \\ 4x-4 & 2\le x\le 3 \\ -\frac{8x}{\pi-3}+\frac{24}{\pi-3}+8 & 3\le x\le \pi \end{cases} & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=1. mediante un desarrollo en serie de Fourier de orden 9.

1) u(1,1.) = -0.961163

2) u(1,1.) = -2.11879

3) u(1,1.) = 2.44457

4) u(1,1.) = 3.36319

5) u(1,1.) = 2.58193

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 24

Exercise 1

Given the system

$x z^2 + 3 z u_1 u_4 == 124$
$-2 x z u_3 == -128$
$-y u_1 - 2 z u_4 == -18$

determine if it is possible to solve for variables x,y,z in terms of variables u_1,u_2,u_3,u_4 arround the point p=(x,y,z,u_1,u_2,u_3,u_4)=(4,2,-4,5,4,-4,-1). Compute if possible $\frac{\partial y}{\partial u_4}(5,4,-4,-1)$.

1) $\frac{\partial y}{\partial u_4}(5,4,-4,-1) = \frac{131}{155}$

2) $\frac{\partial y}{\partial u_4}(5,4,-4,-1) = \frac{26}{31}$

3) $\frac{\partial y}{\partial u_4}(5,4,-4,-1) = \frac{129}{155}$

4) $\frac{\partial y}{\partial u_4}(5,4,-4,-1) = \frac{128}{155}$

5) $\frac{\partial y}{\partial u_4}(5,4,-4,-1) = \frac{132}{155}$

Exercise 2

Compute the volume of the domain limited by the plane $8 x + 8 z == 3$ and the paraboloid $z == 7 x^2 + 7 y^2$.

1) 0.0504834

2) 0.0378531

3) 0.110999

4) 0.0783491

5) 0.0887118

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \longrightarrow R^2$
$r(t) = \{(7 t + 5) \sin(2 t) (2 \cos(20 t) + 4), (6 t + 3) \sin(t) (2 \cos(20 t) + 4)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 8027.51 2) 1505.41 3) 5017.31 4) 6522.41

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=2\,(x-1)^2\left(x-\frac{7}{10}\right)x & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} \frac{15x}{4} & 0\le x\le \frac{4}{5} \\ 15-15x & \frac{4}{5}\le x\le 1 \end{cases} & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{3}{5}$ en el instante $t=0.3$ mediante un desarrollo en serie de Fourier de orden 11.

1) $u\left(\frac{3}{5},0.3\right) = -8.72651$

2) $u\left(\frac{3}{5},0.3\right) = -6.03954$

3) $u\left(\frac{3}{5},0.3\right) = 0.708684$

4) $u\left(\frac{3}{5},0.3\right) = -0.00163249$

5) $u\left(\frac{3}{5},0.3\right) = 6.51032$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 25

Exercise 1

Given the function

$f(x,y,z)=14-4x+x^2-6y+y^2+z^2$ defined over the domain $D\equiv \frac{x^2}{16}+\frac{y^2}{4}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-4.32257, -0.889063, -0.4}

2) We have a maximum at {-4.02257, -0.489063, 0.5}

3) We have a maximum at {-3.52257, -0.789063, -0.3}

4) We have a maximum at {2, 3, 0}

5) We have a maximum at {-3.82257, -0.589063, 0.}

Exercise 2

Compute $\int_D (y+y^2)\,dx\,dy\,dz$ for $D=$
$\{4z^7 \leq x^8 y^2 \leq 8z^7,\ 8y^7 z^4 \leq x^8 \leq 9y^7 z^4,\ 7z^2 \leq x^4 \leq 13z^2,\ x>0,\ y>0,\ z>0\}$

1) 0.730729

2) 0.830729

3) 0.0307293

4) 0.230729

5) -0.869271

Exercise 3

Consider the vectorial field $F(x,y,z)=$
$\{8-4x+\text{Cos}[y^2],\ e^{-x^2-z^2}+4xy,\ -4y+yz+\text{Cos}[x^2+y^2]\}$ and the surface

$S\equiv \left(\frac{7+x}{3}\right)^2+\left(\frac{4+y}{3}\right)^2+\left(\frac{7+z}{8}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -10857.3 2) -41259.7 3) -34744.9 4) -40173.9

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=\begin{cases} \frac{7x}{2} & 0\le x\le 2 \\ 21-7x & 2\le x\le 3 \end{cases} & 0\le x\le 3 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} \frac{9x}{2} & 0\le x\le 2 \\ 27-9x & 2\le x\le 3 \end{cases} & 0.\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.9 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.9) = −5.00362

2) u(1,0.9) = 3.54469

3) u(1,0.9) = −4.80801

4) u(1,0.9) = −0.309949

5) u(1,0.9) = 2.04014

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 26

Exercise 1

Given the functions

$f(x,y) = (2 + 3x - 3x^2 - 2xy - 2y^2, -3 - 2x + 3x^2 - 3y + 2xy - y^2, 2x + 3x^2 - 2y - 3xy)$

and

$g(u,v,w) = (2v, 2v + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2,-1)$.

1) 0.

2) -0.776321

3) -0.196756

4) 0.532358

5) -0.638502

Exercise 2

Compute $\int_D (2x^3 z)\, dx\, dy\, dz$ for D=
$\{6y^9 z \le x^7 \le 7y^9 z,\ 8yz^3 \le x^3 \le 15yz^3,\ 3y^7 \le x^9 \le 10y^7,\ x > 0,\ y > 0,\ z > 0\}$

1) 2.00168

2) -0.998315

3) 0.00168489

4) -0.198315

5) 1.30168

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{\sin(2t)\cos(t)(2\cos(t) + 4), \sin(t)\sin(2t)(2\cos(t) + 4)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 14.1372 2) 26.7372 3) 1.53717 4) 12.7372

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=2\,(x-3)\,x\,(x-\pi) & 0\le x\le \pi \\ \frac{\partial}{\partial t}u(x,0)=(x-1)\,x\,(x-\pi) & 0.\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1
en el instante t=1. mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,1.) = -5.10827

2) u(1,1.) = 3.89517

3) u(1,1.) = -6.64434

4) u(1,1.) = 6.33102

5) u(1,1.) = 3.07022

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 27

Exercise 1

Given the system

$-y^2 u_2 - 3 u_2 u_3^2 + 2 x z u_4 - 3 x u_4^2 == 256$
$-x z + z u_2 u_3 == 57$
$-3 x z^2 + y^2 u_3 - 3 y z u_3 == -161$

determine if it is possible to solve for variables x,y,z in terms of variables u_1,u_2,u_3,u_4 arround the point $p=(x,y,z,u_1,u_2,u_3,u_4)=(3,-4,-3,2,-4,4,-2)$. Compute if possible $\frac{\partial y}{\partial u_2}(2,-4,4,-2)$.

1) $\frac{\partial y}{\partial u_2}(2,-4,4,-2)=-\frac{64}{47}$

2) $\frac{\partial y}{\partial u_2}(2,-4,4,-2)=-\frac{62}{47}$

3) $\frac{\partial y}{\partial u_2}(2,-4,4,-2)=-\frac{65}{47}$

4) $\frac{\partial y}{\partial u_2}(2,-4,4,-2)=-\frac{61}{47}$

5) $\frac{\partial y}{\partial u_2}(2,-4,4,-2)=-\frac{63}{47}$

Exercise 2

Compute $\int_D (x^3 z^3)\, dx dy dz$ for $D=\{1 \le x^8 y^2 z^5 \le 8,\ 7 z^7 \le y^3 \le 11 z^7,\ 9 y^6 z^8 \le x^3 \le 10 y^6 z^8,\ x>0,\ y>0,\ z>0\}$

1) 1.00019

2) 0.00019156

3) 1.70019

4) 0.700192

5) -0.999808

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$

$$r(t) = \left\{\sin(2t)\,(8\cos(t)+8)\left(\frac{\cos(t)}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right),\ \sin(2t)\,(8\cos(t)+8)\left(\frac{\sin(t)}{\sqrt{2}} + \frac{\cos(t)}{\sqrt{2}}\right)\right\}$$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 67.8982   2) 97.8982   3) 75.3982   4) 142.898

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(2,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -9x & 0 \le x \le 1 \\ 9x - 18 & 1 \le x \le 2 \end{cases} & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto $x=\frac{3}{2}$
en el instante t=0.8 mediante un desarrollo en serie de Fourier de orden 11.

1) $u\left(\frac{3}{2},0.8\right) = -1.99187$

2) $u\left(\frac{3}{2},0.8\right) = 2.4096$

3) $u\left(\frac{3}{2},0.8\right) = 4.5799$

4) $u\left(\frac{3}{2},0.8\right) = -4.5$

5) $u\left(\frac{3}{2},0.8\right) = -1.31362$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 28

Exercise 1

Given the system

$u x^2 - v x y + u y^2 + 2 v x z + 2 z^2 == 65$
$3 - 2 v x z == 15$
$-3 u^2 + 3 u v + 2 u v^2 + v^3 - 2 u x^2 - 3 v x y + 2 x^2 y - 2 z == -109$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point p=(x,y,z,u,v)=(-2,3,-3,5,-1). Compute if possible $\frac{\partial z}{\partial u}(5,-1)$.

1) $\frac{\partial z}{\partial u}(5,-1) = -\frac{1677}{782}$

2) $\frac{\partial z}{\partial u}(5,-1) = -\frac{837}{391}$

3) $\frac{\partial z}{\partial u}(5,-1) = -\frac{838}{391}$

4) $\frac{\partial z}{\partial u}(5,-1) = -\frac{1673}{782}$

5) $\frac{\partial z}{\partial u}(5,-1) = -\frac{1675}{782}$

Exercise 2

Compute $\int_D (z^2)\, dx dy dz$ for D=
$\{1 \le x^5 y^3 z^4 \le 9,\ 9 \le x^8 y^5 z^5 \le 18,\ 6 x^2 y^8 z^8 \le 1 \le 10 x^2 y^8 z^8,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.0120077

2) -1.18799

3) 1.81201

4) -1.38799

5) 1.71201

Exercise 3

Consider the vectorial field $F(x,y,z) = \left\{e^{-y^2-z^2} - 8\,x\,y\,z,\ 7 - 8\,y\,z - \mathrm{Sin}\left[x^2 + 2\,z^2\right],\ -2\,x\,y - 4\,x\,z + \mathrm{Cos}\left[y^2\right]\right\}$ and the surface

$$S \equiv \left(\frac{-4+x}{8}\right)^2 + \left(\frac{-3+y}{7}\right)^2 + \left(\frac{7+z}{9}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) 1.00997×10^6 2) 263 472. 3) 439 119. 4) −614 766.

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = 2\,(x-4)^2\,(x-3)\,(x-2)\,x^2 & 0 \le x \le 4 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -x & 0 \le x \le 2 \\ x-4 & 2 \le x \le 4 \end{cases} & 0. \le x \le 4 \\ 0 & \mathrm{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=3 en el instante t= 0.8 mediante un desarrollo en serie de Fourier de orden 11.

1) u(3,0.8) = 1.84338

2) u(3,0.8) = −7.02841

3) u(3,0.8) = −5.48868

4) u(3,0.8) = 3.76332

5) u(3,0.8) = 8.87794

Further Mathematics - 2023/2024 Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 29

Exercise 1

Given the functions

$f(x,y) = (2 + x + x^2 - y - 2xy + y^2, -1 - x + 2x^2 + y - xy - 3y^2, 1 - 2x + x^2 + 2y - 3xy + 3y^2, -3 + x^2 - 3y - 2xy + 2y^2)$

and

$g(u_1,u_2,u_3,u_4) = (u_1^2 - 3u_2 + 2u_1u_4 - 3u_2u_4, -u_1 + 3u_1^2 + u_3 + 2u_1u_3 + 2u_1u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,3)$.

1) 118 295.

2) 62 476.9

3) 87 444.

4) 53 496.6

5) 74 804.

Exercise 2

Compute the volume of the domain limited by the plane $8x + 6z == 6$ and the paraboloid $z == 3x^2 + 3y^2$.

1) 0.150359

2) 2.8845

3) 0.690231

4) 1.56527

5) 2.31086

Exercise 3

Consider the vectorial field $F(x,y,z) = \{-7y^2 - 9yz^2, -8x^2 - 5yz^2, -9x^2z\}$ and the surface

$S \equiv \left(\frac{-5 + x}{9}\right)^2 + \left(\frac{y}{1}\right)^2 + \left(\frac{8 + z}{3}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) −110 804. 2) −79 145.5 3) −63 316.3 4) −237 438.

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -6x & 0 \le x \le 1 \\ 3x-9 & 1 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.3 mediante un desarrollo en serie de Fourier de orden 8.

1) u(1,0.3) = -4.25803

2) u(1,0.3) = 4.48653

3) u(1,0.3) = -4.90492

4) u(1,0.3) = -3.12761

5) u(1,0.3) = 3.93429

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 30

Exercise 1

Given the functions

$f(x,y)=(3-x+x^2-3xy-3y^2,2-2x-3x^2-y+3xy+3y^2)$

and

$g(u,v)=(-3+u-2u^2+v-uv-v^2,1-u^2-2v+v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,-3)$.

1) $-622680.$

2) -5.3727×10^6

3) -4.75539×10^6

4) -2.84543×10^6

5) -3.69427×10^6

Exercise 2

Compute $\int_D (2x^4)\,dx\,dy\,dz$ for D=

$\{5\le x^3y^2z^8\le 6,\ 5x^4\le y^2z^5\le 8x^4,\ 5x^9yz\le 1\le 11x^9yz,\ x>0,\ y>0,\ z>0\}$

1) 1.50049

2) 0.000485734

3) -1.79951

4) -0.499514

5) 1.20049

Exercise 3

Consider the vectorial field $F(x,y,z)=$
$\{1+e^{y^2+z^2}-5x,\ 4xy+3z+\text{Sin}[x^2],\ -3z+\text{Cos}[x^2-2y^2]\}$ and the surface

$S\equiv\left(\frac{-7+x}{6}\right)^2+\left(\frac{-7+y}{6}\right)^2+\left(\frac{7+z}{2}\right)^2=1$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) 18093.9 2) 6031.86 3) -9045.64 4) 3619.46

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 1 \\ -\frac{2x}{\pi-1} + \frac{2}{\pi-1} + 2 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = -3(x-2)x^2(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2
en el instante t=0.9 mediante un desarrollo en serie de Fourier de orden 9.

1) u(2,0.9) = -7.38568

2) u(2,0.9) = 1.13028

3) u(2,0.9) = -7.96405

4) u(2,0.9) = -7.04412

5) u(2,0.9) = -7.21094

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 31

Exercise 1

Given the functions

$f(x,y)=(-x+3x^2-3y-xy-2y^2, 1+x-x^2-y+3xy-3y^2, 1-x-3x^2-y-3y^2)$

and

$g(u,v,w)=(2uv+3v^2+w^2, -2-3u^2-3v-v^2-vw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(-3,2).

1) -2.02335×10^7

2) -8.07151×10^6

3) -9.55277×10^6

4) -5.29998×10^6

5) -2.74031×10^7

Exercise 2

Compute $\int_D (y^3 z)\,dx\,dy\,dz$ for D=

$\{7x^8 \le z^4 \le 15x^8,\ x^7y^7 \le z^4 \le 3x^7y^7,\ 6y^9z^8 \le x^8 \le 13y^9z^8,\ x>0,\ y>0,\ z>0\}$

1) -1.1999

2) 0.0000954953

3) 1.7001

4) 1.9001

5) 0.700095

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$
$r(t)=\{\sin(2t)\cos(t)(2\cos(t)+8), \sin(t)\sin(2t)(2\cos(t)+8)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 67.1363 2) 97.7363 3) 51.8363 4) 72.2363

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=3\,(x-1)\,x^2\,(x-\pi) & 0\le x\le \pi \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} x & 0\le x\le 3 \\ -\frac{3x}{\pi-3}+\frac{9}{\pi-3}+3 & 3\le x\le \pi \end{cases} & 0.\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2
en el instante t=0.4 mediante un desarrollo en serie de Fourier de orden 8.

1) u(2,0.4) = 4.53093

2) u(2,0.4) = 8.44906

3) u(2,0.4) = 8.19138

4) u(2,0.4) = 7.58448

5) u(2,0.4) = -7.38867

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 32

Exercise 1

Given the function

$f(x,y,z) = -15 + 2x - x^2 + 4y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0.798251, 2.13127, -0.387504}

2) We have a maximum at {0.992003, 1.93752, 0.}

3) We have a maximum at {1, 2, 0}

4) We have a maximum at {1.57326, 1.16251, -0.193752}

5) We have a maximum at {0.410747, 1.16251, 0.193752}

Exercise 2

Compute $\int_D (2xy)\,dx\,dy\,dz$ for D≡

$\{7y^9z^3 \le x^7 \le 14y^9z^3,\ 7y^3z^9 \le x^5 \le 14y^3z^9,\ 7z^5 \le x^4y \le 10z^5,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.89517

2) -0.795173

3) 0.604827

4) 0.00482696

5) 1.90483

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] \dashrightarrow R^2$

$r(t) = \left\{\frac{\cos(t)\,(2\cos(t)+8)}{\sin^2(t)+1},\ \frac{\sin(t)\cos(t)\,(2\cos(t)+8)}{\sin^2(t)+1}\right\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 67.4336 2) 33.9336 3) 47.3336 4) 74.1336

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -((x-3)(x-1)\,x\,(x-\pi)) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.2 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.2) = −2.29011

2) u(1,0.2) = −0.52593

3) u(1,0.2) = 0.766445

4) u(1,0.2) = −3.06777

5) u(1,0.2) = 1.64106

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 33

Exercise 1

Given the system

$2x^2 + 3x^3 - 2y^2 + 3uy^2 - 2x^2 z == -578$
$-2ux^2 - uy - 3uz^2 + 3xz^2 - 3z^3 == -160$
$-u^2 + x^3 + uy - 3u^2 y + 3x^2 y - 3z^2 == 138$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point p=(x,y,z,u)=(-3,-5,5,-5). Compute if possible $\frac{\partial y}{\partial u}(-5)$.

1) $\frac{\partial y}{\partial u}(-5) == -\frac{31896}{32059}$

2) $\frac{\partial y}{\partial u}(-5) == -\frac{159482}{160295}$

3) $\frac{\partial y}{\partial u}(-5) == -\frac{159481}{160295}$

4) $\frac{\partial y}{\partial u}(-5) == -\frac{159484}{160295}$

5) $\frac{\partial y}{\partial u}(-5) == -\frac{159483}{160295}$

Exercise 2

Compute the volume of the domain limited by the plane $9x + 5z == 5$ and the paraboloid $z == 4x^2 + 4y^2$.

1) 1.50786

2) 0.476216

3) 0.64592

4) 0.444582

5) 0.567845

Exercise 3

Consider the vectorial field $F(x,y,z) = \{-7x^2 y z^2, 3x^2, -5y + 8xy\}$ and the surface

$S \equiv \left(\frac{-7+x}{6}\right)^2 + \left(\frac{-8+y}{3}\right)^2 + \left(\frac{8+z}{7}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -3.05374×10^7 2) -2.13762×10^7 3) -1.06881×10^8 4) -7.02359×10^7

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=(x-1)^2\left(x-\frac{3}{10}\right)x^2 & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} -8x & 0\le x\le \frac{1}{2} \\ -\frac{15x}{2}-\frac{1}{4} & \frac{1}{2}\le x\le \frac{9}{10} \\ 70x-70 & \frac{9}{10}\le x\le 1 \end{cases} & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{2}{5}$ en el instante t=0.9 mediante un desarrollo en serie de Fourier de orden 9.

1) $u(\frac{2}{5},0.9) = -8.14406$

2) $u(\frac{2}{5},0.9) = -4.89536$

3) $u(\frac{2}{5},0.9) = -0.475091$

4) $u(\frac{2}{5},0.9) = -1.93552$

5) $u(\frac{2}{5},0.9) = -6.37975$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 34

Exercise 1

Given the system

$-3u^2 v + 2uvx + 2y + 3vz^2 == -177$

$3u^3 + uvy - 2xz^2 == -168$

$2 + 3xy^2 - 2yz == 25$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point p=(x,y,z,u,v)=(5,-1,4,-1,-5). Compute if possible $\frac{\partial y}{\partial u}(-1,-5)$.

1) $\frac{\partial y}{\partial u}(-1,-5) = \frac{4853}{43693}$

2) $\frac{\partial y}{\partial u}(-1,-5) = \frac{4850}{43693}$

3) $\frac{\partial y}{\partial u}(-1,-5) = \frac{4854}{43693}$

4) $\frac{\partial y}{\partial u}(-1,-5) = \frac{4852}{43693}$

5) $\frac{\partial y}{\partial u}(-1,-5) = \frac{4851}{43693}$

Exercise 2

Compute $\int_D (2y + z^3)\, dx\, dy\, dz$ for D=

$\{6x^9 z^9 \le y^8 \le 9x^9 z^9,\ 5x^3 y^2 \le z^7 \le 6x^3 y^2,\ 3xy^3 z^8 \le 1 \le 11xy^3 z^8,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.99984

2) -0.0998438

3) 0.0001562

4) -1.99984

5) 0.800156

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \longrightarrow R^2$

$r(t) = \left\{\sin(2t)\,(6\cos(t)+9)\left(-\frac{\sin(t)}{2} - \frac{1}{2}\sqrt{3}\cos(t)\right),\ \sin(2t)\,(6\cos(t)+9)\left(\frac{\cos(t)}{2} - \frac{1}{2}\sqrt{3}\sin(t)\right)\right\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 77.7544 2) 93.1544 3) 131.654 4) 39.2544

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 7x & 0 \le x \le 1 \\ -\frac{7x}{\pi-1} + \frac{7}{\pi-1} + 7 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = -((x-3)\,x\,(x-\pi)) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.8 mediante un desarrollo en serie de Fourier de orden 11.

1) u(1,0.8) = -0.136945

2) u(1,0.8) = -2.02578

3) u(1,0.8) = -0.111127

4) u(1,0.8) = -0.97534

5) u(1,0.8) = -3.25811

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 35

Exercise 1

Given the function

$f(x,y,z) = -21 + 4x - x^2 + 4y - y^2 + 6z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.62621, 1.41982, 1.56298}

2) We have a maximum at {1.13835, 1.74506, 0.749874}

3) We have a maximum at {0.813106, 1.74506, 1.40036}

4) We have a maximum at {2, 2, 3}

5) We have a maximum at {1.95145, 0.769338, 2.37609}

Exercise 2

Compute $\int_D (x^2)\, dx\, dy\, dz$ for $D = \{2 \le x^9 y \le 7,\ x^4 z \le y^5 \le 3 x^4 z,\ 6 x^6 y^5 z^2 \le 1 \le 10 x^6 y^5 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) -0.998026

2) 1.70197

3) 0.00197422

4) -1.29803

5) 0.601974

Exercise 3

Compute the area of the domain whose boundary is the curve

$r : [0, 2\pi] \dashrightarrow R^2$

$$r(t) = \left\{ \frac{\left(-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+10)}{\sin^2(t)+1},\ \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+10)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 73.0602 2) 97.2602 3) 121.46 4) 24.6602

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(5,t)=0 & 0\le t \\ u(x,0)=\begin{cases} \frac{5x}{3} & 0\le x\le 3 \\ \frac{25}{2}-\frac{5x}{2} & 3\le x\le 5 \end{cases} & 0\le x\le 5 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.9 mediante un desarrollo en serie de Fourier de orden 9.

1) u(2,0.9) = -3.28513

2) u(2,0.9) = -4.94667

3) u(2,0.9) = 2.49998

4) u(2,0.9) = 1.33141

5) u(2,0.9) = -3.00886

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 36

Exercise 1

Given the function

$f(x,y,z) = -12 - x^2 + 4y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-0.8, 1.2, -0.4\}$

2) We have a maximum at $\{0, 2, 0\}$

3) We have a maximum at $\{-1., 2.2, -0.8\}$

4) We have a maximum at $\{0.8, 1.4, -0.8\}$

5) We have a maximum at $\{0.8, 2.8, -0.2\}$

Exercise 2

Compute $\int_D (x^2)\, dx\, dy\, dz$ for D=

$\{8 y^7 z^4 \leq x^9 \leq 14 y^7 z^4,\ 7 x y^3 z^3 \leq 1 \leq 12 x y^3 z^3,\ 7 \leq x^2 y^7 z^4 \leq 12,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.20239

2) -1.79761

3) 0.902385

4) 0.00238548

5) -1.09761

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] ----\to R^2$

$$r(t) = \left\{ \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+6)}{\sin^2(t)+1},\ \frac{\left(\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+6)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 23.2602   2) 34.6602   3) 57.4602   4) 80.2602

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(\pi,t)=0 & 0\le t \\ u(x,0)=-2\,(x-3)\,x\,(x-\pi)^2 & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.5 mediante un desarrollo en serie de Fourier de orden 10.

1) u(1,0.5) = 3.35921

2) u(1,0.5) = -4.11456

3) u(1,0.5) = 1.90052

4) u(1,0.5) = 9.00922

5) u(1,0.5) = 2.52986

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 37

Exercise 1

Given the functions

$f(x,y)=(1+x+x\,y+3\,y^2, 3-2\,x^2+2\,x\,y+3\,y^2)$

and

$g(u,v)=(3-2\,u-u^2-2\,v+2\,u\,v+2\,v^2, 1-3\,u+u^2-2\,v+2\,u\,v-3\,v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(3,-1).

1) 498 006.

2) 360 565.

3) 142 130.

4) 414 363.

5) 306 180.

Exercise 2

Compute the volume of the domain limited by the plane $2\,x+2\,z==10$ and the paraboloid $z==2\,x^2+2\,y^2$.

1) 26.9024

2) 16.5355

3) 91.9849

4) 86.7218

5) 20.629

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\text{----}\to R^2$
$r(t)=\{(3\,t+4)\sin(2\,t)\,(4\cos(9\,t)+9), (3\,t+4)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 573.139 2) 955.139 3) 859.639 4) 1814.64

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 2\,(x-2)\,x\,(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -8\,x & 0 \le x \le 1 \\ \frac{8\,x}{\pi-1} - \frac{8}{\pi-1} - 8 & 1 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.9 mediante un desarrollo en serie de Fourier de orden 10.

1) u(1,0.9) = 5.0705

2) u(1,0.9) = 7.5591

3) u(1,0.9) = -4.03127

4) u(1,0.9) = 3.60014

5) u(1,0.9) = 3.99687

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 38

Exercise 1

Given the functions

$f(x,y)=(-1+2x+3x^2-3y+3y^2, 1+x-2x^2+2y+2xy-2y^2)$

and

$g(u,v)=(2u+2u^2+3uv, 3+2u+3u^2+uv-v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(3,3).

1) 43992.5

2) 25665.

3) 297802.

4) 249254.

5) 161760.

Exercise 2

Compute $\int_D (3z+z^2)\,dx\,dy\,dz$ for D=

$\{6x^4y^4z^4 \leq 1 \leq 15x^4y^4z^4,\ y^6z^2 \leq x \leq 9y^6z^2,\ 5xy^8 \leq z^5 \leq 13xy^8,\ x>0,\ y>0,\ z>0\}$

1) -1.98278

2) -1.08278

3) -0.582777

4) -0.882777

5) 0.0172226

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$

$r(t)=\{\sin(2t)(-\cos(t))(\cos(t)+6), -(\sin(t)\sin(2t)(\cos(t)+6))\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 14.667 2) 37.067 3) 51.067 4) 28.667

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = -(x-3)^2 (x-2)(x-1) x^2 & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.8 mediante un desarrollo en serie de Fourier de orden 10.

1) u(2,0.8) = 2.18619

2) u(2,0.8) = 1.10632

3) u(2,0.8) = -0.180199

4) u(2,0.8) = -2.33377

5) u(2,0.8) = 3.5361

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 39

Exercise 1

Given the functions

$f(x,y) = (-2 + 3x - 2x^2 + y - 3xy, -1 - 2x^2 - y + 2xy + 2y^2)$

and

$g(u,v) = (2 - 3u - u^2 - v - 3uv + 3v^2, 3 + 2u + 2u^2 - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,2)$.

1) $-42884.$

2) -38190.6

3) -10203.2

4) -32446.6

5) $-68678.$

Exercise 2

Compute the volume of the domain limited by the plane
$7x + 2z == 2$ and the paraboloid $z == x^2 + y^2$.

1) 21.1005

2) 119.283

3) 25.9243

4) 10.6598

5) 75.2882

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0, \pi] \dashrightarrow R^2$
$r(t) = \{(4t + 4)\sin(2t)(8\cos(13t) + 9), (2t + 5)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 1044.69 2) 313.889 3) 627.089 4) 209.489

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(\pi,t)=0 & 0\leq t \\ u(x,0)=-2\ (x-3)\ x^2\ (x-\pi)^2 & 0\leq x\leq \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.6 mediante un desarrollo en serie de Fourier de orden 8.

1) u(1,0.6) = 1.02601

2) u(1,0.6) = 9.47671

3) u(1,0.6) = 3.53536

4) u(1,0.6) = 2.01575

5) u(1,0.6) = -2.7996

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 40

Exercise 1

Given the system

$$2v - 3x^2 - 3y - 3uxz - vz^2 == 14$$
$$2u^2 + 2v - 3uxz == 90$$
$$2u^2 v - 2v^3 + vxy - 3xy^2 - 3uvz - 3uxz - xz^2 == 70$$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point p=(x,y,z,u,v)=(-4,-3,-1,-4,5). Compute if possible $\frac{\partial x}{\partial v}(-4,5)$.

1) $\frac{\partial x}{\partial v}(-4,5) = -\frac{31}{222}$

2) $\frac{\partial x}{\partial v}(-4,5) = -\frac{449}{3219}$

3) $\frac{\partial x}{\partial v}(-4,5) = -\frac{1795}{12876}$

4) $\frac{\partial x}{\partial v}(-4,5) = -\frac{599}{4292}$

5) $\frac{\partial x}{\partial v}(-4,5) = -\frac{1799}{12876}$

Exercise 2

Compute the volume of the domain limited by the plane $x + 8z == 7$ and the paraboloid $z == 2x^2 + 2y^2$.

1) 0.604008

2) 0.81542

3) 0.500092

4) 0.419482

5) 1.72489

Exercise 3

Consider the vectorial field $F(x,y,z) = \{-2x^2 - 3x^2 y, -9x^2 y^2 z - 8yz^2, 7y^2\}$ and the surface

$$S \equiv \left(\frac{8+x}{8}\right)^2 + \left(\frac{8+y}{1}\right)^2 + \left(\frac{8+z}{5}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -1.49754×10^7 2) 3.59409×10^7 3) -1.64729×10^7 4) 7.48768×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -\left((x-3)(x-2)\,x\,(x-\pi)^2\right) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.8 mediante un desarrollo en serie de Fourier de orden 8.

1) u(2,0.8) = 3.11373

2) u(2,0.8) = -4.36671

3) u(2,0.8) = -3.39962

4) u(2,0.8) = -1.58315

5) u(2,0.8) = 0.568963

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 41

Exercise 1

Given the functions

$f(x,y)=(3-x+x^2-y-3xy-3y^2, 2x-x^2+3y+2xy, -2+3x-2x^2-3y+2y^2, 2-3x-2x^2+3y-2xy+2y^2)$

and

$g(u_1,u_2,u_3,u_4)=(-3u_4+2u_3u_4, -3+2u_2^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(0,2).

1) -13 863.1

2) -9864.

3) -2519.67

4) -4007.93

5) -3897.93

Exercise 2

Compute the volume of the domain limited by the plane $6x+10z==1$ and the paraboloid $z==5x^2+5y^2$.

1) 0.0214171

2) 0.00719543

3) 0.0204681

4) 0.0122183

5) 0.00437435

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$

$r(t)=\{(3t+1)\sin(2t)(8\cos(10t)+8), (9t+5)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 1765.84 2) 2118.94 3) 2001.24 4) 1177.34

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-3)\,x\,(x-\pi)^2 & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = -3\,(x-1)\,x\,(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2 en el instante t= 0.6 mediante un desarrollo en serie de Fourier de orden 12.

1) u(2,0.6) = 4.38509

2) u(2,0.6) = -2.33559

3) u(2,0.6) = 6.48542

4) u(2,0.6) = 7.03304

5) u(2,0.6) = 4.38054

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 42

Exercise 1

Given the function

$f(x,y,z) = -23 + 2x - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at $\{-1.20808, -3.62423, -0.889145\}$

2) We have a minimum at $\{-1.10808, -3.42423, -1.28914\}$

3) We have a minimum at $\{1, 3, 2\}$

4) We have a minimum at $\{-1.40808, -3.72423, -1.28914\}$

5) We have a minimum at $\{-1.30808, -3.12423, -1.18914\}$

Exercise 2

Compute $\int_D (x\,y)\,dx\,dy\,dz$ for D=

$\{7z^8 \leq x^7 \leq 9z^8,\ 9y^7z^6 \leq x^8 \leq 10y^7z^6,\ 8y^7z^9 \leq x^5 \leq 10y^7z^9,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.69998

2) 0.0000204674

3) -1.89998

4) -0.19998

5) -0.19998

Exercise 3

Consider the vectorial field F(x,y,z) = $\{e^{-2y^2-z^2} + 5yz,\ -2 - \mathrm{Sin}[2x^2 + 2z^2],\ e^{-2x^2-2y^2}\}$ and the surface

$S \equiv \left(\frac{x}{1}\right)^2 + \left(\frac{-6+y}{5}\right)^2 + \left(\frac{9+z}{5}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 2.4 2) -0.5 3) 0. 4) -3.4

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(4,t)=0 & 0\leq t \\ u(x,0)=-2\ (x-4)\ (x-3)\ (x-1)\ x & 0\leq x\leq 4 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.5 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.5) = 1.2516

2) u(1,0.5) = −2.50606

3) u(1,0.5) = 2.59729

4) u(1,0.5) = −1.06667

5) u(1,0.5) = 4.93463

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 43

Exercise 1

Given the functions

$f(x,y) = (2 + 3x + 2x^2 + 3y - xy - 3y^2, -3 + x - 2x^2 - 3y - xy - 3y^2, 3 - 3x^2 - 2y - 2xy, 3 + x - 3x^2 + 3y + xy + y^2)$

and

$g(u_1,u_2,u_3,u_4) = (-2u_1^2 - u_3 + u_1 u_4, -3u_1 u_2 + 2u_2^2 - 2u_3 - 2u_1 u_4 - u_3 u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,-3)$.

1) −638 690.

2) −391 078.

3) −553 664.

4) −982 881.

5) −271 053.

Exercise 2

Compute the volume of the domain limited by the plane $6x + 8z == 1$ and the paraboloid $z == 9x^2 + 9y^2$.

1) 0.0151487

2) 0.00345146

3) 0.0370753

4) 0.00778258

5) 0.0145624

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$

$r(t) = \{(2t+6)\sin(2t)(5\cos(9t)+7), (t+1)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 229.997 2) 46.7974 3) 298.697 4) 436.097

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = -(x-3)^2 (x-2)(x-1)x & 0 \le x \le 3 \\ \frac{\partial}{\partial t}u(x,0) = -3(x-3)(x-2)(x-1)x^2 & 0. \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.1 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.1) = 0.688231

2) u(1,0.1) = −3.05993

3) u(1,0.1) = −6.30849

4) u(1,0.1) = −0.175437

5) u(1,0.1) = −1.6411

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 44

Exercise 1

Given the functions

$f(x,y)=(-2x^2-2y-2xy+y^2,-2+2x-2x^2-2y+2xy-3y^2)$

and

$g(u,v)=(-3-3u+3u^2+3v-2v^2,1+3u+u^2-3v-2uv+2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,-2)$.

1) 2.46292×10^6

2) 2.71843×10^6

3) 1.64549×10^6

4) 2.74138×10^6

5) 466 954.

Exercise 2

Compute the volume of the domain limited by the plane $8x+9z==7$ and the paraboloid $z==8x^2+8y^2$.

1) 0.468839

2) 0.112228

3) 0.126441

4) 0.107883

5) 0.52426

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]---\to R^2$
$r(t)=\{(2t+9)\sin(2t)(9\cos(20t)+10),(3t+5)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 1579.79 2) 316.59 3) 1737.69 4) 1106.09

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -50x & 0\le x\le \frac{1}{10} \\ \frac{70x}{3}-\frac{22}{3} & \frac{1}{10}\le x\le \frac{2}{5} \\ \frac{10}{3}-\frac{10x}{3} & \frac{2}{5}\le x\le 1 \\ 0 & \text{True} \end{cases} & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=(x-1)^2\left(x-\frac{9}{10}\right)\left(x-\frac{1}{2}\right)x & 0.\le x\le 1 \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{1}{2}$

en el instante t=0.3 mediante un desarrollo en serie de Fourier de orden 8.

1) $u\left(\frac{1}{2},0.3\right) = -5.5986$

2) $u\left(\frac{1}{2},0.3\right) = -4.14423$

3) $u\left(\frac{1}{2},0.3\right) = 1.93908$

4) $u\left(\frac{1}{2},0.3\right) = 0.599049$

5) $u\left(\frac{1}{2},0.3\right) = 1.13074$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 45

Exercise 1

Given the system

$2x - y u_2 - 3 u_4^3 == 363$
$3 y z u_4 == 90$
$3 x y u_2 == -48$

determine if it is possible to solve for variables x,y,z in terms of variables u_1, u_2, u_3, u_4 arround the point $p=(x,y,z,u_1,u_2,u_3,u_4)=(-4,-2,3,3,-2,4,-5)$. Compute if possible $\frac{\partial z}{\partial u_1}(3,-2,4,-5)$.

1) $\frac{\partial z}{\partial u_1}(3,-2,4,-5)=2$

2) $\frac{\partial z}{\partial u_1}(3,-2,4,-5)=3$

3) $\frac{\partial z}{\partial u_1}(3,-2,4,-5)=0$

4) $\frac{\partial z}{\partial u_1}(3,-2,4,-5)=1$

5) $\frac{\partial z}{\partial u_1}(3,-2,4,-5)=4$

Exercise 2

Compute $\int_D (3 y^2 z)\, dx\, dy\, dz$ for D=
$\{5 y^5 z^4 \le x^8 \le 10 y^5 z^4,\ 4 y^3 z^5 \le x^3 \le 7 y^3 z^5,\ 9 x^8 y^5 z^4 \le 1 \le 11 x^8 y^5 z^4,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.30033

2) -1.99967

3) -1.19967

4) 0.000329778

5) 0.50033

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$

$r(t)=\left\{\sin(2t)\,(8\cos(t)+8)\left(\frac{\cos(t)}{2}-\frac{1}{2}\sqrt{3}\sin(t)\right),\ \sin(2t)\,(8\cos(t)+8)\left(\frac{\sin(t)}{2}+\frac{1}{2}\sqrt{3}\cos(t)\right)\right\}$

Indication: it is necessary to represent
 the curve to check whether it has intersection points.

1) 105.398   2) 75.3982   3) 30.3982   4) 60.3982

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t)=u(5,t)=0 & 0\le t \\ u(x,0)=3\,(x-5)\,(x-1)\,x^2 & 0\le x\le 5 \\ \frac{\partial}{\partial t}u(x,0)=(x-5)^2\,(x-3)\,(x-1)\,x & 0.\le x\le 5 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1
 en el instante t=0.5 mediante un desarrollo en serie de Fourier de orden 8.

1) u(1,0.5) = -4.72116

2) u(1,0.5) = 2.207

3) u(1,0.5) = 5.17261

4) u(1,0.5) = -0.190872

5) u(1,0.5) = 6.26094

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 46

Exercise 1

Given the functions

$f(x,y)=(2-2x+x^2+2xy-y^2, 2-3x+2x^2-2y+3xy-3y^2, 3+x+2y)$

and

$g(u,v,w)=(1-3u^2, 3u-v+2vw+w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,2)$.

1) -487.389

2) -1016.87

3) -295.758

4) -270.671

5) -672.

Exercise 2

Compute $\int_D (y^3 z^3)\,dx\,dy\,dz$ for D=
$\{7x^3z^5 \le y^6 \le 10x^3z^5,\ 8x^8y^6 \le z^5 \le 9x^8y^6,\ 2z^8 \le x^5y^3 \le 5z^8,\ x>0,\ y>0,\ z>0\}$

1) -1.1

2) 1.2

3) 4.49371×10^{-6}

4) -0.199996

5) 2.

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$

$r(t)=\left\{\sin(2t)\,(2\cos(t)+3)\left(-\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}-\frac{(1+\sqrt{3})\cos(t)}{2\sqrt{2}}\right), \sin(2t)\,(2\cos(t)+3)\left(\frac{(\sqrt{3}-1)\cos(t)}{2\sqrt{2}}-\frac{(1+\right.\right.$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 8.63938 2) 5.43938 3) 14.2394 4) 11.0394

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -3x & 0\le x\le 1 \\ 3x-6 & 1\le x\le 3 \\ -\frac{3x}{\pi-3}+\frac{9}{\pi-3}+3 & 3\le x\le \pi \end{cases} & 0\le x\le \pi \\ \frac{\partial}{\partial t}u(x,0)=2(x-1)x^2(x-\pi)^2 & 0.\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2 en el instante t= 0.4 mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,0.4) = -3.43523

2) u(2,0.4) = 3.18246

3) u(2,0.4) = -5.50553

4) u(2,0.4) = 5.55574

5) u(2,0.4) = -6.17408

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 47

Exercise 1

Given the functions

$f(x,y)=(-1-3x^2-2y+xy-3y^2,-2-x+3x^2+3y+xy-3y^2,-1-3x+3x^2+y+xy-2y^2)$

and

$g(u,v,w)=(2vw,-2v^2+w+vw-2w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(2,2).

1) 4370.15

2) 5053.49

3) 2664.

4) 4371.59

5) 1759.14

Exercise 2

Compute the volume of the domain limited by the plane $3x+9z==9$ and the paraboloid $z==8x^2+8y^2$.

1) 0.197715

2) 0.135438

3) 0.401303

4) 0.168268

5) 0.610669

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-xy^2z-xyz^2, 6z+4z^2, -5y\}$ and the surface

$S\equiv\left(\frac{4+x}{1}\right)^2+\left(\frac{7+y}{2}\right)^2+\left(\frac{6+z}{4}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 15366.5 2) 19208.1 3) 65307.3 4) -7683.08

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 5x & 0 \le x \le \frac{1}{5} \\ \frac{5}{4} - \frac{5x}{4} & \frac{1}{5} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto $x=\frac{1}{5}$ en el instante $t=0.8$ mediante un desarrollo en serie de Fourier de orden 8.

1) $u(\frac{1}{5},0.8) = 0.50008$

2) $u(\frac{1}{5},0.8) = -1.06556$

3) $u(\frac{1}{5},0.8) = -2.77996$

4) $u(\frac{1}{5},0.8) = 3.22496$

5) $u(\frac{1}{5},0.8) = -2.06174$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 48

Exercise 1

Given the function

$f(x,y,z)=7-4x+x^2-2y+y^2-2z+z^2$ defined over the domain $D\equiv \frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at $\{-1.84149, -4.30674, -0.0484624\}$

2) We have a maximum at $\{-2.24149, -4.10674, 0.0515376\}$

3) We have a maximum at $\{-2.14149, -4.20674, -0.148462\}$

4) We have a maximum at $\{-2.44149, -4.10674, -0.0484624\}$

5) We have a maximum at $\{2, 1, 1\}$

Exercise 2

Compute $\int_D (2y^3)\,dx\,dy\,dz$ for $D=\{9z^6\le y\le 14z^6,\ 6x^7z^6\le y^7\le 11x^7z^6,\ 2\le x^8y^7z^8\le 7,\ x>0,\ y>0,\ z>0\}$

1) -0.897049

2) 1.80295

3) 1.20295

4) 0.00295089

5) 1.10295

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,2\pi]\dashrightarrow R^2$

$r(t)=\left\{\frac{\cos(t)\,(6\cos(t)+9)}{\sin^2(t)+1},\ \frac{\sin(t)\cos(t)\,(6\cos(t)+9)}{\sin^2(t)+1}\right\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 111.903 2) 123.003 3) 178.503 4) 145.203

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t)=u(5,t)=0 & 0\le t \\ u(x,0)=\begin{cases} \frac{x}{3} & 0\le x\le 3 \\ \frac{5}{2}-\frac{x}{2} & 3\le x\le 5 \end{cases} & 0\le x\le 5 \\ \frac{\partial}{\partial t}u(x,0)=-\left((x-5)(x-4)x^2\right) & 0.\le x\le 5 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.2 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.2) = -7.89868

2) u(1,0.2) = -3.48955

3) u(1,0.2) = 2.9648

4) u(1,0.2) = -0.308682

5) u(1,0.2) = -2.04126

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 49

Exercise 1

Given the system

$x y^2 + 2 z^2 == 2$
$-y z^2 == 36$
$-3 x y z == 36$

determine if it is possible to solve for variables x,y,z in terms of variables u_1,u_2,u_3,u_4 arround the point $p=(x,y,z,u_1,u_2,u_3,u_4)=(-1,-4,-3,4,-5,-2,-3)$. Compute if possible $\frac{\partial x}{\partial u_4}(4,-5,-2,-3)$.

1) $\frac{\partial x}{\partial u_4}(4,-5,-2,-3)=0$

2) $\frac{\partial x}{\partial u_4}(4,-5,-2,-3)=3$

3) $\frac{\partial x}{\partial u_4}(4,-5,-2,-3)=1$

4) $\frac{\partial x}{\partial u_4}(4,-5,-2,-3)=4$

5) $\frac{\partial x}{\partial u_4}(4,-5,-2,-3)=2$

Exercise 2

Compute the volume of the domain limited by the plane $5 x + 5 z == 10$ and the paraboloid $z == 3 x^2 + 3 y^2$.

1) 2.27256

2) 9.52921

3) 8.74042

4) 0.992645

5) 1.9547

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-8y^2 z-3x^2 z^2, -7x^2 y, -7x^2\}$ and the surface

$S\equiv\left(\frac{2+x}{6}\right)^2+\left(\frac{5+y}{3}\right)^2+\left(\frac{5+z}{5}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 509 569.   2) 106 161.   3) 0.69895   4) 159 241.

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=2\,(x-2)\,(x-1)\,x\,(x-\pi)^2 & 0\le x\le\pi \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} 8x & 0\le x\le 1 \\ 15-7x & 1\le x\le 3 \\ \frac{6x}{\pi-3}-\frac{18}{\pi-3}-6 & 3\le x\le\pi \end{cases} & 0.\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2 en el instante t= 0.9 mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,0.9) = -1.48475

2) u(2,0.9) = 3.2608

3) u(2,0.9) = 2.28083

4) u(2,0.9) = 2.42434

5) u(2,0.9) = 5.97016

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 50

Exercise 1

Given the system

$3\,y\,z == -60$
$-x\,z\,u_1 == -20$
$-3\,y^2 - y\,z\,u_4 == -35$

determine if it is possible to solve for variables x,y,z in terms of variables u_1,u_2,u_3,u_4 arround the point $p=(x,y,z,u_1,u_2,u_3,u_4)=(5,5,-4,-1,4,-2,2)$. Compute if possible $\frac{\partial z}{\partial u_2}(-1,4,-2,2)$.

1) $\frac{\partial z}{\partial u_2}(-1,4,-2,2)=1$

2) $\frac{\partial z}{\partial u_2}(-1,4,-2,2)=3$

3) $\frac{\partial z}{\partial u_2}(-1,4,-2,2)=4$

4) $\frac{\partial z}{\partial u_2}(-1,4,-2,2)=0$

5) $\frac{\partial z}{\partial u_2}(-1,4,-2,2)=2$

Exercise 2

Compute the volume of the domain limited by the plane $7\,x + 10\,z == 6$ and the paraboloid $z == 2\,x^2 + 2\,y^2$.

1) 0.834791

2) 0.139583

3) 0.281889

4) 0.343417

5) 1.19771

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-2xy^2-8x^2y^2z^2,\ 2x^2yz,\ -4x^2y^2\}$ and the surface

$S\equiv(\frac{-1+x}{3})^2+(\frac{-8+y}{1})^2+(\frac{-2+z}{7})^2=1$

Compute $\int_S F.$

Indication: Use Stoke's Theorem if it is necessary.

1) -6.03097×10^6 2) 2.7642×10^6 3) -1.25645×10^6 4) -3.26677×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t)=u(5,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -2x & 0\le x\le 1 \\ 8x-10 & 1\le x\le 2 \\ 10-2x & 2\le x\le 5 \end{cases} & 0\le x\le 5 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} 9x & 0\le x\le 1 \\ \frac{40}{3}-\frac{13x}{3} & 1\le x\le 4 \\ 4x-20 & 4\le x\le 5 \end{cases} & 0.\le x\le 5 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=2 en el instante t= 0.8 mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,0.8) = 6.72957

2) u(2,0.8) = 0.561519

3) u(2,0.8) = -0.832864

4) u(2,0.8) = 3.6878

5) u(2,0.8) = -5.27536

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 51

Exercise 1

Given the system

$2v^2 - 2xy^2 - yz == 212$

$2uxy - x^2 z == -285$

$-3uv^2 + 3v^2 w + uw^2 - 3x + 3wxz == 137$

determine if it is possible to solve for variables x, y,z in terms of variables u,v,w arround the point p=(x,y,z,u,v,w)=(-5,-4,5,-4,-4,2). Compute if possible $\frac{\partial z}{\partial v}(-4,-4,2)$.

1) $\frac{\partial z}{\partial v}(-4,-4,2) = -\frac{53469}{7843}$

2) $\frac{\partial z}{\partial v}(-4,-4,2) = -\frac{53470}{7843}$

3) $\frac{\partial z}{\partial v}(-4,-4,2) = -\frac{53472}{7843}$

4) $\frac{\partial z}{\partial v}(-4,-4,2) = -\frac{53468}{7843}$

5) $\frac{\partial z}{\partial v}(-4,-4,2) = -\frac{4861}{713}$

Exercise 2

Compute $\int_D (5z)\,dx\,dy\,dz$ for D=
$\{8z^2 \le x^4 y^2 \le 13z^2,\ 3x^2 y^6 z^6 \le 1 \le 4x^2 y^6 z^6,\ 3y^9 \le x^2 z^2 \le 9y^9,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.00200873

2) -0.397991

3) -0.597991

4) -0.897991

5) 1.20201

Exercise 3

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$

$r(t)=\left\{\sin(2t)\,(7\cos(t)+10)\left(\frac{(\sqrt{3}-1)\cos(t)}{2\sqrt{2}}-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}}\right),\ \sin(2t)\,(7\cos(t)+10)\left(\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}+\frac{(1+}{}\right.\right.$

Indication: it is necessary to represent
 the curve to check whether it has intersection points.

1) 10.4821 2) 78.3821 3) 97.7821 4) 39.5821

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -9x & 0\le x\le 1 \\ 9x-18 & 1\le x\le 2 \end{cases} & 0\le x\le 2 \\ \frac{\partial}{\partial t}u(x,0)=-(x-2)^2(x-1)x & 0.\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{3}{10}$
 en el instante t=0.6 mediante un desarrollo en serie de Fourier de orden 10.

1) $u\left(\frac{3}{10},0.6\right) = -4.3308$

2) $u\left(\frac{3}{10},0.6\right) = -1.87023$

3) $u\left(\frac{3}{10},0.6\right) = 2.65851$

4) $u\left(\frac{3}{10},0.6\right) = -4.16473$

5) $u\left(\frac{3}{10},0.6\right) = -3.93945$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 52

Exercise 1

Given the functions

$f(x,y)=(-2x+2x^2+y-3xy-3y^2, -3+3x-2x^2+2y-xy+3y^2, 3x-3x^2+2xy-y^2)$

and

$g(u,v,w)=(-2+3u+3u^2+uv-2uw+vw+2w^2, -3-3u-3uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(0,0)$.

1) −14.0958

2) −21.3143

3) −54.

4) −94.1995

5) −88.0633

Exercise 2

Compute the volume of the domain limited by the plane $3x+z==3$ and the paraboloid $z==3x^2+3y^2$.

1) 21.5528

2) 19.5677

3) 3.77974

4) 32.9373

5) 7.36311

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-7x^2+9xy^2z^2, 5x^2y^2z^2, -xz^2\}$ and the surface

$S\equiv\left(\frac{4+x}{9}\right)^2+\left(\frac{8+y}{9}\right)^2+\left(\frac{7+z}{7}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -3.76942×10^8 2) -2.51295×10^8 3) -1.25647×10^8 4) -2.01036×10^8

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-2)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.9 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.9) = -0.552294

2) u(1,0.9) = 3.13283

3) u(1,0.9) = -0.919978

4) u(1,0.9) = 4.14631

5) u(1,0.9) = 0.6513

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 53

Exercise 1

Given the function

$f(x,y,z) = 1 + 4x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain $D \equiv \frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {2.6, 1.4, 0.8}

2) We have a maximum at {1.4, 0.4, 0.4}

3) We have a maximum at {2, 1, 1}

4) We have a maximum at {1.2, 0.4, 1.6}

5) We have a maximum at {3., 1.6, 0.4}

Exercise 2

Compute $\int_D (y^6)\, dx\, dy\, dz$ for $D = \{6 \le y^5 z^3 \le 15,\ 8 z^5 \le x^3 \le 10 z^5,\ 4 x z^4 \le 1 \le 11 x z^4,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.757295

2) 0.257295

3) −0.0427052

4) 1.95729

5) 1.05729

Exercise 3

Consider the vectorial field $F(x,y,z) = \{4x + \mathrm{Cos}[y^2 - 2z^2],\ -6xy - \mathrm{Sin}[2x^2 - 2z^2],\ -9 + e^{-2x^2 - y^2} + 2xyz\}$ and the surface

$S \equiv \left(\frac{-5 + x}{5}\right)^2 + \left(\frac{-9 + y}{5}\right)^2 + \left(\frac{-9 + z}{3}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 20 106.2 2) 58 307.6 3) 32 169.8 4) 38 201.6

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{3x}{2} & 0 \le x \le 2 \\ -\frac{3x}{\pi-2} + \frac{6}{\pi-2} + 3 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=1
en el instante t=0.5 mediante un desarrollo en serie de Fourier de orden 10.

1) u(1,0.5) = 4.5119

2) u(1,0.5) = -3.37279

3) u(1,0.5) = -4.92068

4) u(1,0.5) = 1.5

5) u(1,0.5) = 0.4054

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 54

Exercise 1

Given the system

$u - 3z + 2uz - 3xyz - y^2 z == -37$
$-2u^3 - 3uz + u^2 z - vxz == 72$
$2 - xy - 3y^3 == -373$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point p=(x,y,z,u,v)=(0,5,1,-3,5). Compute if possible $\frac{\partial z}{\partial u}(-3,5)$.

1) $\frac{\partial z}{\partial u}(-3,5)=\frac{237}{109}$

2) $\frac{\partial z}{\partial u}(-3,5)=\frac{238}{109}$

3) $\frac{\partial z}{\partial u}(-3,5)=\frac{473}{218}$

4) $\frac{\partial z}{\partial u}(-3,5)=\frac{475}{218}$

5) $\frac{\partial z}{\partial u}(-3,5)=\frac{477}{218}$

Exercise 2

Compute the volume of the domain limited by the plane $2x + 8z == 4$ and the paraboloid $z == 8x^2 + 8y^2$.

1) 0.0494716

2) 0.151197

3) 0.0835203

4) 0.127352

5) 0.0931572

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-4xz^2, -2x - 6x^2 yz, -2x - 9y^2 z\}$ and the surface

$S\equiv\left(\frac{-3+x}{4}\right)^2+\left(\frac{-4+y}{8}\right)^2+\left(\frac{2+z}{8}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 96 510.5 2) -636 965. 3) -193 019. 4) 270 229.

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ -\frac{7x}{\pi-2} + \frac{14}{\pi-2} + 7 & 2 \le x \le \pi \\ 0 & \text{True} \end{cases} & 0 \le x \le \pi \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2 en el instante t=0.7 mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,0.7) = −4.04022

2) u(2,0.7) = 3.50067

3) u(2,0.7) = 2.76321

4) u(2,0.7) = 1.35971

5) u(2,0.7) = 0.43494

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 55

Exercise 1

Given the function

$f(x,y,z)=11-4x+x^2-4y+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{16}+\frac{y^2}{4}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {2, 2, 3}

2) We have a maximum at {-1.89355, -0.600885, -3.09032}

3) We have a maximum at {-2.29355, -0.800885, -3.39032}

4) We have a maximum at {-1.79355, 0.0991151, -3.79032}

5) We have a maximum at {-2.19355, -0.300885, -3.29032}

Exercise 2

Compute $\int_D (2x)\,dx\,dy\,dz$ for $D=\{8\le x^8 y\le 12,\ 4x^3y^4\le z^8\le 5x^3y^4,\ y^2z^8\le x^4\le 2y^2z^8,\ x>0,\ y>0,\ z>0\}$

1) 1.90056

2) 1.30056

3) 0.000560148

4) 1.20056

5) -1.29944

Exercise 3

Consider the vectorial field $F(x,y,z)=\{-9xy-7xyz-\mathrm{Sin}[2y^2-z^2],\ 9z-6xyz-\mathrm{Sin}[2x^2+z^2],\ -7y+\mathrm{Sin}[2x^2+2y^2]\}$ and the surface

$S\equiv\left(\frac{7+x}{3}\right)^2+\left(\frac{-9+y}{6}\right)^2+\left(\frac{-4+z}{4}\right)^2=1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -59 715.4 2) -49 762.8 3) -39 810.2 4) -54 739.1

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = (x-3)(x-2)x^2 & 0 \le x \le 3 \\ \frac{\partial}{\partial t} u(x,0) = 2(x-3)(x-2)x^2 & 0. \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.2 mediante un desarrollo en serie de Fourier de orden 10.

1) u(1,0.2) = 4.74439

2) u(1,0.2) = 3.40171

3) u(1,0.2) = 6.14024

4) u(1,0.2) = 3.49251

5) u(1,0.2) = 0.477486

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 56

Exercise 1

Given the functions

$f(x,y) = (-2 - 2x^2 + 2y, 2 + x^2 - y + xy + 3y^2, -2 - 3x - 2y + 2xy - 3y^2)$

and

$g(u,v,w) = (2uv - 3v^2 - w^2, 3 + 2uw + w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,-1)$.

1) -48032.

2) -90197.1

3) -61963.

4) -71052.2

5) -59560.1

Exercise 2

Compute the volume of the domain limited by the plane $7x + 10z == 4$ and the paraboloid $z == 6x^2 + 6y^2$.

1) 0.0981544

2) 0.0462731

3) 0.0596256

4) 0.092533

5) 0.0924039

Exercise 3

Consider the vectorial field $F(x,y,z) = \{5yz, -6y^2z^2 + 8xy^2z^2, 8xy^2z\}$ and the surface

$S \equiv \left(\frac{-4+x}{5}\right)^2 + \left(\frac{3+y}{3}\right)^2 + \left(\frac{3+z}{5}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) -577550. 2) -2.88775×10^6 3) -2.83×10^6 4) -1.44388×10^6

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0\le t \\ u(x,0)=3\ (x-2)^2\ (x-1)\ x^2 & 0\le x\le 2 \\ \frac{\partial}{\partial t}u(x,0)=-2\ (x-2)\ (x-1)\ x^2 & 0.\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{11}{10}$

en el instante t=0.5 mediante un desarrollo en serie de Fourier de orden 11.

1) $u\left(\frac{11}{10},0.5\right) = 4.35082$

2) $u\left(\frac{11}{10},0.5\right) = 3.2345$

3) $u\left(\frac{11}{10},0.5\right) = 0.0179461$

4) $u\left(\frac{11}{10},0.5\right) = -3.11863$

5) $u\left(\frac{11}{10},0.5\right) = -1.08258$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 57

Exercise 1

Given the function

$f(x,y,z) = -5 + 6x - x^2 - y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-2.63443, 0.1, -2.99787}

2) We have a minimum at {3, 0, 1}

3) We have a minimum at {-3.03443, -0.2, -2.99787}

4) We have a minimum at {-2.73443, 0.4, -3.49787}

5) We have a minimum at {-2.93443, 0., -3.39787}

Exercise 2

Compute $\int_D (y^3 + z)\, dx\, dy\, dz$ for D≡

$\{8z^9 \le x^2 y^2 \le 16 z^9,\ 5y^4 z \le x^2 \le 13 y^4 z,\ 6 \le x^9 y^2 z^9 \le 7,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.59971

2) 0.200288

3) 0.00028754

4) 1.10029

5) -0.199712

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] \dashrightarrow R^2$

$r(t) = \left\{-\frac{\cos(t)\,(6\cos(t)+8)}{\sin^2(t)+1},\ -\frac{\sin(t)\cos(t)\,(6\cos(t)+8)}{\sin^2(t)+1}\right\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 94.9027 2) 19.7027 3) 132.503 4) 104.303

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -4x & 0 \le x \le 1 \\ 6x - 10 & 1 \le x \le 2 \\ -\frac{2x}{\pi-2} + \frac{4}{\pi-2} + 2 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=1. mediante un desarrollo en serie de Fourier de orden 11.

1) u(2,1.) = -1.9329

2) u(2,1.) = 0.804931

3) u(2,1.) = 0.00703607

4) u(2,1.) = 3.59712

5) u(2,1.) = -0.591549

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 58

Exercise 1

Given the functions

$f(x,y)=(2+x+x^2-3y+3xy-2y^2, 3x+x^2+2y-xy-3y^2, 3+3x^2-y-2xy-y^2, -2-2x^2+y-xy)$

and

$g(u_1,u_2,u_3,u_4)=(u_1-2u_2-u_2^2-2u_3+u_2u_3-3u_4^2, u_1u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,0)$.

1) 8497.61

2) 2969.33

3) 5206.

4) 4367.86

5) 8171.49

Exercise 2

Compute the volume of the domain limited by the plane $6x+5z==2$ and the paraboloid $z==10x^2+10y^2$.

1) 0.130859

2) 0.0964188

3) 0.0911145

4) 0.0298602

5) 0.0451896

Exercise 3

Consider the vectorial field $F(x,y,z)=\{4xy^2z-6xz^2, -7yz^2, 4y\}$ and the surface

$$S\equiv\left(\frac{-1+x}{4}\right)^2+\left(\frac{-7+y}{7}\right)^2+\left(\frac{-3+z}{7}\right)^2=1$$

Compute $\int_S F$.

Indication: Use Stoke's Theorem if it is necessary.

1) 946 616. 2) 378 647. 3) 1.40099×10^6 4) 454 376.

Ejercicio 4

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = -(x-3)^2 (x-1) x^2 & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la temperatura que tendrá la barra en el punto x=2
en el instante t=0.8 mediante un desarrollo en serie de Fourier de orden 9.

1) u(2,0.8) = 2.68707

2) u(2,0.8) = −3.74138

3) u(2,0.8) = −3.11924

4) u(2,0.8) = −1.37367

5) u(2,0.8) = 2.97505

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Exercise 1

Given the system

$-2u^2 y - 3xy - 2xy^2 + z - 2u^2 z - xz - 3uxz + 3yz == 128$

$3 + 2ux - 3uy - 2xy - 3y^3 == 243$

$-3uy^2 - 2uz + xz == -192$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point p=(x,y,z,u)=(0,-4,0,4). Compute if possible $\frac{\partial z}{\partial u}(4)$.

1) $\frac{\partial z}{\partial u}(4) = -\frac{411}{743}$

2) $\frac{\partial z}{\partial u}(4) = -\frac{414}{743}$

3) $\frac{\partial z}{\partial u}(4) = -\frac{412}{743}$

4) $\frac{\partial z}{\partial u}(4) = -\frac{410}{743}$

5) $\frac{\partial z}{\partial u}(4) = -\frac{413}{743}$

Exercise 2

Compute the volume of the domain limited by the plane $9x + z == 8$ and the paraboloid $z == 5x^2 + 5y^2$.

1) 65.1667

2) 0.283066

3) 88.5372

4) 14.6213

5) 45.6167

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0, \pi] \dashrightarrow R^2$

$r(t) = \{(2t + 9)\sin(2t)(5\cos(11t) + 10), (t + 2)\sin(t)(5\cos(11t) + 10)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 6574.15 2) 1972.35 3) 9203.75 4) 5259.35

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0 \le t \\ u(x,0) = -((x-2)(x-1)x) & 0 \le x \le 2 \\ \frac{\partial}{\partial t}u(x,0) = -2(x-2)^2(x-1)x^2 & 0. \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto $x=\frac{1}{2}$

en el instante t=0.6 mediante un desarrollo en serie de Fourier de orden 9.

1) $u(\frac{1}{2},0.6) = 4.89672$

2) $u(\frac{1}{2},0.6) = -1.18788$

3) $u(\frac{1}{2},0.6) = 7.85262$

4) $u(\frac{1}{2},0.6) = -3.47585$

5) $u(\frac{1}{2},0.6) = -0.35206$

Further Mathematics - 2023/2024
Exam - January Call - Part 2 (to be solved by computer) - training for serial number: 60

Exercise 1

Given the system

$3 v x + 3 y^2 - x z^2 == 122$
$3 v^2 + v^3 + y^2 + z - 3 v^2 z - 2 u y z + 2 x y z - v z^2 == 173$
$3 v - 3 v^2 + 3 v^2 y - 3 y^2 - 3 u z == -93$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point p=(x,y,z,u,v)=(-5,-2,5,5,1). Compute if possible $\frac{\partial z}{\partial u}(5,1)$.

1) $\frac{\partial z}{\partial u}(5,1) = -\frac{197}{304}$

2) $\frac{\partial z}{\partial u}(5,1) = -\frac{25}{38}$

3) $\frac{\partial z}{\partial u}(5,1) = -\frac{99}{152}$

4) $\frac{\partial z}{\partial u}(5,1) = -\frac{201}{304}$

5) $\frac{\partial z}{\partial u}(5,1) = -\frac{199}{304}$

Exercise 2

Compute the volume of the domain limited by the plane $5 x + z == 10$ and the paraboloid $z == 4 x^2 + 4 y^2$.

1) 110.05

2) 52.5005

3) 170.319

4) 24.8214

5) 255.042

Exercise 3

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \longrightarrow R^2$
$r(t) = \{(6 t + 9) \sin(2 t) (3 \cos(18 t) + 6), (2 t + 8) \sin(t) (3 \cos(18 t) + 6)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 5622.57 2) 11 245.1 3) 6747.07 4) 8996.07

Ejercicio 4

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=-(x-3)^2(x-1)\,x & 0\le x\le 3 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} -2x & 0\le x\le 2 \\ 4x-12 & 2\le x\le 3 \end{cases} & 0.\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Calcular la posición de la cuerda en el punto x=1 en el instante t= 0.7 mediante un desarrollo en serie de Fourier de orden 12.

1) u(1,0.7) = 5.94555

2) u(1,0.7) = −0.55884

3) u(1,0.7) = 7.81599

4) u(1,0.7) = −7.83584

5) u(1,0.7) = 3.6378