

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 1

Exercise 1

Compute $\int_D (3x) \, dx \, dy$ for $D = \{0 \leq -x + 7y \leq 4, 0 \leq -3x + y \leq 6\}$

- 1) -5.12
- 2) -3.72
- 3) -4.52
- 4) -3.42
- 5) -4.82

Exercise 2

Compute $\int_D (xy) \, dx \, dy$ for $D = \{5 \leq x^7 y^2 \leq 6, 2 \leq x^{24} y^7 \leq 8, x > 0, y > 0\}$

- 1) -0.2
- 2) -0.3
- 3) 5.41449×10^{-18}
- 4) -1.
- 5) -0.3

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 4, z \geq \sqrt{x^2 + y^2}\}$

- 1) 4.90747
- 2) -0.490747
- 3) -4.90747
- 4) -4.90747
- 5) 1.47224

Exercise 4

Compute the volume of the domain limited by the plane $5x + 5z = 4$ and the paraboloid $z = 7x^2 + 7y^2$.

- 1) 0.395233
- 2) 0.121221
- 3) 0.186268
- 4) 0.156725
- 5) 0.112974

Exercise 5

Compute $\int_D (2yz^2) \, dx \, dy \, dz$ for $D =$

$$\{2xz^3 \leq y^7 \leq 9xz^3, 6y^9 \leq xz^2 \leq 9y^9, 8x^9y^5 \leq z \leq 12x^9y^5, x > 0, y > 0, z > 0\}$$

- 1) -0.999997
- 2) $-2.$
- 3) 3.15221×10^{-6}
- 4) -1.6
- 5) -1.7

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Exam - 2 - Multivariate Integration for serial number: 2

Exercise 1

Compute $\int_D (2x + y) \, dx \, dy$ for $D = \{0 \leq -2x + 7y \leq 2, 0 \leq x - 3y \leq 5\}$

- 1) 141.
- 2) 470.
- 3) -282.
- 4) 47.
- 5) 1269.

Exercise 2

Compute $\int_D (x + 3y) \, dx \, dy$ for $D = \{5x^7y^2 \leq 1 \leq 11x^7y^2, 4x^{10}y^3 \leq 1 \leq 10x^{10}y^3, x > 0, y > 0\}$

- 1) -3.10125×10^8
- 2) -3.10125×10^8
- 3) -1.77214×10^8
- 4) 4.43036×10^8
- 5) 5.31643×10^8

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 1, z \geq 6(x^2 + y^2)\}$

- 1) 1.84116
- 2) -0.158843
- 3) -0.658843
- 4) -0.658843
- 5) 0.241157

Exercise 4

Compute the volume of the domain limited by the plane $4x + 2z = 6$ and the paraboloid $z = 4x^2 + 4y^2$.

- 1) 9.06746
- 2) 2.81022
- 3) 4.14788
- 4) 16.6321
- 5) 2.71811

Exercise 5

Compute $\int_D (x z^3) \, dx \, dy \, dz$ for $D = \{9x^2 \leq y^7 z \leq 17x^2, x^3 \leq y^7 z^8 \leq 4x^3, 8y^8 \leq z^7 \leq 14y^8, x > 0, y > 0, z > 0\}$

1) 1.79796×10^{-10}

2) -1.1

3) -1.1

4) 0.7

5) -1.5

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Exam - 2 - Multivariate Integration for serial number: 3

Exercise 1

Compute $\int_D (4x) \, dx \, dy$ for $D = \{0 \leq 5x + 8y \leq 9, 0 \leq -4x + 6y \leq 3\}$

- 1) 1.92144
- 2) 0.421436
- 3) 1.72144
- 4) -0.878564
- 5) -0.478564

Exercise 2

Compute $\int_D (3x + 3y) \, dx \, dy$ for $D = \{7 \leq x^3 y^5 \leq 12, 7 \leq xy^2 \leq 10, x > 0, y > 0\}$

- 1) 8.73214
- 2) 0.970238
- 3) 12.6131
- 4) 4.85119
- 5) 2.91071

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 196, z \geq 2\sqrt{x^2 + y^2}\}$

- 1) -364.037
- 2) 242.692
- 3) 1395.48
- 4) 606.729
- 5) 1577.5

Exercise 4

Compute the volume of the domain limited by the plane $8x + 5z = 5$ and the paraboloid $z = 8x^2 + 8y^2$.

- 1) 0.418924
- 2) 0.229022
- 3) 0.160911
- 4) 0.272878
- 5) 1.09246

Exercise 5

Compute $\int_D (y^4) dx dy dz$ for $D = \{8z^4 \leq x^5 y^8 \leq 12z^4, 7 \leq x^6 y^6 z^3 \leq 15, 2 \leq y^9 z \leq 10, x > 0, y > 0, z > 0\}$

- 1) -0.296043
- 2) -0.896043
- 3) 1.30396
- 4) 1.00396
- 5) 0.00395719

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Exam - 2 - Multivariate Integration for serial number: 4

Exercise 1

Compute $\int_D (x + y) \, dx \, dy$ for $D = \{0 \leq 3x + 7y \leq 7, 0 \leq -x - 4y \leq 3\}$

- 1) 13.86
- 2) 20.79
- 3) -4.158
- 4) -4.158
- 5) 16.632

Exercise 2

Compute $\int_D (x y^3) \, dx \, dy$ for $D = \{8xy \leq 1 \leq 11xy, 6 \leq x^4 y^3 \leq 14, x > 0, y > 0\}$

- 1) 2.
- 2) 1.8
- 3) 1.01221×10^{-12}
- 4) 1.8
- 5) 0.7

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 144, z \geq 3\sqrt{x^2 + y^2}\}$

- 1) 130.005
- 2) 538.591
- 3) 557.163
- 4) 111.433
- 5) 185.721

Exercise 4

Compute the volume of the domain limited by the plane $3x + 2z = 8$ and the paraboloid $z = 2x^2 + 2y^2$.

- 1) 14.3956
- 2) 34.4227
- 3) 0.211968
- 4) 2.65019
- 5) 10.4633

Exercise 5

Compute $\int_D (2z) \, dx \, dy \, dz$ for $D =$

$$\{2z \leq x^3 y^7 \leq 4z, 7x^4 z^2 \leq y^6 \leq 11x^4 z^2, 9x^6 y^7 \leq z^3 \leq 12x^6 y^7, x > 0, y > 0, z > 0\}$$

- 1) 0.0161798
- 2) 1.91618
- 3) 0.61618
- 4) -0.38382
- 5) -1.18382

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Exam - 2 - Multivariate Integration for serial number: 5

Exercise 1

Compute $\int_D (3x + y) \, dx \, dy$ for $D = \{0 \leq 4x - y \leq 6, 0 \leq -2x + y \leq 9\}$

- 1) 1192.73
- 2) -62.775
- 3) 0.
- 4) 627.75
- 5) 1255.5

Exercise 2

Compute $\int_D (y^3) \, dx \, dy$ for $D = \{7x^3y \leq 1 \leq 10x^3y, 6 \leq x^4y \leq 9, x > 0, y > 0\}$

- 1) 1.1
- 2) -0.5
- 3) 3.4615×10^{-24}
- 4) -1.1
- 5) 0.8

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 144, z \geq 9\sqrt{x^2 + y^2}\}$

- 1) 42.0574
- 2) -2.21355
- 3) 4.42709
- 4) 48.698
- 5) 22.1355

Exercise 4

Compute the volume of the domain limited by the plane $7x + 4z = 1$ and the paraboloid $z = 9x^2 + 9y^2$.

- 1) 0.076145
- 2) 0.0590706
- 3) 0.0195951
- 4) 0.0222472
- 5) 0.0324898

Exercise 5

Compute $\int_D (y + 2z) \, dx \, dy \, dz$ for $D =$

$$\{5y z^2 \leq x^4 \leq 9y z^2, 6z^4 \leq x y^2 \leq 12z^4, 9 \leq x^9 y z^9 \leq 18, x > 0, y > 0, z > 0\}$$

- 1) 0.00986614
- 2) 1.80987
- 3) -1.59013
- 4) -0.690134
- 5) 1.10987

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Exam - 2 - Multivariate Integration for serial number: 6

Exercise 1

Compute $\int_D (2x + 2y) \, dx \, dy$ for $D = \{0 \leq -5x + 8y \leq 8, 0 \leq 2x - 5y \leq 5\}$

- 1) -59.7531
- 2) -60.0531
- 3) -60.5531
- 4) -61.5531
- 5) -59.6531

Exercise 2

Compute $\int_D (y^2) \, dx \, dy$ for $D = \{6x^6 \leq y^7 \leq 7x^6, 2x \leq y \leq 11x, x > 0, y > 0\}$

- 1) 0.4
- 2) 1.7
- 3) 3.29316×10^{-7}
- 4) -1.8
- 5) -1.8

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 100, z \geq 2(x^2 + y^2)\}$

- 1) -68.9479
- 2) 76.6087
- 3) -38.3044
- 4) 38.3044
- 5) 168.539

Exercise 4

Compute the volume of the domain limited by the plane $3x + 5z = 6$ and the paraboloid $z = 7x^2 + 7y^2$.

- 1) 1.0553
- 2) 0.615906
- 3) 0.330097
- 4) 1.57653
- 5) 0.148757

Exercise 5

Compute $\int_D (x^2 + z) \, dx \, dy \, dz$ for $D =$

$$\{7x^5 \leq y^8 z^2 \leq 8x^5, x^5 \leq y^5 z^9 \leq 4x^5, 4y^6 \leq x^7 z \leq 11y^6, x > 0, y > 0, z > 0\}$$

- 1) -1.65958
- 2) 1.94042
- 3) 0.0404182
- 4) -0.959582
- 5) -0.859582

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Exam - 2 - Multivariate Integration for serial number: 7

Exercise 1

Compute $\int_D (x + 3y) \, dx \, dy$ for $D = \{0 \leq x + 4y \leq 9, 0 \leq 7x \leq 1\}$

- 1) 2.39923
- 2) 1.09056
- 3) -0.654337
- 4) 1.19962
- 5) 2.39923

Exercise 2

Compute $\int_D (x + 3y) \, dx \, dy$ for $D = \{2y^7 \leq x^{23} \leq 6y^7, 8y^3 \leq x^{10} \leq 11y^3, x > 0, y > 0\}$

- 1) 1.37519×10^{45}
- 2) 4.58395×10^{45}
- 3) 1.05431×10^{46}
- 4) 8.70951×10^{45}
- 5) 8.70951×10^{45}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 49, z \geq 12\sqrt{x^2 + y^2}\}$

- 1) 4.9629
- 2) 6.45177
- 3) 2.48145
- 4) 3.22588
- 5) 1.98516

Exercise 4

Compute the volume of the domain limited by the plane $7x + 8z = 3$ and the paraboloid $z = 10x^2 + 10y^2$.

- 1) 0.119141
- 2) 0.0244018
- 3) 0.0446663
- 4) 0.091841
- 5) 0.0637487

Exercise 5

Compute $\int_D (x^3 y^2) \, dx \, dy \, dz$ for $D =$

$$\{8 \leq x y^4 z^7 \leq 13, 2 y z^3 \leq 1 \leq 3 y z^3, 9 x^9 y^3 z^6 \leq 1 \leq 14 x^9 y^3 z^6, x > 0, y > 0, z > 0\}$$

- 1) 0.78102
- 2) 1.28102
- 3) 0.98102
- 4) -0.41898
- 5) 1.58102

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Exam - 2 - Multivariate Integration for serial number: 8

Exercise 1

Compute $\int_D (3x + 3y) \, dx \, dy$ for $D = \{0 \leq 4x + 2y \leq 6, 0 \leq -5x + y \leq 6\}$

- 1) 17.1918
- 2) 13.2245
- 3) 7.93469
- 4) -6.61224
- 5) 25.1265

Exercise 2

Compute $\int_D (2x^3) \, dx \, dy$ for $D = \{5 \leq x^9 y^4 \leq 8, 4 \leq x^{11} y^5 \leq 10, x > 0, y > 0\}$

- 1) 255.859
- 2) 460.547
- 3) 690.82
- 4) -127.93
- 5) 51.1718

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 144, z \geq 11\sqrt{x^2 + y^2}\}$

- 1) 32.6985
- 2) 7.43148
- 3) 31.2122
- 4) -2.97259
- 5) 14.863

Exercise 4

Compute the volume of the domain limited by the plane $7x + 10z = 8$ and the paraboloid $z = 10x^2 + 10y^2$.

- 1) 0.103633
- 2) 0.490337
- 3) 0.09513
- 4) 0.0987507
- 5) 0.244306

Exercise 5

Compute $\int_D (x^3 z) \, dx \, dy \, dz$ for $D = \{z^5 \leq x^4 y^2 \leq 3z^5, 8 \leq x y^2 z \leq 11, 3z^8 \leq x^2 y^9 \leq 4z^8, x > 0, y > 0, z > 0\}$

1) -0.438138

2) 0.561862

3) -0.138138

4) 0.061862

5) 1.66186

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Exam - 2 - Multivariate Integration for serial number: 9

Exercise 1

Compute $\int_D (4y) \, dx \, dy$ for $D = \{0 \leq -6x + 3y \leq 5, 0 \leq 3x + 5y \leq 5\}$

- 1) 1.47929
- 2) 4.43787
- 3) 1.0355
- 4) -0.295858
- 5) 3.40237

Exercise 2

Compute $\int_D (5y) \, dx \, dy$ for $D = \{9y \leq x^4 \leq 11y, 3x^7 \leq y^2 \leq 4x^7, x > 0, y > 0\}$

- 1) 4.45489×10^{20}
- 2) 3.71241×10^{20}
- 3) 1.48496×10^{20}
- 4) -1.11372×10^{20}
- 5) 1.0766×10^{21}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 49, z \geq 13(x^2 + y^2)\}$

- 1) 10.0101
- 2) -5.88828
- 3) 14.7207
- 4) -1.76648
- 5) 5.88828

Exercise 4

Compute the volume of the domain limited by the plane $2x + 2z = 10$ and the paraboloid $z = 6x^2 + 6y^2$.

- 1) 2.03366
- 2) 21.6339
- 3) 2.85594
- 4) 6.65452
- 5) 1.12014

Exercise 5

Compute $\int_D (x + y) \, dx \, dy \, dz$ for $D =$

$$\{2x^6 \leq y^3 z^3 \leq 10x^6, 6y^3 \leq x^5 z^6 \leq 13y^3, 6x^7 \leq y^3 z^2 \leq 13x^7, x > 0, y > 0, z > 0\}$$

- 1) -0.991889
- 2) -1.09189
- 3) 1.70811
- 4) 0.0081105
- 5) 0.808111

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Exam - 2 - Multivariate Integration for serial number: 10

Exercise 1

Compute $\int_D (2x) \, dx \, dy$ for $D = \{0 \leq 6x - 6y \leq 8, 0 \leq -9x \leq 6\}$

- 1) -0.692593
- 2) -2.29259
- 3) -0.592593
- 4) 1.20741
- 5) 0.607407

Exercise 2

Compute $\int_D (6x^2) \, dx \, dy$ for $D = \{x^{71} \leq y^{26} \leq 5x^{71}, 3y^{11} \leq x^{30} \leq 5y^{11}, x > 0, y > 0\}$

- 1) 0.3
- 2) 5.18272×10^{-75}
- 3) 1.2
- 4) -0.2
- 5) -0.2

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 169, z \geq 9\sqrt{x^2 + y^2}\}$

- 1) 73.1726
- 2) 75.9869
- 3) 28.1433
- 4) 8.44299
- 5) 5.62866

Exercise 4

Compute the volume of the domain limited by the plane $8x + 3z = 4$ and the paraboloid $z = 4x^2 + 4y^2$.

- 1) 5.67346
- 2) 0.865983
- 3) 2.87318
- 4) 3.9425
- 5) 1.24112

Exercise 5

Compute $\int_D (x + 2y) \, dx \, dy \, dz$ for $D =$

$$\{6y^6z^9 \leq x^6 \leq 11y^6z^9, 2y^9z^2 \leq 1 \leq 7y^9z^2, 4y^3 \leq x^4z^3 \leq 12y^3, x > 0, y > 0, z > 0\}$$

- 1) 0.00673642
- 2) -0.193264
- 3) 1.70674
- 4) -0.693264
- 5) 1.70674

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Exam - 2 - Multivariate Integration for serial number: 11

Exercise 1

Compute $\int_D (2x + 3y) \, dx \, dy$ for $D = \{0 \leq -6x \leq 6, 0 \leq 6x + 8y \leq 5\}$

- 1) 0.664063
- 2) 2.56406
- 3) 0.0640625
- 4) 2.16406
- 5) 0.564063

Exercise 2

Compute $\int_D (3x + y^3) \, dx \, dy$ for $D = \{8x^3y^{10} \leq 1 \leq 11x^3y^{10}, 5 \leq xy^3 \leq 14, x > 0, y > 0\}$

- 1) 2.34691×10^{23}
- 2) -5.52215×10^{22}
- 3) 2.62302×10^{23}
- 4) 1.51859×10^{23}
- 5) 1.38054×10^{23}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 64, z \geq 10(x^2 + y^2)\}$

- 1) -5.99432
- 2) -7.99242
- 3) 19.9811
- 4) 9.99053
- 5) 27.9735

Exercise 4

Compute the volume of the domain limited by the plane $2x + 6z = 3$ and the paraboloid $z = 9x^2 + 9y^2$.

- 1) 0.0822134
- 2) 0.124267
- 3) 0.0441736
- 4) 0.099899
- 5) 0.153532

Exercise 5

Compute $\int_D (3x) \, dx \, dy \, dz$ for $D = \{8x^6 \leq z^6 \leq 17x^6, 5 \leq x^8 y^8 z \leq 8, 2y^7 \leq z^4 \leq 6y^7, x > 0, y > 0, z > 0\}$

- 1) 0.104236
- 2) 0.00423611
- 3) 1.60424
- 4) -1.89576
- 5) -1.09576

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Exam - 2 - Multivariate Integration for serial number: 12

Exercise 1

Compute $\int_D (3x + y) \, dx \, dy$ for $D = \{0 \leq -5x + 8y \leq 5, 0 \leq -7x + 8y \leq 2\}$

- 1) 2.65234
- 2) 4.92578
- 3) 3.2207
- 4) 1.89453
- 5) 3.41016

Exercise 2

Compute $\int_D (2x^2) \, dx \, dy$ for $D = \{7y^{13} \leq x^3 \leq 10y^{13}, 8x^2 \leq y^9 \leq 17x^2, x > 0, y > 0\}$

- 1) 7.83833×10^{76}
- 2) 1.95958×10^{78}
- 3) 2.03797×10^{78}
- 4) 1.25413×10^{78}
- 5) 7.83833×10^{77}

Exercise 3

Compute the volume of $D = \{5(x^2 + y^2) \leq z \leq 49 - x^2 - y^2\}$

- 1) 817.154
- 2) 1760.02
- 3) 1760.02
- 4) 1760.02
- 5) 628.58

Exercise 4

Compute the volume of the domain limited by the plane $10x + 10z = 3$ and the paraboloid $z = 3x^2 + 3y^2$.

- 1) 0.0769399
- 2) 0.0702141
- 3) 0.234836
- 4) 0.248877
- 5) 0.0951021

Exercise 5

Compute $\int_D (2y + z^3) \, dx \, dy \, dz$ for $D =$

$$\{4x^5z^7 \leq y^8 \leq 13x^5z^7, 5 \leq x^8y^8 \leq 10, 4 \leq x^9y^6z^5 \leq 11, x > 0, y > 0, z > 0\}$$

- 1) 0.0148578
- 2) 0.914858
- 3) 1.81486
- 4) 1.01486
- 5) 0.814858

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Exam - 2 - Multivariate Integration for serial number: 13

Exercise 1

Compute $\int_D (x + y) \, dx \, dy$ for $D = \{0 \leq -7x + 3y \leq 7, 0 \leq 8x - 4y \leq 6\}$

- 1) -188.
- 2) -188.2
- 3) -189.
- 4) -188.3
- 5) -190.4

Exercise 2

Compute $\int_D (x^2 + y) \, dx \, dy$ for $D = \{8y^7 \leq x^4 \leq 10y^7, 5y^5 \leq x^3 \leq 12y^5, x > 0, y > 0\}$

- 1) 0.
- 2) 9.24552×10^7
- 3) -9.24552×10^7
- 4) -4.62276×10^7
- 5) 1.15569×10^8

Exercise 3

Compute the volume of $D = \{2(x^2 + y^2) \leq z \leq 196 - x^2 - y^2\}$

- 1) 34194.8
- 2) 38217.7
- 3) 56320.8
- 4) 20114.6
- 5) -18103.1

Exercise 4

Compute the volume of the domain limited by the plane $5x + 5z = 9$ and the paraboloid $z = 10x^2 + 10y^2$.

- 1) 1.43496
- 2) 0.523173
- 3) 0.643689
- 4) 1.81483
- 5) 0.10464

Exercise 5

Compute $\int_D (2x + x^2) \, dx \, dy \, dz$ for $D =$

$$\{6x^7 \leq y^3 z^8 \leq 12x^7, 5x^4 y^6 \leq z^8 \leq 8x^4 y^6, 2y^3 \leq x^9 z^3 \leq 5y^3, x > 0, y > 0, z > 0\}$$

- 1) 1.60191
- 2) -0.698088
- 3) 0.401912
- 4) 0.901912
- 5) 0.00191216

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Exam - 2 - Multivariate Integration for serial number: 14

Exercise 1

Compute $\int_D (6x) \, dx \, dy$ for $D = \{0 \leq 5x + y \leq 9, 0 \leq -2x + 6y \leq 5\}$

- 1) 9.04395
- 2) 11.6279
- 3) 1.29199
- 4) 6.45996
- 5) 10.9819

Exercise 2

Compute $\int_D (y + y^2) \, dx \, dy$ for $D = \{2 \leq x^{23} y^{11} \leq 10, 8 \leq x^2 y \leq 16, x > 0, y > 0\}$

- 1) 2.00741×10^{66}
- 2) -5.20439×10^{65}
- 3) 1.93306×10^{66}
- 4) -5.20439×10^{65}
- 5) 7.43485×10^{65}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 64, z \geq 7\sqrt{x^2 + y^2}\}$

- 1) 19.3994
- 2) -2.15549
- 3) 17.2439
- 4) 18.3217
- 5) 10.7775

Exercise 4

Compute the volume of the domain limited by the plane $8x + 6z = 10$ and the paraboloid $z = 4x^2 + 4y^2$.

- 1) 5.86674
- 2) 1.91943
- 3) 1.24112
- 4) 0.99408
- 5) 4.16376

Exercise 5

Compute $\int_D (2y) \, dx \, dy \, dz$ for $D = \{4 \leq x^6 y^4 z^6 \leq 13, 2 \leq x^4 y^5 z \leq 5, 8x^2 \leq y^6 \leq 13x^2, x > 0, y > 0, z > 0\}$

- 1) 1.71404
- 2) 1.61404
- 3) -0.285962
- 4) 0.0140384
- 5) 1.11404

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Exam - 2 - Multivariate Integration for serial number: 15

Exercise 1

Compute $\int_D (3x + 3y) \, dx \, dy$ for $D = \{0 \leq 2x - 7y \leq 4, 0 \leq 4x + 4y \leq 8\}$

- 1) 1.6
- 2) -2.4
- 3) -1.86667
- 4) 2.66667
- 5) 0.533333

Exercise 2

Compute $\int_D (3x^3 y) \, dx \, dy$ for $D = \{7 \leq x^5 y^{14} \leq 15, 9x^4 y^{11} \leq 1 \leq 10x^4 y^{11}, x > 0, y > 0\}$

- 1) -2.
- 2) 1.8
- 3) -0.5
- 4) 8.62881×10^{-78}
- 5) 0.6

Exercise 3

Compute the volume of $D = \{12(x^2 + y^2) \leq z \leq 81 - x^2 - y^2\}$

- 1) 554.938
- 2) -317.108
- 3) -317.108
- 4) -158.554
- 5) 792.769

Exercise 4

Compute the volume of the domain limited by the plane $3x + 3z = 8$ and the paraboloid $z = 4x^2 + 4y^2$.

- 1) 0.436906
- 2) 5.38485
- 3) 2.92496
- 4) 8.5599
- 5) 12.8539

Exercise 5

Compute $\int_D (x^3 + y) \, dx \, dy \, dz$ for $D =$

$$\{5y^7 \leq x^2 z^4 \leq 10y^7, 5z^3 \leq x^4 y^2 \leq 7z^3, y^9 \leq x^3 z^6 \leq 4y^9, x > 0, y > 0, z > 0\}$$

- 1) -0.499622
- 2) 0.000378392
- 3) -0.399622
- 4) -0.0996216
- 5) -1.59962

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 16

Exercise 1

Compute $\int_D (2x + 2y) \, dx \, dy$ for $D = \{0 \leq x - 9y \leq 7, 0 \leq x - 3y \leq 4\}$

- 1) -8.4
- 2) 28.
- 3) 6.53333
- 4) 15.8667
- 5) 9.33333

Exercise 2

Compute $\int_D (2xy) \, dx \, dy$ for $D = \{3y^3 \leq x^4 \leq 11y^3, 3y^8 \leq x^{11} \leq 7y^8, x > 0, y > 0\}$

- 1) -1.4
- 2) 1.4
- 3) -1.8
- 4) -1.1
- 5) 1.88747×10^{-9}

Exercise 3

Compute the volume of $D = \{11(x^2 + y^2) \leq z \leq 225 - x^2 - y^2\}$

- 1) -662.68
- 2) -2650.72
- 3) 11265.6
- 4) 7289.48
- 5) 6626.8

Exercise 4

Compute the volume of the domain limited by the plane $9x + 2z = 3$ and the paraboloid $z = 8x^2 + 8y^2$.

- 1) 1.06499
- 2) 2.78747
- 3) 0.893172
- 4) 0.747449
- 5) 0.65224

Exercise 5

Compute $\int_D (z + z^2) \, dx \, dy \, dz$ for $D =$

$$\{9y^9 \leq xz \leq 15y^9, 5x^3 \leq y^4z \leq 10x^3, 6x^2z^2 \leq y^8 \leq 12x^2z^2, x > 0, y > 0, z > 0\}$$

- 1) 0.500014
- 2) -1.89999
- 3) 0.00001422
- 4) 0.400014
- 5) -0.299986

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 17

Exercise 1

Compute $\int_D (2x) \, dx \, dy$ for $D = \{0 \leq 9x - y \leq 5, 0 \leq -x - 4y \leq 8\}$

- 1) -0.0493791
- 2) -0.949379
- 3) 0.350621
- 4) 1.15062
- 5) -1.24938

Exercise 2

Compute $\int_D (4x) \, dx \, dy$ for $D = \{2y^7 \leq x^2 \leq 10y^7, 3y^3 \leq x \leq 11y^3, x > 0, y > 0\}$

- 1) -1.28206×10^{13}
- 2) 1.28206×10^{13}
- 3) -2.56413×10^{12}
- 4) 2.56413×10^{12}
- 5) 1.79489×10^{13}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 25, z \geq 11\sqrt{x^2 + y^2}\}$

- 1) 1.07516
- 2) 1.3977
- 3) 0.96764
- 4) 0.860125
- 5) -1.07516

Exercise 4

Compute the volume of the domain limited by the plane $5x + 9z = 5$ and the paraboloid $z = 2x^2 + 2y^2$.

- 1) 0.598749
- 2) 0.559584
- 3) 0.277243
- 4) 0.154928
- 5) 0.242972

Exercise 5

Compute $\int_D (2xy^2) \, dx \, dy \, dz$ for $D =$

$$\{4y^5 \leq x^9 z^5 \leq 13y^5, 3x^5 z \leq y^5 \leq 5x^5 z, 7x^9 \leq y^2 z^4 \leq 13x^9, x > 0, y > 0, z > 0\}$$

- 1) -1.9473
- 2) 1.1527
- 3) -1.3473
- 4) 0.0527045
- 5) -0.347295

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 18

Exercise 1

Compute $\int_D (4y) \, dx \, dy$ for $D = \{0 \leq -7x - 9y \leq 1, 0 \leq -6x - 5y \leq 3\}$

- 1) 1.34931
- 2) 2.24931
- 3) 1.74931
- 4) 0.249307
- 5) -1.15069

Exercise 2

Compute $\int_D (3xy) \, dx \, dy$ for $D = \{5x^{11} \leq y^{29} \leq 8x^{11}, 8y^8 \leq x^3 \leq 9y^8, x > 0, y > 0\}$

- 1) 1.3
- 2) 4.04595×10^{-91}
- 3) -2.
- 4) 1.1
- 5) -1.9

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 36, z \geq 9(x^2 + y^2)\}$

- 1) 9.33805
- 2) 18.6761
- 3) 5.60283
- 4) 8.71551
- 5) 6.22537

Exercise 4

Compute the volume of the domain limited by the plane $2x + 7z = 3$ and the paraboloid $z = x^2 + y^2$.

- 1) 1.32193
- 2) 0.848821
- 3) 0.36858
- 4) 0.265681
- 5) 0.316645

Exercise 5

Compute $\int_D (x + y) \, dx \, dy \, dz$ for $D =$

$$\{9x^7y^9z^6 \leq 1 \leq 16x^7y^9z^6, 2x^5z^8 \leq y^2 \leq 9x^5z^8, 9x^3y^4 \leq 1 \leq 11x^3y^4, x > 0, y > 0, z > 0\}$$

- 1) -0.498688
- 2) 1.50131
- 3) 0.601312
- 4) 0.701312
- 5) 0.00131229

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 19

Exercise 1

Compute $\int_D (2x + 2y) \, dx \, dy$ for $D = \{0 \leq -6y \leq 4, 0 \leq -2x + 3y \leq 6\}$

- 1) -9.23333
- 2) -7.33333
- 3) -9.33333
- 4) -9.13333
- 5) -9.93333

Exercise 2

Compute $\int_D (2x^2) \, dx \, dy$ for $D = \{8x^5y^3 \leq 1 \leq 11x^5y^3, 6x^{23}y^{14} \leq 1 \leq 13x^{23}y^{14}, x > 0, y > 0\}$

- 1) -1.
- 2) 1.2
- 3) -1.1
- 4) -0.8
- 5) 4.96692×10^{-15}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 9, z \geq 4\sqrt{x^2 + y^2}\}$

- 1) 1.6884
- 2) 3.88332
- 3) 3.03912
- 4) 2.87028
- 5) -1.18188

Exercise 4

Compute the volume of the domain limited by the plane $5x + 9z = 7$ and the paraboloid $z = 9x^2 + 9y^2$.

- 1) 0.0994897
- 2) 0.325816
- 3) 0.0979995
- 4) 0.125799
- 5) 0.107922

Exercise 5

Compute $\int_D (2x^3) \, dx \, dy \, dz$ for $D =$

$$\{7y^6 z^2 \leq 1 \leq 9y^6 z^2, 6y^3 \leq x^3 z^7 \leq 13y^3, 6x^7 z^5 \leq y^3 \leq 10x^7 z^5, x > 0, y > 0, z > 0\}$$

- 1) 0.0000306391
- 2) 1.60003
- 3) -1.59997
- 4) 0.700031
- 5) 2.00003

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 20

Exercise 1

Compute $\int_D (3x + 2y) \, dx \, dy$ for $D = \{0 \leq 5x + 4y \leq 3, 0 \leq -3x - 7y \leq 7\}$

- 1) -0.351323
- 2) -0.468431
- 3) 0.702647
- 4) 2.57637
- 5) 1.17108

Exercise 2

Compute $\int_D (3x + y^3) \, dx \, dy$ for $D = \{7x^3y^7 \leq 1 \leq 13x^3y^7, 9x^2y^5 \leq 1 \leq 15x^2y^5, x > 0, y > 0\}$

- 1) -25302.9
- 2) 116393.
- 3) 25302.9
- 4) 50605.7
- 5) 5060.57

Exercise 3

Compute the volume of $D = \{7(x^2 + y^2) \leq z \leq 144 - x^2 - y^2\}$

- 1) 1221.45
- 2) 2850.05
- 3) -1628.6
- 4) 4071.5
- 5) 10178.8

Exercise 4

Compute the volume of the domain limited by the plane $10x + z = 3$ and the paraboloid $z = 9x^2 + 9y^2$.

- 1) 3.46753
- 2) 5.82638
- 3) 20.1379
- 4) 13.4544
- 5) 12.5462

Exercise 5

Compute $\int_D (z + z^2) dx dy dz$ for $D =$

$$\{9z^3 \leq x^5 y^4 \leq 15z^3, x^7 y^4 z^2 \leq 1 \leq 10x^7 y^4 z^2, 2x^4 z^3 \leq y^8 \leq 6x^4 z^3, x > 0, y > 0, z > 0\}$$

- 1) -1.29851
- 2) 0.901493
- 3) 0.00149291
- 4) -1.59851
- 5) -1.59851

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 21

Exercise 1

Compute $\int_D (3x + y) \, dx \, dy$ for $D = \{0 \leq 4x + 2y \leq 3, 0 \leq -8x - 7y \leq 8\}$

- 1) -1.83333
- 2) 4.58333
- 3) -2.75
- 4) 1.375
- 5) 12.375

Exercise 2

Compute $\int_D (2y^2) \, dx \, dy$ for $D = \{6y^3 \leq x \leq 13y^3, 6x^2 \leq y^7 \leq 10x^2, x > 0, y > 0\}$

- 1) 7.40338×10^{18}
- 2) 1.55471×10^{19}
- 3) 1.25857×10^{19}
- 4) -1.48068×10^{18}
- 5) -7.40338×10^{18}

Exercise 3

Compute the volume of $D = \{6(x^2 + y^2) \leq z \leq 196 - x^2 - y^2\}$

- 1) -6034.37
- 2) 18965.2
- 3) 8620.53
- 4) 862.053
- 5) -6896.42

Exercise 4

Compute the volume of the domain limited by the plane $6x + 7z = 2$ and the paraboloid $z = 2x^2 + 2y^2$.

- 1) 0.505158
- 2) 0.100331
- 3) 0.235299
- 4) 0.111954
- 5) 0.100045

Exercise 5

Compute $\int_D (x + z^2) \, dx \, dy \, dz$ for $D = \{z^8 \leq x^4 y^2 \leq 10 z^8, 4 x^7 \leq y^8 z \leq 8 x^7, 4 \leq x y^4 \leq 9, x > 0, y > 0, z > 0\}$

1) -1.67724

2) 0.72276

3) -1.57724

4) 0.0227604

5) -0.97724

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 22

Exercise 1

Compute $\int_D (3y) \, dx \, dy$ for $D = \{0 \leq -2x + y \leq 7, 0 \leq -9x + 5y \leq 8\}$

- 1) -3949.5
- 2) -3949.
- 3) -3947.7
- 4) -3948.
- 5) -3946.

Exercise 2

Compute $\int_D (2xy) \, dx \, dy$ for $D = \{8 \leq x^5 y^{11} \leq 14, 5 \leq x^4 y^9 \leq 10, x > 0, y > 0\}$

- 1) -5.9005
- 2) 23.602
- 3) -7.86734
- 4) 19.6683
- 5) -7.86734

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 36, z \geq 5\sqrt{x^2 + y^2}\}$

- 1) 17.5702
- 2) 24.5983
- 3) 5.27106
- 4) -1.75702
- 5) 8.7851

Exercise 4

Compute the volume of the domain limited by the plane $6x + 5z = 9$ and the paraboloid $z = 4x^2 + 4y^2$.

- 1) 6.33043
- 2) 5.52691
- 3) 3.71432
- 4) 1.40276
- 5) 0.689872

Exercise 5

Compute $\int_D (x + y^3) \, dx \, dy \, dz$ for $D = \{8x^2z^9 \leq y \leq 15x^2z^9, 6 \leq x^5yz^3 \leq 7, x^3 \leq y^6z^2 \leq 5x^3, x > 0, y > 0, z > 0\}$

- 1) -0.793952
- 2) 0.406048
- 3) -0.593952
- 4) 0.00604784
- 5) 0.106048

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 23

Exercise 1

Compute $\int_D (5y) \, dx \, dy$ for $D = \{0 \leq 5x + y \leq 5, 0 \leq -x - 2y \leq 2\}$

- 1) -2.62963
- 2) -2.62963
- 3) -3.72963
- 4) -2.72963
- 5) -4.62963

Exercise 2

Compute $\int_D (x^3 + y) \, dx \, dy$ for $D = \{2x^2y \leq 1 \leq 9x^2y, 7x^3y^2 \leq 1 \leq 15x^3y^2, x > 0, y > 0\}$

- 1) 1.97823
- 2) -0.791293
- 3) 3.363
- 4) 0.197823
- 5) 3.16517

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 49, z \geq 11(x^2 + y^2)\}$

- 1) 20.8558
- 2) 7.64714
- 3) 13.9039
- 4) 14.5991
- 5) 6.95194

Exercise 4

Compute the volume of the domain limited by the plane $7x + 6z = 3$ and the paraboloid $z = x^2 + y^2$.

- 1) 1.99596
- 2) 0.994604
- 3) 0.493906
- 4) 1.10909
- 5) 3.98194

Exercise 5

Compute $\int_D (2y + 2z) \, dx \, dy \, dz$ for $D =$

$$\{8z \leq x^5 y \leq 13z, 6x^2 z^3 \leq y^3 \leq 11x^2 z^3, 4x^7 z^2 \leq y^8 \leq 13x^7 z^2, x > 0, y > 0, z > 0\}$$

- 1) 0.0162968
- 2) 0.116297
- 3) 0.516297
- 4) 1.2163
- 5) 0.916297

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 24

Exercise 1

Compute $\int_D (2y) \, dx \, dy$ for $D = \{0 \leq 5x + 7y \leq 9, 0 \leq 6x - 9y \leq 6\}$

- 1) -1.02878
- 2) 0.771225
- 3) 0.171225
- 4) 2.17122
- 5) 1.47122

Exercise 2

Compute $\int_D (6x^2) \, dx \, dy$ for $D = \{8 \leq x^5 y^2 \leq 17, 3 \leq x^{12} y^5 \leq 11, x > 0, y > 0\}$

- 1) 2133.82
- 2) 213.382
- 3) 4481.02
- 4) 3627.49
- 5) 640.145

Exercise 3

Compute the volume of $D = \{12(x^2 + y^2) \leq z \leq 100 - x^2 - y^2\}$

- 1) 1208.3
- 2) 3141.59
- 3) 3624.91
- 4) 3262.42
- 5) -604.152

Exercise 4

Compute the volume of the domain limited by the plane $7x + 10z = 4$ and the paraboloid $z = x^2 + y^2$.

- 1) 1.06857
- 2) 0.428837
- 3) 0.30129
- 4) 0.187564
- 5) 1.22306

Exercise 5

Compute $\int_D (3x + 3z) \, dx \, dy \, dz$ for $D =$

$$\{1 \leq x^5 y^7 z^6 \leq 3, 4x^9 y^5 z^4 \leq 1 \leq 6x^9 y^5 z^4, 8z^4 \leq x^2 y^3 \leq 16z^4, x > 0, y > 0, z > 0\}$$

- 1) 0.00482577
- 2) 1.40483
- 3) 1.70483
- 4) 1.00483
- 5) -1.39517

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 25

Exercise 1

Compute $\int_D (3x + 2y) \, dx \, dy$ for $D = \{0 \leq 5x - 7y \leq 8, 0 \leq 2x - 6y \leq 6\}$

- 1) -1.8375
- 2) -0.2375
- 3) -0.9375
- 4) -1.9375
- 5) -2.1375

Exercise 2

Compute $\int_D (x + y^3) \, dx \, dy$ for $D = \{7y^3 \leq x^{13} \leq 13y^3, y^5 \leq x^{22} \leq 7y^5, x > 0, y > 0\}$

- 1) 0.2
- 2) -1.9
- 3) 0.8
- 4) 1.69755×10^{-14}
- 5) 1.5

Exercise 3

Compute the volume of $D = \{13(x^2 + y^2) \leq z \leq 64 - x^2 - y^2\}$

- 1) 321.699
- 2) 643.398
- 3) -229.785
- 4) -91.914
- 5) 459.57

Exercise 4

Compute the volume of the domain limited by the plane $10x + 4z = 7$ and the paraboloid $z = 2x^2 + 2y^2$.

- 1) 16.3057
- 2) 8.97951
- 3) 0.388098
- 4) 2.41741
- 5) 5.03222

Exercise 5

Compute $\int_D (x^2 z) \, dx \, dy \, dz$ for $D = \{7y^7 \leq x^5 z^2 \leq 13y^7, 4x^6 \leq yz^4 \leq 13x^6, 9x \leq y^2 \leq 15x, x > 0, y > 0, z > 0\}$

- 1) 114 602.
- 2) 38 200.6
- 3) 725 811.
- 4) 382 006.
- 5) 0.

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 26

Exercise 1

Compute $\int_D (4y) \, dx \, dy$ for $D = \{0 \leq x + 2y \leq 4, 0 \leq -4x + 2y \leq 8\}$

- 1) -9.216
- 2) -13.824
- 3) 13.824
- 4) 43.008
- 5) 15.36

Exercise 2

Compute $\int_D (2x^2y) \, dx \, dy$ for $D = \{8x^3 \leq y^4 \leq 16x^3, 2x^7 \leq y^9 \leq 11x^7, x > 0, y > 0\}$

- 1) 6.52705×10^{41}
- 2) 2.41742×10^{41}
- 3) 5.56008×10^{41}
- 4) 5.80182×10^{41}
- 5) -2.41742×10^{41}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 36, z \geq 6\sqrt{x^2 + y^2}\}$

- 1) -1.23105
- 2) 2.4621
- 3) 6.15524
- 4) -3.07762
- 5) 13.5415

Exercise 4

Compute the volume of the domain limited by the plane $6x + 6z = 8$ and the paraboloid $z = 8x^2 + 8y^2$.

- 1) 0.929668
- 2) 0.36562
- 3) 0.163712
- 4) 0.17994
- 5) 0.217349

Exercise 5

Compute $\int_D (y^3 + 2z) \, dx \, dy \, dz$ for $D =$

$$\{3z^4 \leq xy^2 \leq 9z^4, 7z^3 \leq x^2y^5 \leq 14z^3, 8y^5z^6 \leq 1 \leq 16y^5z^6, x > 0, y > 0, z > 0\}$$

- 1) 0.312466
- 2) 0.812466
- 3) -1.08753
- 4) -1.38753
- 5) -1.18753

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 27

Exercise 1

Compute $\int_D (2y) \, dx \, dy$ for $D = \{0 \leq -2x - 8y \leq 2, 0 \leq -2y \leq 9\}$

- 1) -19.25
- 2) -18.65
- 3) -18.55
- 4) -20.25
- 5) -21.75

Exercise 2

Compute $\int_D (x y^2) \, dx \, dy$ for $D = \{9y^2 \leq x^5 \leq 13y^2, y^5 \leq x^{13} \leq 9y^5, x > 0, y > 0\}$

- 1) 1.4
- 2) -1.5
- 3) 1.5
- 4) 2.53382×10^{-32}
- 5) 0.8

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 225, z \geq 2(x^2 + y^2)\}$

- 1) 208.562
- 2) 173.802
- 3) 34.7604
- 4) 504.025
- 5) 347.604

Exercise 4

Compute the volume of the domain limited by the plane $4x + 9z = 7$ and the paraboloid $z = 10x^2 + 10y^2$.

- 1) 0.138852
- 2) 0.096234
- 3) 0.425928
- 4) 0.0877669
- 5) 0.0953827

Exercise 5

Compute $\int_D (x^3 y) \, dx \, dy \, dz$ for $D =$

$$\{5x^2y^8 \leq 1 \leq 9x^2y^8, 4x^2z^8 \leq y^9 \leq 11x^2z^8, 3z^8 \leq x^7y^9 \leq 7z^8, x > 0, y > 0, z > 0\}$$

- 1) 0.000294743
- 2) 0.900295
- 3) 1.30029
- 4) 1.90029
- 5) -1.49971

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 28

Exercise 1

Compute $\int_D (3x) \, dx \, dy$ for $D = \{0 \leq 7x - y \leq 9, 0 \leq -5x - 4y \leq 9\}$

- 1) 7.53099
- 2) -2.40992
- 3) 2.40992
- 4) 3.0124
- 5) 2.71116

Exercise 2

Compute $\int_D (x + y) \, dx \, dy$ for $D = \{2 \leq x y^2 \leq 4, 5 x^4 y^7 \leq 1 \leq 11 x^4 y^7, x > 0, y > 0\}$

- 1) 1.57691
- 2) 3.6269
- 3) 3.46921
- 4) -0.473074
- 5) 4.25767

Exercise 3

Compute the volume of $D = \{13(x^2 + y^2) \leq z \leq 144 - x^2 - y^2\}$

- 1) 1163.29
- 2) -1628.6
- 3) 232.657
- 4) 2326.57
- 5) 6979.72

Exercise 4

Compute the volume of the domain limited by the plane $3x + z = 8$ and the paraboloid $z = 3x^2 + 3y^2$.

- 1) 64.5424
- 2) 40.088
- 3) 7.06703
- 4) 168.423
- 5) 184.424

Exercise 5

Compute $\int_D (3y + z) \, dx \, dy \, dz$ for $D =$

$$\{5y z^5 \leq x^7 \leq 11y z^5, 6x^4 y^5 \leq z^6 \leq 8x^4 y^5, 6y^5 z^9 \leq x \leq 14y^5 z^9, x > 0, y > 0, z > 0\}$$

- 1) 0.101069
- 2) -0.798931
- 3) -1.89893
- 4) -0.398931
- 5) 0.00106868

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Exam - 2 - Multivariate Integration for serial number: 29

Exercise 1

Compute $\int_D (5y) \, dx \, dy$ for $D = \{0 \leq 9y \leq 8, 0 \leq -6x - 9y \leq 5\}$

- 1) -0.164609
- 2) -1.15226
- 3) 1.64609
- 4) 1.97531
- 5) -0.164609

Exercise 2

Compute $\int_D (y^3) \, dx \, dy$ for $D = \{5 \leq xy^3 \leq 12, 2 \leq xy^4 \leq 8, x > 0, y > 0\}$

- 1) 5.25281
- 2) -1.57584
- 3) 5.77809
- 4) 6.82866
- 5) 6.82866

Exercise 3

Compute the volume of $D = \{10(x^2 + y^2) \leq z \leq 100 - x^2 - y^2\}$

- 1) -856.798
- 2) 3141.59
- 3) 1142.4
- 4) -999.598
- 5) 1428.

Exercise 4

Compute the volume of the domain limited by the plane $7x + 5z = 4$ and the paraboloid $z = 9x^2 + 9y^2$.

- 1) 0.450988
- 2) 0.103523
- 3) 0.113826
- 4) 0.107493
- 5) 0.127422

Exercise 5

Compute $\int_D (3xz) \, dx \, dy \, dz$ for $D = \{z^6 \leq y^9 \leq 10z^6, 5x^9z^3 \leq y^6 \leq 13x^9z^3, 1 \leq xy^7z \leq 3, x > 0, y > 0, z > 0\}$

- 1) 1.51212
- 2) 0.012123
- 3) -1.88788
- 4) -1.28788
- 5) -0.887877

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 30

Exercise 1

Compute $\int_D (3x + y) \, dx \, dy$ for $D = \{0 \leq 4x - 9y \leq 1, 0 \leq -9x - 3y \leq 1\}$

- 1) 0.898208
- 2) -1.90179
- 3) 1.69821
- 4) -0.00179211
- 5) -1.90179

Exercise 2

Compute $\int_D (xy) \, dx \, dy$ for $D = \{x^{14} \leq y^{37} \leq 8x^{14}, 8y^8 \leq x^3 \leq 17y^8, x > 0, y > 0\}$

- 1) 3.41821×10^{-96}
- 2) -0.9
- 3) -0.5
- 4) -1.
- 5) -1.

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 121, z \geq \sqrt{x^2 + y^2}\}$

- 1) 816.481
- 2) -81.6481
- 3) 1388.02
- 4) 898.129
- 5) -816.481

Exercise 4

Compute the volume of the domain limited by the plane $7x + 9z = 1$ and the paraboloid $z = 2x^2 + 2y^2$.

- 1) 0.110959
- 2) 0.0273849
- 3) 0.0514921
- 4) 0.0924135
- 5) 0.0668905

Exercise 5

Compute $\int_D (4xy) \, dx \, dy \, dz$ for $D =$

$$\{3y^9z^9 \leq x^4 \leq 7y^9z^9, 5x^3z^9 \leq y \leq 7x^3z^9, 9z \leq x^6y^7 \leq 17z, x > 0, y > 0, z > 0\}$$

- 1) -1.79885
- 2) 0.801146
- 3) 0.00114603
- 4) -0.198854
- 5) 1.70115

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 31

Exercise 1

Compute $\int_D (5x) \, dx \, dy$ for $D = \{0 \leq 4x - 2y \leq 9, 0 \leq -8x - 7y \leq 3\}$

- 1) -1.39114
- 2) 1.39114
- 3) 1.98735
- 4) 0.596204
- 5) 1.19241

Exercise 2

Compute $\int_D (y + y^2) \, dx \, dy$ for $D = \{9 \leq x^7 y^3 \leq 11, 9x^{26} y^{11} \leq 1 \leq 12x^{26} y^{11}, x > 0, y > 0\}$

- 1) 3.51668×10^{86}
- 2) 1.69322×10^{86}
- 3) 1.30247×10^{86}
- 4) 3.90742×10^{85}
- 5) 1.82346×10^{86}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 144, z \geq 5\sqrt{x^2 + y^2}\}$

- 1) 70.2808
- 2) 7.02808
- 3) 56.2246
- 4) 14.0562
- 5) 49.1965

Exercise 4

Compute the volume of the domain limited by the plane $7x + 4z = 3$ and the paraboloid $z = 8x^2 + 8y^2$.

- 1) 0.686373
- 2) 0.392711
- 3) 0.107817
- 4) 0.140432
- 5) 0.101386

Exercise 5

Compute $\int_D (z + z^2) \, dx \, dy \, dz$ for $D =$

$$\{9y^2z^8 \leq x^4 \leq 14y^2z^8, 6x^7y \leq z^7 \leq 10x^7y, 2y^7z^6 \leq x^2 \leq 3y^7z^6, x > 0, y > 0, z > 0\}$$

- 1) -0.599971
- 2) -0.999971
- 3) 0.0000288205
- 4) -1.59997
- 5) -1.09997

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 32

Exercise 1

Compute $\int_D (2x + y) \, dx \, dy$ for $D = \{0 \leq 7x - 2y \leq 3, 0 \leq -9x + 2y \leq 6\}$

- 1) -59.0625
- 2) -57.7625
- 3) -57.0625
- 4) -61.0625
- 5) -58.0625

Exercise 2

Compute $\int_D (2x + x^3) \, dx \, dy$ for $D = \{3 \leq x y^3 \leq 9, 2x^6 y^{17} \leq 1 \leq 11x^6 y^{17}, x > 0, y > 0\}$

- 1) 2.
- 2) 1.95106×10^{-17}
- 3) 0.3
- 4) -0.7
- 5) -0.8

Exercise 3

Compute the volume of $D = \{11(x^2 + y^2) \leq z \leq 4 - x^2 - y^2\}$

- 1) 1.0472
- 2) 2.30383
- 3) 4.60767
- 4) 3.76991
- 5) 2.0944

Exercise 4

Compute the volume of the domain limited by the plane $9x + 7z = 9$ and the paraboloid $z = x^2 + y^2$.

- 1) 0.984678
- 2) 4.53415
- 3) 0.428633
- 4) 10.7769
- 5) 11.1903

Exercise 5

Compute $\int_D (3z^2) \, dx \, dy \, dz$ for $D =$

$$\{9y^2z^9 \leq x^7 \leq 17y^2z^9, 5y^5 \leq x^2z^9 \leq 11y^5, x^3y^5 \leq z^7 \leq 7x^3y^5, x > 0, y > 0, z > 0\}$$

- 1) -0.593411
- 2) 2.00659
- 3) 1.80659
- 4) 0.706589
- 5) 0.00658944

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 33

Exercise 1

Compute $\int_D (2x) \, dx \, dy$ for $D = \{0 \leq 9x + 6y \leq 6, 0 \leq -3x - 3y \leq 4\}$

- 1) 17.4222
- 2) 37.3333
- 3) 16.1778
- 4) 8.71111
- 5) 12.4444

Exercise 2

Compute $\int_D (x + 3y) \, dx \, dy$ for $D = \{3xy \leq 1 \leq 9xy, 5 \leq x^2y \leq 12, x > 0, y > 0\}$

- 1) 11.5417
- 2) 7.69445
- 3) 20.775
- 4) 13.85
- 5) -1.53889

Exercise 3

Compute the volume of $D = \{4(x^2 + y^2) \leq z \leq 25 - x^2 - y^2\}$

- 1) -196.35
- 2) 392.699
- 3) 196.35
- 4) 215.984
- 5) 294.524

Exercise 4

Compute the volume of the domain limited by the plane $10x + 4z = 4$ and the paraboloid $z = 4x^2 + 4y^2$.

- 1) 3.07078
- 2) 0.759416
- 3) 2.9209
- 4) 0.454975
- 5) 3.09073

Exercise 5

Compute $\int_D (x + z^3) \, dx \, dy \, dz$ for $D = \{z^3 \leq x^9 y^6 \leq 6z^3, 7x^3 y^9 \leq 1 \leq 16x^3 y^9, 8 \leq x y^5 z^3 \leq 9, x > 0, y > 0, z > 0\}$

- 1) 1.95992
- 2) -0.440082
- 3) 1.35992
- 4) 0.359918
- 5) 2.05992

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 34

Exercise 1

Compute $\int_D (4y) \, dx \, dy$ for $D = \{0 \leq -3x - 4y \leq 3, 0 \leq 3y \leq 2\}$

- 1) 0.288889
- 2) 1.18889
- 3) 0.688889
- 4) -0.411111
- 5) 0.888889

Exercise 2

Compute $\int_D (x + y^3) \, dx \, dy$ for $D = \{3y^2 \leq x^9 \leq 5y^2, x^{13} \leq y^3 \leq 2x^{13}, x > 0, y > 0\}$

- 1) 3.65043×10^{46}
- 2) -1.46017×10^{46}
- 3) 9.85616×10^{46}
- 4) 4.38051×10^{46}
- 5) 3.65043×10^{45}

Exercise 3

Compute the volume of $D = \{7(x^2 + y^2) \leq z \leq 81 - x^2 - y^2\}$

- 1) 1288.25
- 2) 0.
- 3) -386.475
- 4) 2962.97
- 5) 515.3

Exercise 4

Compute the volume of the domain limited by the plane $2x + 9z = 8$ and the paraboloid $z = 5x^2 + 5y^2$.

- 1) 0.187498
- 2) 0.792649
- 3) 0.153686
- 4) 0.249606
- 5) 0.750939

Exercise 5

Compute $\int_D (x + y) \, dx \, dy \, dz$ for $D =$

$$\{3x^3y^4 \leq 1 \leq 5x^3y^4, z^7 \leq x^3y^3 \leq 4z^7, x^7y^5 \leq z^2 \leq 8x^7y^5, x > 0, y > 0, z > 0\}$$

- 1) -0.282325
- 2) 0.0176754
- 3) -1.88232
- 4) 1.31768
- 5) -1.28232

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 35

Exercise 1

Compute $\int_D (2x + 2y) \, dx \, dy$ for $D = \{0 \leq -9x + 2y \leq 9, 0 \leq 6x + 7y \leq 2\}$

- 1) 1.4416
- 2) 0.0416
- 3) 0.4416
- 4) 0.5416
- 5) -1.5584

Exercise 2

Compute $\int_D (x^2 + 3y) \, dx \, dy$ for $D = \{9x^7 \leq y^9 \leq 11x^7, 6x^4 \leq y^5 \leq 13x^4, x > 0, y > 0\}$

- 1) 0.7
- 2) 1.3
- 3) -1.
- 4) 0.8
- 5) 4.06637×10^{-7}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 1, z \geq 12(x^2 + y^2)\}$

- 1) 0.125595
- 2) -0.374405
- 3) 1.72559
- 4) 1.82559
- 5) 1.52559

Exercise 4

Compute the volume of the domain limited by the plane $x + 3z = 1$ and the paraboloid $z = 8x^2 + 8y^2$.

- 1) 0.0649569
- 2) 0.0255218
- 3) 0.0222735
- 4) 0.0675239
- 5) 0.0264655

Exercise 5

Compute $\int_D (3x + 2y) \, dx \, dy \, dz$ for $D =$

$$\{4x^9 \leq y^2 z^6 \leq 12x^9, 5y^5 \leq x^7 z^4 \leq 9y^5, 8x^6 \leq y^4 z^5 \leq 13x^6, x > 0, y > 0, z > 0\}$$

- 1) 1.51844
- 2) 0.818435
- 3) 0.0184351
- 4) -1.48156
- 5) 0.618435

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Exam - 2 - Multivariate Integration for serial number: 36

Exercise 1

Compute $\int_D (3x + 2y) \, dx \, dy$ for $D = \{0 \leq 4x - 8y \leq 5, 0 \leq -x - 3y \leq 3\}$

- 1) 0.15625
- 2) 0.55625
- 3) -0.34375
- 4) -1.14375
- 5) -1.74375

Exercise 2

Compute $\int_D (x + y) \, dx \, dy$ for $D = \{2x^9 \leq y^{13} \leq 7x^9, 3x^7 \leq y^{10} \leq 10x^7, x > 0, y > 0\}$

- 1) 2529.45
- 2) -361.35
- 3) 1806.75
- 4) 5239.58
- 5) -542.025

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 144, z \geq 10\sqrt{x^2 + y^2}\}$

- 1) 34.1259
- 2) 17.961
- 3) 21.5532
- 4) 5.38829
- 5) 43.1063

Exercise 4

Compute the volume of the domain limited by the plane $7x + 5z = 4$ and the paraboloid $z = 5x^2 + 5y^2$.

- 1) 0.148419
- 2) 0.8908
- 3) 0.253339
- 4) 1.01065
- 5) 0.904137

Exercise 5

Compute $\int_D (x^3 + 2y) \, dx \, dy \, dz$ for $D =$

$$\{9y^8z \leq x^4 \leq 13y^8z, 8x^3z \leq y^4 \leq 14x^3z, 6z^6 \leq y^7 \leq 13z^6, x > 0, y > 0, z > 0\}$$

- 1) 0.400034
- 2) 0.300034
- 3) -0.0999655
- 4) 0.600034
- 5) 0.0000344699

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 37

Exercise 1

Compute $\int_D (4y) \, dx \, dy$ for $D = \{0 \leq 6x - 6y \leq 1, 0 \leq -6x + y \leq 9\}$

- 1) 0.6
- 2) 0.5
- 3) -0.5
- 4) -2.5
- 5) -1.2

Exercise 2

Compute $\int_D (xy) \, dx \, dy$ for $D = \{5y^3 \leq x^4 \leq 8y^3, 4x^5 \leq y^4 \leq 8x^5, x > 0, y > 0\}$

- 1) -2.51449×10^{26}
- 2) 3.14312×10^{26}
- 3) 8.1721×10^{26}
- 4) -3.14312×10^{26}
- 5) 8.48642×10^{26}

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 36, z \geq 2\sqrt{x^2 + y^2}\}$

- 1) 133.728
- 2) -9.552
- 3) 90.744
- 4) 90.744
- 5) 47.76

Exercise 4

Compute the volume of the domain limited by the plane $8x + 5z = 8$ and the paraboloid $z = 2x^2 + 2y^2$.

- 1) 5.89344
- 2) 10.1688
- 3) 1.19194
- 4) 2.89529
- 5) 2.57971

Exercise 5

Compute $\int_D (4y) \, dx \, dy \, dz$ for $D = \{8y^6 z^6 \leq x^6 \leq 14y^6 z^6, 4 \leq x y^7 z^7 \leq 6, z^8 \leq x^3 y^8 \leq 10z^8, x > 0, y > 0, z > 0\}$

- 1) 0.805696
- 2) 1.1057
- 3) 1.1057
- 4) 0.00569566
- 5) -1.0943

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 38

Exercise 1

Compute $\int_D (2x + y) \, dx \, dy$ for $D = \{0 \leq y \leq 7, 0 \leq -6x + 4y \leq 5\}$

- 1) 17.1111
- 2) 115.5
- 3) 64.1667
- 4) -4.27778
- 5) 42.7778

Exercise 2

Compute $\int_D (y^2 + y^3) \, dx \, dy$ for $D = \{4y \leq x \leq 10y, 3y^2 \leq x^3 \leq 7y^2, x > 0, y > 0\}$

- 1) 1.20001
- 2) 1.80001
- 3) 0.0000134521
- 4) -1.59999
- 5) -1.69999

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 9, z \geq 11\sqrt{x^2 + y^2}\}$

- 1) -0.167766
- 2) -1.16777
- 3) 2.23223
- 4) 0.232234
- 5) 0.632234

Exercise 4

Compute the volume of the domain limited by the plane $5x + 7z = 4$ and the paraboloid $z = 7x^2 + 7y^2$.

- 1) 0.0780208
- 2) 0.0977944
- 3) 0.378573
- 4) 0.0706353
- 5) 0.1493

Exercise 5

Compute $\int_D (2xz^2) \, dx \, dy \, dz$ for $D = \{z^6 \leq y \leq 10z^6, 8y^6 \leq x^4z^2 \leq 16y^6, 9z^2 \leq xy^2 \leq 13z^2, x > 0, y > 0, z > 0\}$

- 1) 0.271671
- 2) 1.87167
- 3) -0.0283288
- 4) -1.02833
- 5) 2.07167

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 39

Exercise 1

Compute $\int_D (4y) \, dx \, dy$ for $D = \{0 \leq -9y \leq 5, 0 \leq -9x \leq 5\}$

- 1) 0.557064
- 2) -0.342936
- 3) 0.857064
- 4) -0.442936
- 5) 0.157064

Exercise 2

Compute $\int_D (2xy) \, dx \, dy$ for $D = \{2 \leq x^{10} y^{23} \leq 6, 7x^{17} y^{39} \leq 1 \leq 10x^{17} y^{39}, x > 0, y > 0\}$

- 1) 1.05851×10^{-38}
- 2) 1.1
- 3) -0.5
- 4) 0.3
- 5) -1.7

Exercise 3

Compute the volume of $D = \{x^2 + y^2 + z^2 \leq 16, z \geq \sqrt{x^2 + y^2}\}$

- 1) 39.2598
- 2) 106.001
- 3) 58.8897
- 4) 102.075
- 5) -3.92598

Exercise 4

Compute the volume of the domain limited by the plane $5x + 2z = 2$ and the paraboloid $z = 5x^2 + 5y^2$.

- 1) 0.796481
- 2) 0.541188
- 3) 0.380054
- 4) 1.95457
- 5) 0.217419

Exercise 5

Compute $\int_D (x^3 + z) \, dx \, dy \, dz$ for $D =$

$$\{2y^9 z^9 \leq 1 \leq 10y^9 z^9, 6y \leq z^8 \leq 7y, 2y^9 z \leq x \leq 11y^9 z, x > 0, y > 0, z > 0\}$$

- 1) 0.00173939
- 2) 0.601739
- 3) -0.998261
- 4) 1.10174
- 5) 1.50174

Further Mathematics - 2022/2023

Exam - 2 - Multivariate Integration for serial number: 40

Exercise 1

Compute $\int_D (2x + y) \, dx \, dy$ for $D = \{0 \leq 7x + 8y \leq 5, 0 \leq -x + 7y \leq 6\}$

- 1) 0.896953
- 2) -1.20305
- 3) -0.103047
- 4) 0.396953
- 5) 0.0969529

Exercise 2

Compute $\int_D (2x) \, dx \, dy$ for $D = \{2 \leq x^3 y^{13} \leq 6, 5xy^4 \leq 1 \leq 14xy^4, x > 0, y > 0\}$

- 1) -1.3
- 2) -0.9
- 3) -0.6
- 4) -1.7
- 5) 8.1374×10^{-21}

Exercise 3

Compute the volume of $D = \{8(x^2 + y^2) \leq z \leq 4 - x^2 - y^2\}$

- 1) 7.53982
- 2) -2.23402
- 3) 5.3058
- 4) 2.79253
- 5) 0.837758

Exercise 4

Compute the volume of the domain limited by the plane $9x + 6z = 2$ and the paraboloid $z = 5x^2 + 5y^2$.

- 1) 0.0889126
- 2) 0.0664774
- 3) 0.0624446
- 4) 0.078524
- 5) 0.189765

Exercise 5

Compute $\int_D (y + z) \, dx \, dy \, dz$ for $D =$

$$\{3z \leq x^7 y^8 \leq 11z, 8y^3 \leq x^7 z^4 \leq 16y^3, 9y^4 \leq x^9 z^3 \leq 17y^4, x > 0, y > 0, z > 0\}$$

- 1) 0.0145698
- 2) 1.51457
- 3) -1.68543
- 4) 1.71457
- 5) -0.98543