

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 1

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-yz(3xyz-x)\sin(xyz) + (3yz-1)\cos(xyz) + 4x - 3y, -xz(3xyz-x)\sin(xyz) + 3xz\cos(xyz) - 3x, 3xy\cos(xyz) - xy(3xyz-x)\sin(xyz))$ . Compute the potential function for this field whose potential at the origin is -3.

. Calculate the value of the potential at the point  $p=(1,-7,2)$ .

- 1)  $\frac{97}{5} - 43 \cos[14]$     2)  $20 - 43 \cos[14]$     3)  $\frac{173}{10} - 43 \cos[14]$     4)  $23 - 43 \cos[14]$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \left\{ \frac{\left( \frac{1+\sqrt{3}}{2\sqrt{2}} \sin(t) - \frac{\sqrt{3}-1}{2\sqrt{2}} \right) \cos(t) (8 \cos(t)+9)}{\sin^2(t)+1}, \frac{\left( \frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1) \sin(t)}{2\sqrt{2}} \right) \cos(t) (8 \cos(t)+9)}{\sin^2(t)+1} \right\}$$

$$\left\{ \frac{\cos(t) (9+8 \cos(t)) \left( -\frac{-1+\sqrt{3}}{2\sqrt{2}} - \frac{(1+\sqrt{3}) \sin(t)}{2\sqrt{2}} \right)}{1+\sin^2(t)}, \frac{\cos(t) (9+8 \cos(t)) \left( \frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(-1+\sqrt{3}) \sin(t)}{2\sqrt{2}} \right)}{1+\sin^2(t)} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 135.338    2) 135.938    3) 133.238    4) 138.938

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{e^{-z^2} + 3x - 7yz, xz + 2xyz + \cos[z^2], yz + \cos[2y^2]\}$  and the surface

$$S \equiv \left( \frac{2+x}{1} \right)^2 + \left( \frac{-2+y}{3} \right)^2 + \left( \frac{3+z}{3} \right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) 640.285    2) 640.885    3) 638.185    4) 643.885

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 2

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (3y^2z \cos(xy z) + 4xy, 2x^2 + 3\sin(xy z) + 3xyz \cos(xy z), 3xy^2 \cos(xy z))$ . Compute the potential function for this field whose potential at the origin is 1. Calculate the value of the potential at the point  $p=(-6,8,3)$ .

- 1)  $283 - 24 \sin[144]$     2)  $\frac{9059}{5} - 24 \sin[144]$     3)  $-\frac{7111}{5} - 24 \sin[144]$     4)  $577 - 24 \sin[144]$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(3t+4)\sin(2t)(3\cos(5t)+9), (7t+4)\sin(t)(3\cos(5t)+9)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 13680.9    2) 19760.9    3) 15200.9    4) 25840.9

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{4z + 9z^2, -4x^2yz^2, 6x^2y^2z\}$  and the surface

$$S \equiv \left(\frac{-9+x}{7}\right)^2 + \left(\frac{8+y}{7}\right)^2 + \left(\frac{-2+z}{5}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1)  $-9.07097 \times 10^7$     2)  $1.17167 \times 10^8$     3)  $3.77957 \times 10^7$     4)  $1.88978 \times 10^7$

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 3

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-6xy^2 + y e^{xy})(x+yz) + e^{xy}, -6x^2y + x e^{xy}(x+yz) + z e^{xy}$ ,  $y e^{xy}$ ). Compute the potential function for this field whose potential at the origin is 1. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) 1.74409    2) -1.55591    3) 1.94409    4) 3.44409

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, 2\pi] \dashrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \left\{ \frac{\left( -\frac{1}{2}\sqrt{3} \sin(t) - \frac{1}{2} \right) \cos(t) (4 \cos(t) + 8)}{\sin^2(t) + 1}, \frac{\left( \frac{\sqrt{3}}{2} - \frac{\sin(t)}{2} \right) \cos(t) (4 \cos(t) + 8)}{\sin^2(t) + 1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 116.235    2) 77.7345    3) 85.4345    4) 131.635

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{e^{2y^2-2z^2} + 2xz, -9x + 8xz - \text{Sin}[2x^2 - z^2], 4 + y + \text{Cos}[2x^2 - 2y^2]\}$  and the surface

$$S \equiv \left( \frac{6+x}{6} \right)^2 + \left( \frac{-9+y}{4} \right)^2 + \left( \frac{3+z}{4} \right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) 5550.16    2) -2412.74    3) -2895.34    4) -6514.84

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 4

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (2xy, x^2, 0)$

). Compute the potential function for this field whose potential at the origin is 4.

. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) 4.16667    2) -4.63333    3) -7.03333    4) 17.7667

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(4t+7) \sin(2t) (2 \cos(17t) + 8), (9t+8) \sin(t) (2 \cos(17t) + 8)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 15 991.4    2) 10 661.2    3) 5330.96    4) 26 651.8

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{2xy - 3xz^2, -8xyz + 5y^2z^2, 9z\}$  and the surface

$$S \equiv \left(\frac{3+x}{8}\right)^2 + \left(\frac{-3+y}{2}\right)^2 + \left(\frac{-5+z}{7}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1)  $-6.83304 \times 10^6$     2)  $1.86356 \times 10^6$     3)  $-1.55296 \times 10^6$     4)  $2.79533 \times 10^6$

## Further Mathematics - Grado en Ingeniería - 2019/2020

### 04-Line-surface integral-Test 1 for serial number: 5

#### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-2xyz + yz(3z-2x) - 2y^2 + 2y, -4xy + xz(3z-2x) + 2x, 3xyz + xy(3z-2x))$ . Compute the potential function for this field whose potential at the origin is 1. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) -2.65    2) 5.15    3) 1.25    4) -1.75

#### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(7t+5)\sin(2t), (6\cos(5t)+6), (7t+6)\sin(t)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 4154.23    2) 4984.93    3) 2769.73    4) 277.632

#### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{4x^2, -9x^2yz^2, -7\}$  and the surface

$$S \equiv \left(\frac{-3+x}{8}\right)^2 + \left(\frac{5+y}{7}\right)^2 + \left(\frac{-1+z}{2}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) -142070.    2) 412007.    3) 284143.    4) -696147.

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 6

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (2 - 2y^2, 1 - 4xy, 0)$

- ). Compute the potential function for this field whose potential at the origin is -5.
- . Calculate the value of the potential at the point  $p=(6, -4, 0)$ .

1)  $\frac{3969}{10}$     2)  $-\frac{3024}{5}$     3)  $-189$     4)  $-\frac{567}{10}$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(4t+3) \sin(2t), (4 \cos(19t) + 10), (5t+3) \sin(t)(4 \cos(19t) + 10)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 7607.42    2) 28907.    3) 4564.62    4) 15214.4

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{1, -8x, 3x^2y^2 - 3z^2\}$  and the surface

$$S \equiv \left(\frac{4+x}{2}\right)^2 + \left(\frac{8+y}{5}\right)^2 + \left(\frac{6+z}{3}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

1) -9497.41    2) 14474.5    3) 1357.79    4) 4523.89

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 7

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (6x^2y^3z^3 + 2xy, 6x^3y^2z^3 + x^2, 6x^3y^3z^2)$ .  
 ). Compute the potential function for this field whose potential at the origin is 3.  
 . Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) 3.19792    2) 7.99792    3) 5.59792    4) -3.40208

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(8t+4)\sin(2t), (2\cos(20t)+10), (7t+5)\sin(t)\}$$

Indication: it is necessary to represent  
 the curve to check whether it has intersection points.

- 1) 5191.28    2) 4820.48    3) 2224.88    4) 3708.08

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-4xy^2 - 4xz^2, -7z - 3xyz^2, 2xy\}$  and the surface

$$S \equiv \left(\frac{2+x}{4}\right)^2 + \left(\frac{-3+y}{2}\right)^2 + \left(\frac{-9+z}{5}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) 37397.2    2) 20776.4    3) 54018.    4) -24930.8

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 8

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = \left( \frac{yz(3xz - 2x)}{xyz + 1} + (3z - 2) \log(xyz + 1) + 2xy + y^2, x^2 + \frac{xz(3xz - 2x)}{xyz + 1} + 2xy, \frac{xy(3xz - 2x)}{xyz + 1} + 3x \log(xyz + 1) \right)$

). Compute the potential function for this field whose potential at the origin is -4.

. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) 3.93098    2) -11.269    3) 7.13098    4) -3.66902

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, 2\pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \left\{ \frac{\left(\frac{1}{2} - \frac{1}{2}\sqrt{3}\sin(t)\right)\cos(t)(6\cos(t)+9)}{\sin^2(t)+1}, \frac{\left(\frac{\sin(t)}{2} + \frac{\sqrt{3}}{2}\right)\cos(t)(6\cos(t)+9)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 123.003    2) 167.403    3) 23.1027    4) 111.903

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{e^{-2z^2} + 9yz, e^{2x^2-z^2} - 5y, xyz - \sin[y^2]\}$  and the surface

$$S \equiv \left( \frac{-3+x}{3} \right)^2 + \left( \frac{-9+y}{9} \right)^2 + \left( \frac{8+z}{3} \right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) 7464.42    2) -12688.4    3) 21646.    4) -6717.18

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 9

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-6xy^2, -6x^2y, 0)$

- ). Compute the potential function for this field whose potential at the origin is 3.
  - . Calculate the value of the potential at the point  $p=(7,7,-9)$ .
- 1) -19 440    2) 5760    3) 14 400    4) -7200

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(3t+8)\sin(2t)(7\cos(8t)+7), (2t+6)\sin(t)(7\cos(8t)+7)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 10 419.5    2) 11 577.2    3) 13 892.6    4) 17 365.7

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-4y^2 - yz^2, 5xy^2, 9y + 7xyz\}$  and the surface

$$S \equiv \left(\frac{-9+x}{1}\right)^2 + \left(\frac{1+y}{5}\right)^2 + \left(\frac{-1+z}{8}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) 20 509.4    2) -69 216.6    3) -25 635.4    4) 51 272.6

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 10

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = \left( \frac{z(2x-2yz)}{xz+1} + 2\log(xz+1), 2y - 2z\log(xz+1), \frac{x(2x-2yz)}{xz+1} - 2y\log(xz+1) \right)$

- ). Compute the potential function for this field whose potential at the origin is -4.
- . Calculate the value of the potential at the point  $p=(-9,-8,-4)$ .

1)  $-\frac{174}{5} - 82 \log[37]$     2)  $60 - 82 \log[37]$     3)  $-\frac{2007}{10} - 82 \log[37]$     4)  $-\frac{3492}{5} - 82 \log[37]$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(4t+7)\sin(2t)(8\cos(6t)+9), (5t+2)\sin(t)(8\cos(6t)+9)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 16611.    2) 8305.82    3) 20763.6    4) 24916.2

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-6x^2y, 4z^2 - 5x^2y^2z^2, -7x\}$  and the surface

$$S \equiv \left( \frac{3+x}{8} \right)^2 + \left( \frac{-1+y}{8} \right)^2 + \left( \frac{8+z}{7} \right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1)  $-1.23694 \times 10^8$     2)  $-6.18471 \times 10^7$     3)  $-4.12314 \times 10^7$     4)  $-2.9451 \times 10^7$

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 11

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = ((2y-3)\sin(xy) + yz(2xy-3x)\cos(xy))\mathbf{i} + 2x\sin(xy) + xz(2xy-3x)\cos(xy)\mathbf{j} + xy(2xy-3x)\cos(xy)\mathbf{k}$ .  
 ) . Compute the potential function for this field whose potential at the origin is -3.  
 . Calculate the value of the potential at the point  $p=(-7,-7,9)$ .

1)  $\frac{1204}{5} + 119 \sin[441]$     2)  $-3 + 119 \sin[441]$     3)  $50 + 119 \sin[441]$     4)  $-\frac{1923}{5} + 119 \sin[441]$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, \pi] \rightarrow \mathbb{R}^2$

$$r(t) = \left\{ \sin(2t)(5\cos(t) + 10) \left( \frac{(1+\sqrt{3})\cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} \right), \sin(2t)(5\cos(t) + 10) \left( \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} + \frac{(\sqrt{3}-1)\cos(t)}{2\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent  
 the curve to check whether it has intersection points.

1) 167.557    2) 88.3573    3) 17.9573    4) 26.7573

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-4x - \sin[2y^2 - z^2], e^{-2x^2+2z^2} - 8z, xz + \cos[x^2 - 2y^2]\}$  and the surface  
 $S \equiv \left( \frac{-5+x}{4} \right)^2 + \left( \frac{-5+y}{1} \right)^2 + \left( \frac{-9+z}{8} \right)^2 = 1$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

1) 201.041    2) -348.359    3) 442.241    4) 134.041

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 12

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (y-3)\sin(xz) + z(xy-3x)\cos(xz), x\sin(xz), x(xy-3x)\cos(xz)$

. Compute the potential function for this field whose potential at the origin is -3.

. Calculate the value of the potential at the point  $p=(1,-7,-5)$ .

1)  $-3 + 10 \sin[5]$     2)  $\frac{102}{5} + 10 \sin[5]$     3)  $-\frac{407}{10} + 10 \sin[5]$     4)  $-\frac{277}{10} + 10 \sin[5]$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$r: [0, 2\pi] \longrightarrow \mathbb{R}^2$$

$$r(t) = \left\{ \frac{\left(-\frac{1}{2}\sqrt{3}\sin(t) - \frac{1}{2}\right)\cos(t)(7\cos(t)+10)}{\sin^2(t)+1}, \frac{\left(\frac{\sqrt{3}}{2} - \frac{\sin(t)}{2}\right)\cos(t)(7\cos(t)+10)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 213.062    2) 142.062    3) 42.662    4) 85.262

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{8y - 4yz - \sin(z^2), -5 + \sin(x^2 + z^2), e^{x^2+2y^2} - 4y - 6yz\}$  and the surface  $S \equiv \left(\frac{3+x}{1}\right)^2 + \left(\frac{-6+y}{3}\right)^2 + \left(\frac{8+z}{1}\right)^2 = 1$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

1) -1766.09    2) -452.389    3) 363.011    4) -1313.09

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 13

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (3x^2y^3z^3(2z-3yz), -3x^3y^3z^4 + 3x^3y^2z^3(2z-3yz) + 3, x^3(2-3y)y^3z^3 + 3x^3y^3z^2(2z-3yz))$ .  
 ). Compute the potential function for this field whose potential at the origin is -3.  
 . Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) -9.105    2) 5.295    3) -1.505    4) -8.505

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(2t+7)\sin(2t)(\cos(6t)+7), (8t+6)\sin(t)(\cos(6t)+7)\}$$

Indication: it is necessary to represent  
the curve to check whether it has intersection points.

- 1) 5090.09    2) 19086.5    3) 16541.7    4) 12724.5

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-xy^2z + 2x^2yz^2, 9x, 8y^2z\}$  and the surface

$$S \equiv \left(\frac{7+x}{7}\right)^2 + \left(\frac{4+y}{2}\right)^2 + \left(\frac{8+z}{2}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1)  $-2.11858 \times 10^6$     2) 882742.    3)  $4.23716 \times 10^6$     4)  $3.97234 \times 10^6$

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 14

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-6xy^2 - yz(3xy - 3)\sin(xy) + 3y\cos(xy), -6x^2y - xz(3xy - 3)\sin(xy) + 3x\cos(xy) - 2y, -xy(3xy - 3)\sin(xy))$ .  
 . Compute the potential function for this field whose potential at the origin is -2.  
 . Calculate the value of the potential at the point  $p=(-6,-5,2)$ .

1)  $\frac{25678}{5} + 87 \cos[60]$     2)  $-2724 + 87 \cos[60]$     3)  $-\frac{131099}{10} + 87 \cos[60]$     4)  $\frac{14201}{2} + 87 \cos[60]$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, \pi] \rightarrow \mathbb{R}^2$   
 $r(t) = \{\sin(t) \sin(2t) (-(\cos(t) + 1)), \sin(2t) \cos(t) (\cos(t) + 1)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 1.1781    2) 1.4781    3) 0.878097    4) 1.7781

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{6z + \cos[z^2], e^{2x^2-2z^2} - 3z, e^{x^2+2y^2} - 4x\}$  and the surface  $S \equiv \left(\frac{-7+x}{1}\right)^2 + \left(\frac{8+y}{7}\right)^2 + \left(\frac{-8+z}{3}\right)^2 = 1$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

1) -3.5    2) -2.8    3) 0.    4) 3.7

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 15

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-6xy^2 + \frac{y(-3yz-y)}{xy+1} - 2xy, -6x^2y - x^2 + \frac{x(-3yz-y)}{xy+1} + (-3z-1)\log(xy+1), -3y\log(xy+1))$

). Compute the potential function for this field whose potential at the origin is 5.

. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) -0.240734    2) 4.15927    3) 2.15927    4) -0.640734

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(2t+6)\sin(2t)(8\cos(19t)+8), (5t+7)\sin(t)(8\cos(19t)+8)\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 30078.5    2) 15924.1    3) 17693.4    4) 26539.9

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{5xyz - 8x^2y^2z, 9xy - 6x^2y^2z, -8xy\}$  and the surface

$$S \equiv \left(\frac{4+x}{7}\right)^2 + \left(\frac{-1+y}{8}\right)^2 + \left(\frac{-4+z}{9}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1)  $2.40502 \times 10^6$     2) -481004.    3) -962008.    4)  $4.81004 \times 10^6$

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 16

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-2xy^2 + z \sin(xy z) + yz(xz + yz) \cos(xy z), -2x^2y + z \sin(xy z) + xz(xz + yz) \cos(xy z), (x+y) \sin(xy z) + xy(xz + yz) \cos(xy z))$ . Compute the potential function for this field whose potential at the origin is 8.

. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) 10.0967    2) 17.7967    3) 7.99672    4) -17.2033

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, 2\pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \left\{ -\frac{\cos(t)(3\cos(t)+3)}{\sin^2(t)+1}, -\frac{\sin(t)\cos(t)(3\cos(t)+3)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 18.3257    2) 11.9257    3) 16.7257    4) 23.1257

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-5xy + \cos[2y^2 - z^2], -xz + \cos[2x^2 - z^2], -6xy + \cos[2x^2 - 2y^2]\}$  and the surface  $S \equiv \left( \frac{-2+x}{8} \right)^2 + \left( \frac{1+y}{1} \right)^2 + \left( \frac{-3+z}{6} \right)^2 = 1$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) -7838.07    2) 7236.93    3) 3920.43    4) 3015.93

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 17

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-2y, -2x - 2z \cos(yz), -2y \cos(yz))$

). Compute the potential function for this field whose potential at the origin is -4.

. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) -22.4796    2) -0.979624    3) -4.97962    4) 14.5204

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \longrightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(7t+4) \sin(2t), (3 \cos(14t) + 7), (3t+8) \sin(t)\}$$

Indication: it is necessary to represent  
the curve to check whether it has intersection points.

- 1) 3441.85    2) 1811.95    3) 182.053    4) 2355.25

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{6xy^2 + 4x^2z, 8xz^2 + 2x^2z^2, 5xy^2z\}$  and the surface

$$S \equiv \left(\frac{3+x}{4}\right)^2 + \left(\frac{2+y}{5}\right)^2 + \left(\frac{-8+z}{1}\right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) -102919.    2) -4573.99    3) 66326.1    4) -22870.8

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 18

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (4xy^2 + \frac{yz(3xyz+3z)}{xyz+1} + 3yz\log(xyz+1), 4x^2y + \frac{xz(3xyz+3z)}{xyz+1} + 3xz\log(xyz+1), \frac{xy(3xyz+3z)}{xyz+1} + (3xy+3)\log(xyz+1))$ .

). Compute the potential function for this field whose potential at the origin is 1.

. Calculate the value of the potential at the point  $p=(7,0,0)$ .

1)  $\frac{2}{5}$     2) 1    3) 2    4)  $-\frac{11}{5}$

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \left\{ \sin(2t)(3\cos(t) + 8) \left( \frac{\cos(t)}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}} \right), \sin(2t)(3\cos(t) + 8) \left( \frac{\sin(t)}{\sqrt{2}} + \frac{\cos(t)}{\sqrt{2}} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 69.6998    2) 96.1998    3) 90.8998    4) 53.7998

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-4xz + \cos[y^2 + z^2], e^{-x^2} - 6xz, 9xz + \sin[2x^2 - 2y^2]\}$  and the surface

$$S \equiv \left( \frac{3+x}{6} \right)^2 + \left( \frac{-8+y}{2} \right)^2 + \left( \frac{-1+z}{7} \right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

1) -21815.6    2) -10907.6    3) 23998.    4) -4362.81

# Further Mathematics - Grado en Ingeniería - 2019/2020

## 04-Line-surface integral-Test 1 for serial number: 19

### Exercise 1

Consider the vectorial field  $F(x,y,z) = (-x^2 y^2 z^3 + 2xy^2 z^2 (-xz - 2yz) + 2xy^2$   
 $, -2x^2 y^2 z^3 + 2x^2 yz^2 (-xz - 2yz) + 2x^2 y, x^2 y^2 z^2 (-x - 2y) + 2x^2 y^2 z (-xz - 2yz)$   
). Compute the potential function for this field whose potential at the origin is -3.  
. Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

- 1) -2.95139    2) -2.05139    3) 8.14861    4) 4.54861

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [0, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \{(3t+8)\sin(2t)(8\cos(19t)+9), (2t+8)\sin(t)(8\cos(19t)+9)\}$$

Indication: it is necessary to represent  
the curve to check whether it has intersection points.

- 1) 17248.3    2) 19404.3    3) 15092.3    4) 21560.3

### Exercise 3

Consider the vectorial field  $F(x,y,z) = \{-9y^2 + 8x^2yz^2, -6xz, -6xy^2z - 2x^2z^2\}$  and the surface

$$S \equiv \left(\frac{4+x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{-5+z}{4}\right)^2 = 1$$

Compute  $\int_S F$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) -68521.9    2) 185009.    3) 102783.    4) -47965.3

## Further Mathematics - Grado en Ingeniería - 2019/2020 04-Line-surface integral-Test 1 for serial number: 20

### Exercise 1

Consider the vectorial field  $\mathbf{F}(x,y,z) = (-4xy^2 + 2y^2z - 1, 4xyz - 4x^2y, 2xy^2)$ .  
 ). Compute the potential function for this field whose potential at the origin is -3.  
 . Calculate the integral of the potential function  $\phi$  over the domain  $[0,1]^3$ .

1) -1.55556    2) -19.1556    3) -7.15556    4) -3.55556

### Exercise 2

Compute the area of the domain whose boundary is the curve

$$\mathbf{r}: [\theta, \pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{r}(t) = \left\{ \sin(2t)(7\cos(t) + 10) \left( \frac{1}{2}\sqrt{3}\cos(t) - \frac{\sin(t)}{2} \right), \sin(2t)(7\cos(t) + 10) \left( \frac{1}{2}\sqrt{3}\sin(t) + \frac{\cos(t)}{2} \right) \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

- 1) 58.9821    2) 39.5821    3) 97.7821    4) 126.882

### Exercise 3

Consider the vectorial field  $\mathbf{F}(x,y,z) = \{-8 + 9z + \cos[2y^2 + z^2], -3xz - \sin[x^2 + 2z^2], 5y - xy - \sin[2x^2 + y^2]\}$  and the surface

$$S \equiv \left( \frac{-4+x}{3} \right)^2 + \left( \frac{-4+y}{8} \right)^2 + \left( \frac{6+z}{3} \right)^2 = 1$$

Compute  $\int_S \mathbf{F} \cdot d\mathbf{S}$ .

Indication: Use Stoke's Theorem if it is necessary.

- 1) -4825.49    2) -9168.89    3) -2412.49    4) -23646.9