Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 1

Exercise 1

Given the functions

$f(x,y,z) = (-3 - 3y - 2xy - 2z + yz - 3z^2, 3xz, 2y^2, 2x^2 + 2z - 2xz)$

and

$g(u_1,u_2,u_3,u_4) = (-3 - 2u_3 - 2u_1u_3, -2u_1 + 2u_1^2 - 2u_2 - 3u_1u_2 + 2u_1u_3 + u_1u_4 + 3u_2u_4, -3u_1^2 - u_3 + u_1u_3 + u_2u_4 - 3u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0,-1,3)$.

1) -3.639×10^9

2) -2.74136×10^9

3) -2.36375×10^9

4) -2.92134×10^9

5) -1.92035×10^9

Exercise 2

Given the system

$2xy^2 == 128$
$-2vy^2 + y^2z == 48$
$2uvz - vxz == -280$

determine if it is possible to solve for variables x, y,z in terms of variables u,v,w arround the point $p = (x,y,z,u,v,w) = (4,4,-5,-5,-4,1)$. Compute if possible $\frac{\partial z}{\partial w}(-5,-4,1)$.

1) $\frac{\partial z}{\partial w}(-5,-4,1) = 0$

2) $\frac{\partial z}{\partial w}(-5,-4,1) = 3$

3) $\frac{\partial z}{\partial w}(-5,-4,1) = 1$

4) $\frac{\partial z}{\partial w}(-5,-4,1) = 4$

5) $\frac{\partial z}{\partial w}(-5,-4,1) = 2$

Exercise 3

Given the function

$f(x,y,z) = 1 - x^2 + 2y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-4.41132, -0.9625, 0.1}

2) We have a minimum at {-4.91132, -0.5625, 0.}

3) We have a minimum at {-4.71132, -0.9625, -0.3}

4) We have a minimum at {-4.61132, -0.6625, 0.2}

5) We have a minimum at {0, 1, 0}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{3\left(x^3 + y^3\right)}{3x + 3x^2 + 4x^3 - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 4x^3 - 2y^3$ defined over the domain D≡ $30x^2 + 9y^2 \leq 831$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.9****

3) The value of the minimum is ****.4****

4) The value of the minimum is ****.0****

5) The value of the minimum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 2

Exercise 1

Given the functions

$f(x,y) = (1 - x - x y, -2 + 2 x - 2 x^2 + 3 y + x y, 3 + 3 x - 2 x^2 - 2 y + 2 x y + 3 y^2, -1 - x - 3 x^2 - 3 y - 2 y^2)$

and

$g(u_1,u_2,u_3,u_4) = (2 - 3 u_1 + 2 u_3 - 3 u_2 u_3 - u_4^2, -2 u_2 + u_1 u_3 - 2 u_4 - 2 u_2 u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,2)$.

1) -1.98277×10^6

2) -1.21423×10^6

3) -2.1477×10^6

4) $-959260.$

5) -2.05504×10^6

Exercise 2

Given the system

$u + 3 u^2 v - 2 v^2 - v^2 x + u x^2 - u^2 y - x^2 y == -6$
$2 u^3 + 3 x - v y + 2 u x y + x^2 y == -27$

determine if it is possible to solve for variables x,y in terms of variables u, v arround the point $p=(x,y,u,v)=(4,-3,0,3)$. Compute if possible $\frac{\partial y}{\partial u}(0,3)$.

1) $\frac{\partial y}{\partial u}(0,3) = \frac{2}{47}$

2) $\frac{\partial y}{\partial u}(0,3) = 0$

3) $\frac{\partial y}{\partial u}(0,3) = \frac{3}{47}$

4) $\frac{\partial y}{\partial u}(0,3) = -\frac{1}{47}$

5) $\frac{\partial y}{\partial u}(0,3) = \frac{1}{47}$

Exercise 3

Given the function

$f(x,y,z) = -3 + 2x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1, 3, 1}

2) We have a maximum at {-0.5, 3.6, 0.4}

3) We have a maximum at {0.4, 3.9, -0.2}

4) We have a maximum at {0.4, 2.1, 1.3}

5) We have a maximum at {-0.2, 3.6, -0.2}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2(x^2 - y^2)}{-9x + 8x^2 - 18x^3 + 18x^4 + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -x^3 + y^3$ defined over the domain D≡ $6x^2 + 3y^2 \leq 108$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.6****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 3

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (-3 - 3 x_1^2 + 3 x_2 - 2 x_3 + 2 x_3^2 + 2 x_1 x_4, -3 x_1 - 3 x_2 x_3 + 2 x_1 x_4)$

and

$g(u,v) = (1 - u^2 + 2 v - u v + 3 v^2, -1 - 2 u + 3 u^2 + 2 v + 3 u v - 3 v^2, 3 + 3 u - u^2 - 2 v + 2 u v, -1 - u - 2 u^2 - 3 v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,-1,-1,3)$.

1) -0.275285

2) -0.179419

3) 0.

4) 0.487057

5) -0.505845

Exercise 2

Given the system

$2 x_1^2 x_2 - x_2^2 - 2 x_3 x_4 == -15$
$-3 x_2 x_3 == -9$
$-x_1^2 x_3 == 48$
$-3 x_1^2 - x_1 x_2 x_4 - 3 x_3^2 x_4 == -117$

determine if it is possible to solve for variables x_1,x_2,x_3,x_4 in terms of variable u arround the point $p=(x_1,x_2,x_3,x_4,u)=(4,-1,-3,3,-4)$. Compute if possible $\frac{\partial x_2}{\partial u}(-4)$.

1) $\frac{\partial x_2}{\partial u}(-4)=4$

2) $\frac{\partial x_2}{\partial u}(-4)=0$

3) $\frac{\partial x_2}{\partial u}(-4)=3$

4) $\frac{\partial x_2}{\partial u}(-4)=2$

5) $\frac{\partial x_2}{\partial u}(-4)=1$

Exercise 3

Given the function

$f(x,y,z)=7+x^2-2y+y^2+z^2$ defined over the domain D≡ $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.3, 0.9, -0.1}

2) We have a minimum at {-0.4, 0.8, 0.4}

3) We have a minimum at {-0.4, 0.5, 0.3}

4) We have a minimum at {0, 1, 0}

5) We have a minimum at {0.2, 0.6, 0.3}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3y^2}{\left(x^2+y^2\right)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-3x^3+4y^3$ defined over the domain D≡ $9x^2+6y^2\leqslant 42$, compute its absolute maxima and minima.

1) The value of the minimum is ****.8****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.4****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 4

Exercise 1

Given the functions

$f(x,y) = (-1 + 3x + 2x^2 + xy - 2y^2, 1 + 2x - 3x^2 + 3y + 3xy + 3y^2, -3 + 3x - x^2 - 2y + 2xy + y^2)$

and

$g(u,v,w) = (3 - 2u^2 - 3v - 3v^2, -2w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1,2)$.

1) $-18\,792.$

2) $-34\,100.8$

3) $-30\,370.$

4) $-14\,375.7$

5) $-12\,698.$

Exercise 2

Given the system

$3x^2 y - 2 y u_1 u_3 - u_3^3 + 2 y u_1 u_4 == -324$
$2 x y^2 == -160$

determine if it is possible to solve for variables x,y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (-5, -4, 5, 4, 4, 3)$. Compute if possible $\frac{\partial x}{\partial u_1}(5,4,4,3)$.

1) $\frac{\partial x}{\partial u_1}(5,4,4,3) = 0$

2) $\frac{\partial x}{\partial u_1}(5,4,4,3) = -\frac{3}{47}$

3) $\frac{\partial x}{\partial u_1}(5,4,4,3) = -\frac{1}{47}$

4) $\frac{\partial x}{\partial u_1}(5,4,4,3) = -\frac{4}{47}$

5) $\frac{\partial x}{\partial u_1}(5,4,4,3) = -\frac{2}{47}$

Exercise 3

Given the function

$f(x,y,z)=21-4x+x^2-4y+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{25}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.719908, -0.0736363, -4.77685}

2) We have a maximum at {0.0800915, -0.673636, -4.97685}

3) We have a maximum at {2, 2, 3}

4) We have a maximum at {-0.119908, -0.373636, -5.37685}

5) We have a maximum at {-0.219908, -0.573636, -4.87685}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} 0$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-x^3-3y^3$ defined over the domain D≡ $6x^2+27y^2\leqslant 1068$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.1****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 5

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (-3 x_3 - 3 x_1 x_3 - x_2 x_4 + 3 x_3 x_4 - x_4^2, x_1 + 3 x_1 x_2 + 2 x_3 - x_1 x_3, -x_3 + 2 x_3^2 + 2 x_4)$

and

$g(u,v,w) = (-3 u - 3 v - 2 u w, 3 + u^2 - 3 u v + 2 w^2, 2 u + w + 3 v w, -2 - 3 u + 3 v w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,0,2,2)$.

1) 0.718569

2) 0.495537

3) 0.482535

4) 0.240056

5) 0.

Exercise 2

Given the system

$2 x_1^2 x_3 == 128$
$3 x_1 x_3 x_4 == -48$
$3 x_3^2 x_4 == 48$
$x_1 x_2^2 == -36$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u,v arround the point $p=(x_1,x_2,x_3,x_4,u,v)=(-4,3,4,1,5,-3)$. Compute if possible $\frac{\partial x_4}{\partial v}(5,-3)$.

1) $\frac{\partial x_4}{\partial v}(5,-3)=1$

2) $\frac{\partial x_4}{\partial v}(5,-3)=3$

3) $\frac{\partial x_4}{\partial v}(5,-3)=4$

4) $\frac{\partial x_4}{\partial v}(5,-3)=0$

5) $\frac{\partial x_4}{\partial v}(5,-3)=2$

Exercise 3

Given the function

$f(x,y,z) = -3 + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.75453, -3.35122, 0.641404}

2) We have a maximum at {2.14442, -1.59669, -0.723228}

3) We have a maximum at {2.33937, -3.15627, -0.528281}

4) We have a maximum at {1.94948, -2.57143, -0.333333}

5) We have a maximum at {0, 2, 1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x - 3y^2}{2y + 3xy + 2y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -3x^3 + 4y^3$ defined over the domain D≡ $27x^2 + 18y^2 \leqslant 1134$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.2****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 6

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4)=(2-3x_1-3x_1^2-2x_2-x_1x_3-x_3^2-x_1x_4+2x_2x_4$
$,-2x_2^2+x_3-2x_1x_3+x_3x_4+x_4^2,x_2-3x_2^2-x_4^2,-2-2x_1^2-3x_1x_2-2x_3+3x_3^2)$

and

$g(u_1,u_2,u_3,u_4)=(-3u_1+u_1^2+u_4^2,-3u_2-u_2^2,$
$u_1^2+3u_2-u_2u_3+2u_3^2+2u_1u_4+u_2u_4,3+2u_1^2+3u_2-u_3^2+3u_4-u_2u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(0,3,-2,3)$.

1) -1.38472×10^{11}

2) -1.09128×10^{11}

3) -4.17435×10^{10}

4) -1.45001×10^{11}

5) -7.8354×10^{10}

Exercise 2

Given the system

$-x_3x_4==-4$
$2x_1^2x_2==-64$
$-2x_1^2x_2-x_2x_4^2==96$
$3x_1^3-2u\,x_2x_4==208$

determine if it is possible to solve for variables x_1,x_2,x_3 $,x_4$ in terms of variables u,v,w arround the point $p=(x_1,x_2,x_3,x_4$ $,u,v,w)=(4,-2,1,4,1,2,-1)$. Compute if possible $\frac{\partial x_2}{\partial u}(1,2,-1)$.

1) $\frac{\partial x_2}{\partial u}(1,2,-1)=-\frac{1}{35}$

2) $\frac{\partial x_2}{\partial u}(1,2,-1)=0$

3) $\frac{\partial x_2}{\partial u}(1,2,-1)=-\frac{2}{35}$

4) $\frac{\partial x_2}{\partial u}(1,2,-1)=-\frac{3}{35}$

5) $\frac{\partial x_2}{\partial u}(1,2,-1)=-\frac{4}{35}$

Exercise 3

Given the function

$f(x,y,z)=-28+4x-x^2+6y-y^2+6z-z^2$ defined over the domain D≡ $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.30907, 3.4203, 3.9465}

2) We have a maximum at {1.30907, 1.5786, 2.3679}

3) We have a maximum at {2.62457, 2.8941, 3.6834}

4) We have a maximum at {1.83527, 2.631, 2.631}

5) We have a maximum at {2, 3, 3}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=2x^3-5y^3$ defined over the domain D≡ $9x^2+30y^2\leqslant 561$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.3****

3) The value of the maximum is ****.4****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.2****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 7

Exercise 1

Given the functions

$f(x,y,z)=(-x y+3 y^2+3 z+3 y z-3 z^2, 3 x+x z+3 z^2, -3 x-x y+y^2+3 z+z^2, -3+x y-2 y^2)$

and

$g(u_1,u_2,u_3,u_4)=(2+u_1-3 u_1^2+2 u_3 u_4, -u_1-2 u_1 u_3+u_2 u_4-2 u_4^2, u_1^2+u_3)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,1,-3)$.

1) 1.65882×10^8

2) 1.49911×10^8

3) 1.52711×10^8

4) 1.86793×10^8

5) 1.21348×10^8

Exercise 2

Given the system

$2 w-2 u v x+3 y==-64$
$-2 v y+z==-5$
$-3 v w y-3 x z^2==225$

determine if it is possible to solve for variables x, y,z in terms of variables u,v,w arround the point $p=(x,y,z,u,v,w)=(-3,0,-5,-4,3,4)$. Compute if possible $\frac{\partial x}{\partial v}(-4,3,4)$.

1) $\frac{\partial x}{\partial v}(-4,3,4)=\frac{1536}{1511}$

2) $\frac{\partial x}{\partial v}(-4,3,4)=\frac{1539}{1511}$

3) $\frac{\partial x}{\partial v}(-4,3,4)=\frac{1537}{1511}$

4) $\frac{\partial x}{\partial v}(-4,3,4)=\frac{1540}{1511}$

5) $\frac{\partial x}{\partial v}(-4,3,4)=\frac{1538}{1511}$

Exercise 3

Given the function

$f(x,y,z)=17-2x+x^2-6y+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {1.49105, 3.00524, 3.22095}

2) We have a minimum at {1.74148, 2.7548, 2.97052}

3) We have a minimum at {0.739735, 2.50437, 2.21921}

4) We have a minimum at {1, 3, 3}

5) We have a minimum at {-0.262012, 1.75306, 2.72008}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^2-2x^2 y}{x^3-3y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-3x^3+4y^3$ defined over the domain D≡ $18x^2+24y^2\leqslant 672$, compute its absolute maxima and minima.

1) The value of the minimum is ****.3****

2) The value of the minimum is ****.6****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 8

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (2 x_1^2 - 2 x_3^2, -3 x_2 + 3 x_1 x_2 - 3 x_2^2 + 3 x_4 + 2 x_2 x_4 + 3 x_3 x_4, -3 x_1^2 - 3 x_2^2 - 2 x_1 x_3 - 3 x_2 x_3 + x_3^2)$

and

$g(u,v,w) = (-3 + 3 w + v w - 3 w^2, -3 u w - 3 v w, 3 u v - 3 u w, 3 u - u v - 2 w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3,1,-2,1)$.

1) 0.169589

2) -0.760244

3) 0.

4) -0.881247

5) 0.473258

Exercise 2

Given the system

$-v^2 x_4 - u x_1 x_4 - 2 x_3 x_4^2 == -8$
$-3 u x_2 x_3 == -30$
$3 x_1 + u v x_1 + 2 u x_1 x_3 == -35$
$-2 x_1 - x_2 == 5$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u,v arround the point $p=(x_1,x_2,x_3,x_4,u,v)=(-5,5,-1,1,-2,0)$. Compute if possible $\frac{\partial x_2}{\partial u}(-2,0)$.

1) $\frac{\partial x_2}{\partial u}(-2,0)=1$

2) $\frac{\partial x_2}{\partial u}(-2,0)=2$

3) $\frac{\partial x_2}{\partial u}(-2,0)=4$

4) $\frac{\partial x_2}{\partial u}(-2,0)=3$

5) $\frac{\partial x_2}{\partial u}(-2,0)=0$

Exercise 3

Given the function

$f(x,y,z)=5-2x+x^2-4y+y^2-4z+z^2$ defined over the domain D≡ $\frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.8, 2.6, 2.2}

2) We have a minimum at {2., 1.2, 3.}

3) We have a minimum at {0.2, 2.4, 2.2}

4) We have a minimum at {1.2, 1.6, 2.6}

5) We have a minimum at {1, 2, 2}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3y-xy-y^2}{3x+2xy}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-4x^3-4y^3$ defined over the domain D≡ $6x^2+6y^2\leqslant 12$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.6****

4) The value of the minimum is ****.8****

5) The value of the minimum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 9

Exercise 1

Given the functions

$f(x,y) = (1 - 2x - 3x^2 + y + 3xy - 3y^2, 2 + 2x + 3x^2 - 3y - 3xy + 2y^2, 1 - 3x - 3x^2 - 3y - xy + 3y^2, -3x - 2x^2 - 2y - 2xy + 3y^2)$

and

$g(u_1,u_2,u_3,u_4) = (-3 + 2u_1 + 2u_2 + u_3, 2u_1u_3 + 2u_2u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,3)$.

1) 16814.1

2) 11900.

3) 6814.14

4) 15993.7

5) 15274.1

Exercise 2

Given the system

$2u^2x - x^2 - 3x^3 + 2uxy - 2y^2 == -8$
$3 - v^2x + vx^2 + 3y + 3uy + 2u^2y - 3uy^2 + 2xy^2 == 49$

determine if it is possible to solve for variables x,y in terms of variables u,v arround the point $p=(x,y,u,v)=(0,2,4,3)$. Compute if possible $\frac{\partial y}{\partial v}(4,3)$.

1) $\frac{\partial y}{\partial v}(4,3)=0$

2) $\frac{\partial y}{\partial v}(4,3)=2$

3) $\frac{\partial y}{\partial v}(4,3)=1$

4) $\frac{\partial y}{\partial v}(4,3)=4$

5) $\frac{\partial y}{\partial v}(4,3)=3$

Exercise 3

Given the function

$f(x,y,z)=-6 - 2x + x^2 + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

RandomChoice: The items for choice {} should be a non-empty list or a rule weights -> choices.

Thread: Objects of unequal length in {} + {−0.5, 0.2, 0.3} cannot be combined.

Thread: Objects of unequal length in {} + {−0.2, −0.4, −0.3} cannot be combined.

Thread: Objects of unequal length in {} + {−0.2, 0.1, −0.2} cannot be combined.

General: Further output of Thread::tdlen will be suppressed during this calculation.

1) We have a maximum at {}

2) We have a maximum at {} + {-0.2, -0.4, -0.3}

3) We have a maximum at {} + {-0.5, 0.2, 0.3}

4) We have a maximum at {} + {-0.2, 0.1, -0.2}

5) We have a maximum at {1, 0, 0}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x + xy - 2y^2}{-x^2 + y + 3xy}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = x^3 + 2y^3$ defined over the domain D≡
$3x^2 + 3y^2 \leq 15$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 10

Exercise 1

Given the functions

$f(x,y) = (2 + 2x + 2x^2 - y^2, -1 + 2x - 3x^2 - 2y - 3y^2)$

and

$g(u,v) = (3 + 3u^2 - v - 2uv - 3v^2, -1 - u - u^2 - v + 2uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(3,3).

1) -1.68202×10^7

2) -5.07675×10^7

3) -2.74822×10^7

4) -4.14791×10^7

5) -4.53175×10^6

Exercise 2

Given the system

$-1 - 2x^3 + 2y - uxy - 3y^3 == 514$
$vx + 2uxy + 2y^3 == -55$

determine if it is possible to solve for variables x,y in terms of variables u, v arround the point p=(x,y,u,v)=(-5,-5,4,1). Compute if possible $\frac{\partial x}{\partial u}(4,1)$.

1) $\frac{\partial x}{\partial u}(4,1) = \frac{7404}{22217}$

2) $\frac{\partial x}{\partial u}(4,1) = \frac{7401}{22217}$

3) $\frac{\partial x}{\partial u}(4,1) = \frac{7400}{22217}$

4) $\frac{\partial x}{\partial u}(4,1) = \frac{7402}{22217}$

5) $\frac{\partial x}{\partial u}(4,1) = \frac{7403}{22217}$

Exercise 3

Given the function

$f(x,y,z)=18-4x+x^2-4y+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{25}+\frac{y^2}{4}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {2, 2, 3}

2) We have a minimum at {1.5683, 1.38836, 0.68849}

3) We have a minimum at {2.61384, 1.73687, 1.21126}

4) We have a minimum at {1.74256, 1.03985, 1.55977}

5) We have a minimum at {1.5683, 1.91112, 2.08254}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2xy-xy^2}{x^2-2y^3}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-x^3-y^3$ defined over the domain D≡ $9x^2+9y^2\leqslant 648$, compute its absolute maxima and minima.

1) The value of the minimum is ****.7****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.4****

5) The value of the minimum is ****.6****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 11

Exercise 1

Given the functions

$f(x,y) = (1 + 2x - 2x^2 + 2xy + 3y^2, 3 - x + x^2 - 2y + xy - y^2)$

and

$g(u,v) = (-3 + 3u^2 + v + 3uv - v^2, 2 - 3u + 2v - uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,3)$.

1) 136 552.

2) 29 233.3

3) 38 760.8

4) 78 804.

5) 125 762.

Exercise 2

Given the system

$-3 + 3u + u^2 - v^3 + vx - vy^2 == 161$
$3u^2 x - 3uvx + vx^2 + vxy == -240$

determine if it is possible to solve for variables x,y in terms of variables u,v

arround the point $p=(x,y,u,v)=(-2,-4,4,-4)$. Compute if possible $\frac{\partial y}{\partial v}(4,-4)$.

1) $\frac{\partial y}{\partial v}(4,-4) = -\frac{259}{127}$

2) $\frac{\partial y}{\partial v}(4,-4) = -\frac{517}{254}$

3) $\frac{\partial y}{\partial v}(4,-4) = -\frac{515}{254}$

4) $\frac{\partial y}{\partial v}(4,-4) = -\frac{258}{127}$

5) $\frac{\partial y}{\partial v}(4,-4) = -\frac{519}{254}$

Exercise 3

Given the function

$f(x,y,z)=-17-x^2+6y-y^2+6z-z^2$ defined over the domain D≡ $\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0., 1.58089, 2.45014}

2) We have a maximum at {-0.735043, 2.56094, 3.67522}

3) We have a maximum at {0, 3, 3}

4) We have a maximum at {0.735043, 1.09086, 2.20513}

5) We have a maximum at {-0.490029, 1.8259, 1.22507}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x+2y}{x+9x\left(1-x+x^2\right)-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=x^3-4y^3$ defined over the domain D≡ $9x^2+24y^2\leqslant 708$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 12

Exercise 1

Given the functions

$f(x,y,z) = (-3y^2 - 2xz - z^2, 3z^2, -1 + 3x - z + xz - yz + z^2, 2x^2 + 3y^2 + 2yz)$

and

$g(u_1,u_2,u_3,u_4) = (2u_2u_4, -2u_1^2 - 2u_1u_2, -u_1^2 + 2u_2^2 + 3u_1u_3 + 3u_2u_3 + u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0,-3,2)$.

1) 1.98018×10^8

2) 5.31024×10^7

3) 4.30204×10^7

4) 1.53592×10^8

5) 2.32054×10^8

Exercise 2

Given the system

$2w^3 - uxy + 3w^2z == -65$
$-vy + 3vwz == -18$
$-vw + 3ux + 3w^2x + 2z == -50$

determine if it is possible to solve for variables x, y,z in terms of variables u,v,w arround the point $p = (x,y,z,u,v,w) = (5,-3,4,-5,2,-1)$. Compute if possible $\frac{\partial x}{\partial u}(-5,2,-1)$.

1) $\frac{\partial x}{\partial u}(-5,2,-1) = \frac{185}{149}$

2) $\frac{\partial x}{\partial u}(-5,2,-1) = \frac{189}{149}$

3) $\frac{\partial x}{\partial u}(-5,2,-1) = \frac{186}{149}$

4) $\frac{\partial x}{\partial u}(-5,2,-1) = \frac{187}{149}$

5) $\frac{\partial x}{\partial u}(-5,2,-1) = \frac{188}{149}$

Exercise 3

Given the function

$f(x,y,z)=4+x^2+y^2-4z+z^2$ defined over the domain D≡ $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0, 0, 2}

2) We have a maximum at {0.5, -0.4, -3.5}

3) We have a maximum at {0., 0., -4.}

4) We have a maximum at {-0.4, 0.4, -3.7}

5) We have a maximum at {-0.5, 0.3, -3.8}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^5-3xy^4+2y^5}{\left(x^2+y^2\right)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-4x^3-y^3$ defined over the domain D≡ $12x^2+9y^2\leqslant 372$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.6****

5) The value of the minimum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 13

Exercise 1

Given the functions

$f(x,y)=(3-x-x^2-2y+3xy+3y^2,-1-3x+x^2-y-xy+y^2,2x^2-3y-2xy-2y^2,1-x+2x^2-3y+2xy-3y^2)$

and

$g(u_1,u_2,u_3,u_4)=(3u_1-3u_1^2+3u_2+3u_4+u_3u_4,-2u_1+3u_1u_2+2u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,-3)$.

1) 2.75007×10^7

2) 9.1886×10^6

3) 6.114×10^6

4) 1.68596×10^7

5) 4.21396×10^7

Exercise 2

Given the system

$-3xyu_4==48$
$1+y^2-2u_2^2-2xu_3-2u_1u_4^2+2xu_5^2==3$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4,u_5 arround the point $p=(x,y,u_1,u_2,u_3,u_4,u_5)=(-2,-2,0,3,4,-4,0)$. Compute if possible $\frac{\partial y}{\partial u_4}(0,3,4,-4,0)$.

1) $\frac{\partial y}{\partial u_4}(0,3,4,-4,0)=3$

2) $\frac{\partial y}{\partial u_4}(0,3,4,-4,0)=0$

3) $\frac{\partial y}{\partial u_4}(0,3,4,-4,0)=-1$

4) $\frac{\partial y}{\partial u_4}(0,3,4,-4,0)=2$

5) $\frac{\partial y}{\partial u_4}(0,3,4,-4,0)=1$

Exercise 3

Given the function

$f(x,y,z) = -15 - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.1, -0.56091, -5.03128}

2) We have a minimum at {0, 1, 3}

3) We have a minimum at {0., -0.66091, -4.93128}

4) We have a minimum at {0.3, -1.06091, -5.33128}

5) We have a minimum at {0.5, -1.16091, -5.03128}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3 y^2 + 2y^5}{(x^2 + y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 5x^3 + 5y^3$ defined over the domain D≡ $45x^2 + 30y^2 \leq 2100$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.2****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.3****

5) The value of the maximum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 14

Exercise 1

Given the functions

$f(x,y) = (1 + 3x + 3x^2 + y + xy + 3y^2, 3 + x - 3y + 2xy - y^2)$

and

$g(u,v) = (-2 - 3u - u^2 + 3v + uv + v^2, -2 + 3u + 2u^2 + 2v - 3uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3,-3)$.

1) 1.98879×10^6

2) 2.59485×10^6

3) 967788.

4) 422821.

5) 1.7698×10^6

Exercise 2

Given the system

$-u - 2u^2 + 3u^3 + 3ux + vx + 3v^2 x + 3x^2 - ux^2 - 3vxy == -125$
$-3u^3 + uv + 2u^2 v + 2ux^2 - 3y^2 == 30$

determine if it is possible to solve for variables x,y in terms of variables u,v arround the point $p=(x,y,u,v)=(-1,-5,-3,2)$. Compute if possible $\frac{\partial y}{\partial u}(-3,2)$.

1) $\frac{\partial y}{\partial u}(-3,2) = \frac{1127}{206}$

2) $\frac{\partial y}{\partial u}(-3,2) = \frac{1691}{309}$

3) $\frac{\partial y}{\partial u}(-3,2) = \frac{3383}{618}$

4) $\frac{\partial y}{\partial u}(-3,2) = \frac{1690}{309}$

5) $\frac{\partial y}{\partial u}(-3,2) = \frac{3379}{618}$

Exercise 3

Given the function

$f(x,y,z)=4-4x+x^2-2y+y^2+z^2$ defined over the domain D≡ $\frac{x^2}{25}+\frac{y^2}{4}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {2, 1, 0}

2) We have a maximum at {-4.7896, -0.228946, 0.5}

3) We have a maximum at {-4.9896, -0.128946, 0.}

4) We have a maximum at {-4.4896, -0.228946, -0.2}

5) We have a maximum at {-5.2896, -0.528946, 0.1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^2-3y^2}{9x-10x^2+18x^3-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=2x^3+3y^3$ defined over the domain D≡ $3x^2+18y^2\leqslant 291$, compute its absolute maxima and minima.

1) The value of the maximum is ****.6****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.7****

4) The value of the maximum is ****.8****

5) The value of the maximum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 15

Exercise 1

Given the functions

$f(x,y)=(-1-x^2+3y+2xy+2y^2, 2-2x-3x^2-y+xy, 1+x+2x^2+2y-xy-3y^2, 3-2x+3x^2-y-2xy)$

and

$g(u_1,u_2,u_3,u_4)=(-u_1^2+u_1u_2+u_2^2-2u_3-u_2u_4-3u_4^2, 1+3u_1^2+2u_2-u_1u_2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(1,2).

1) −46 304.8

2) −30 243.8

3) −5934.47

4) −67 113.4

5) −55 680.

Exercise 2

Given the system

$uvx+2vy-2w^2y+x^2y==37$
$-3uvx+3y^2==27$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point p=(x,y,u,v,w)=(5,2,1,-1,1). Compute if possible $\frac{\partial y}{\partial v}(1,-1,1)$.

1) $\frac{\partial y}{\partial v}(1,-1,1)=\frac{104}{55}$

2) $\frac{\partial y}{\partial v}(1,-1,1)=\frac{107}{55}$

3) $\frac{\partial y}{\partial v}(1,-1,1)=\frac{108}{55}$

4) $\frac{\partial y}{\partial v}(1,-1,1)=\frac{106}{55}$

5) $\frac{\partial y}{\partial v}(1,-1,1)=\frac{21}{11}$

Exercise 3

Given the function

$f(x,y,z)=16-4x+x^2-6y+y^2-4z+z^2$ defined over the domain D≡ $\frac{x^2}{25}+\frac{y^2}{25}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-2.45897, -3.63845, 0.0955153}

2) We have a maximum at {-2.35897, -3.73845, -0.00448472}

3) We have a maximum at {2, 3, 2}

4) We have a maximum at {-3.25897, -4.23845, 0.0955153}

5) We have a maximum at {-2.75897, -4.13845, -0.204485}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)}\frac{y^2+3xy^2+y^3}{x^2+3y^3}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=3x^3+y^3$ defined over the domain D≡ $27x^2+9y^2\leqslant 1296$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.7****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 16

Exercise 1

Given the functions

$f(x,y) = (-1 + x - 3x^2 - y - 2xy + 3y^2, 1 - x + 3x^2 + 3y + xy + y^2, -3 - x - 3x^2 - y + xy - 2y^2)$

and

$g(u,v,w) = (-3 - 2uw + 2vw, u - 3v + v^2 - 2w - 3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2,1)$.

1) -371486.

2) -230443.

3) -199548.

4) -333013.

5) -110579.

Exercise 2

Given the system

$u v w - u w x - v x^2 + w y - y^3 == 8$
$2 w^2 x - 3 v w y == -256$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point $p = (x,y,u,v,w) = (-5,2,3,4,4)$. Compute if possible $\frac{\partial x}{\partial u}(3,4,4)$.

1) $\frac{\partial x}{\partial u}(3,4,4) = -\frac{23}{17}$

2) $\frac{\partial x}{\partial u}(3,4,4) = -\frac{25}{17}$

3) $\frac{\partial x}{\partial u}(3,4,4) = -\frac{27}{17}$

4) $\frac{\partial x}{\partial u}(3,4,4) = -\frac{26}{17}$

5) $\frac{\partial x}{\partial u}(3,4,4) = -\frac{24}{17}$

Exercise 3

Given the function

$f(x,y,z)=-8+2x-x^2+6y-y^2+2z-z^2$ defined over the domain D≡ $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.274397, -2.81914, -0.939712}

2) We have a minimum at {1, 3, 1}

3) We have a minimum at {-0.174397, -3.11914, -1.13971}

4) We have a minimum at {-0.474397, -2.71914, -1.13971}

5) We have a minimum at {-0.174397, -3.31914, -0.539712}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x+5x^2}{-2y+2xy}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-5x^3-4y^3$ defined over the domain D≡ $15x^2+6y^2\leqslant 66$, compute its absolute maxima and minima.

1) The value of the minimum is ****.4****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.0****

4) The value of the minimum is ****.9****

5) The value of the minimum is ****.6****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 17

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (-2 x_1^2 + x_2^2 + 3 x_3 + 2 x_1 x_3 + x_2 x_3 - x_3^2 - 2 x_1 x_4 - x_2 x_4$
$, -1 - 2 x_1 - 3 x_1 x_3 + x_3 x_4, -2 x_1 x_2 - 2 x_1 x_3 - 2 x_3 x_4)$

and

$g(u,v,w) = (3 u v - 3 u w + 2 v w, 2 u + 3 u^2 + 2 v + w - w^2, 2 v^2, -1 + v + 2 v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (0,-1,-3,2)$.

1) -0.150168

2) 0.

3) 0.549397

4) -0.261863

5) -0.289593

Exercise 2

Given the system

$-3 x_3 - 3 v x_3^2 - v x_4 == 113$
$u x_1^2 + 2 x_2 x_3^2 == 42$
$2 v x_2 x_4 == 24$
$-3 v x_2 + 3 u x_3 x_4 == 9$

determine if it is possible to solve for variables x_1, x_2 $, x_3, x_4$ in terms of variables u,v arround the point $p = (x_1, x_2, x_3,$ $x_4, u, v) = (-2,3,-3,-1,-3,-4)$. Compute if possible $\frac{\partial x_2}{\partial u}(-3,-4)$.

1) $\frac{\partial x_2}{\partial u}(-3,-4) = -\frac{222}{529}$

2) $\frac{\partial x_2}{\partial u}(-3,-4) = -\frac{224}{529}$

3) $\frac{\partial x_2}{\partial u}(-3,-4) = -\frac{223}{529}$

4) $\frac{\partial x_2}{\partial u}(-3,-4) = -\frac{221}{529}$

5) $\frac{\partial x_2}{\partial u}(-3,-4) = -\frac{225}{529}$

Exercise 3

Given the function

$f(x,y,z) = -8 - x^2 + 4y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.38242, 1.11113, 2.21187}

2) We have a maximum at {0, 2, 3}

3) We have a maximum at {0.276483, 1.38761, 1.93538}

4) We have a maximum at {-1.10593, 1.6641, 1.93538}

5) We have a maximum at {0., 1.94058, 2.76483}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3 y - 3 y^4}{\left(x^2 + y^2\right)^{3/2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -3x^3 - 4y^3$ defined over the domain D≡ $9x^2 + 30y^2 \leqslant 786$, compute its absolute maxima and minima.

1) The value of the minimum is ****.3****

2) The value of the minimum is ****.5****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 18

Exercise 1

Given the functions

$f(x,y)=(-1-x-2x^2+3y-2xy+3y^2,-1+3x^2+y-3xy-2y^2,1-x+y+2y^2)$

and

$g(u,v,w)=(3u,2u^2+2uw-3vw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,-2)$.

1) -4238.63

2) -10269.6

3) $-7020.$

4) -4942.36

5) -1680.01

Exercise 2

Given the system

$3x+2xyu_1==53$
$-xy^2==-25$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4 arround the point $p=(x,y,u_1,u_2,u_3,u_4)=(1,5,5,0,0,-1)$. Compute if possible $\frac{\partial x}{\partial u_2}(5,0,0,-1)$.

1) $\frac{\partial x}{\partial u_2}(5,0,0,-1)=4$

2) $\frac{\partial x}{\partial u_2}(5,0,0,-1)=3$

3) $\frac{\partial x}{\partial u_2}(5,0,0,-1)=1$

4) $\frac{\partial x}{\partial u_2}(5,0,0,-1)=0$

5) $\frac{\partial x}{\partial u_2}(5,0,0,-1)=2$

Exercise 3

Given the function

$f(x,y,z) = -23 + 6x - x^2 + 4y - y^2 + 4z - z^2$ defined over the domain D≡
$\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {3.57988, 0.181462, 1.32668}

2) We have a maximum at {2.203, 0.456837, 0.225179}

3) We have a maximum at {3, 2, 2}

4) We have a maximum at {3.57988, 1.83371, 2.15281}

5) We have a maximum at {2.75375, 1.28296, 1.60206}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^2 - 3y - 2y^2}{-3x + x^2 + y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -3x^3 - y^3$ defined over the domain D≡
$27x^2 + 6y^2 \leqslant 1068$, compute its absolute maxima and minima.

1) The value of the minimum is ****.8****

2) The value of the minimum is ****.2****

3) The value of the minimum is ****.5****

4) The value of the minimum is ****.9****

5) The value of the minimum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 19

Exercise 1

Given the functions

$f(x,y)=(x-x^2+2y+3xy-y^2, 1-3x^2-y-3xy-y^2, 3+x+3y+3xy, 2+2x-2x^2-2y+3xy-y^2)$

and

$g(u_1,u_2,u_3,u_4)=(3u_1+2u_1^2+3u_2-3u_2^2-u_2u_3+u_4^2, 2u_2u_4-3u_3u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-2,-2)$.

1) 1.32847×10^6

2) 334089.

3) 1.73996×10^6

4) 1.64769×10^6

5) 1.0757×10^6

Exercise 2

Given the system

$xy-3yu_2u_3==48$
$2xu_2^2+2u_1u_3-yu_1u_3+2xu_4u_5==-48$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4,u_5 arround the point $p=(x,y,u_1,u_2,u_3,u_4,u_5)=(-3,-4,1,3,1,0,1)$. Compute if possible $\frac{\partial y}{\partial u_4}(1,3,1,0,1)$.

1) $\frac{\partial y}{\partial u_4}(1,3,1,0,1)=-\frac{3}{55}$

2) $\frac{\partial y}{\partial u_4}(1,3,1,0,1)=-\frac{2}{55}$

3) $\frac{\partial y}{\partial u_4}(1,3,1,0,1)=-\frac{1}{11}$

4) $\frac{\partial y}{\partial u_4}(1,3,1,0,1)=-\frac{6}{55}$

5) $\frac{\partial y}{\partial u_4}(1,3,1,0,1)=-\frac{4}{55}$

Exercise 3

Given the function

$f(x,y,z) = -12 + 6x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0.939797, 1.05826, 1.73036}

2) We have a maximum at {3, 1, 1}

3) We have a maximum at {2.06755, 0.494386, 1.73036}

4) We have a maximum at {1.87959, 0.870305, 0.790558}

5) We have a maximum at {2.25551, 1.8101, 0.22668}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 - 3y^4}{2xy^2 + x^2y^2 - 3y^4}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -2x^3 - 2y^3$ defined over the domain D≡ $6x^2 + 15y^2 \leqslant 399$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.7****

4) The value of the minimum is ****.8****

5) The value of the minimum is ****.5****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 20

Exercise 1

Given the functions

$f(x,y,z) = (3\,x\,y + 3\,z^2, x^2 - 2\,x\,y, 2\,x\,y + 3\,y\,z + 2\,z^2, 3\,x + 3\,x^2)$

and

$g(u_1,u_2,u_3,u_4) = (2\,u_3^2 + u_1\,u_4, -2\,u_1, -1 + 3\,u_1 - 3\,u_1\,u_3 - u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3,-2,-3)$.

1) 8.12882×10^7

2) 7.76972×10^7

3) 7.87434×10^7

4) 6.05012×10^7

5) 3.15286×10^7

Exercise 2

Given the system

$-3\,x\,z - 2\,v\,z^2 == -190$
$-2\,y\,z^2 == 50$
$-w^3 - 3\,u\,y + 3\,v\,z^2 == 430$

determine if it is possible to solve for variables x, y,z in terms of variables u,v,w arround the point $p=(x,y,z,u,v,w)=(-4,-1,5,-3,5,-4)$. Compute if possible $\frac{\partial z}{\partial v}(-3,5,-4)$.

1) $\frac{\partial z}{\partial v}(-3,5,-4) = -\frac{31}{64}$

2) $\frac{\partial z}{\partial v}(-3,5,-4) = -\frac{121}{256}$

3) $\frac{\partial z}{\partial v}(-3,5,-4) = -\frac{125}{256}$

4) $\frac{\partial z}{\partial v}(-3,5,-4) = -\frac{61}{128}$

5) $\frac{\partial z}{\partial v}(-3,5,-4) = -\frac{123}{256}$

Exercise 3

Given the function

$f(x,y,z)=22-6x+x^2+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{9}\leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {-2.52132, 0.4, -2.22132}

2) We have a maximum at {-1.62132, 0.1, -2.22132}

3) We have a maximum at {-2.12132, 0., -2.12132}

4) We have a maximum at {3, 0, 3}

5) We have a maximum at {-1.72132, -0.4, -1.62132}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x+3xy-y^2}{y+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-2x^3+5y^3$ defined over the domain D≡ $9x^2+15y^2\leq 141$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.7****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 21

Exercise 1

Given the functions

$f(x,y) = (3 + 2x + y + 2xy + 3y^2, -1 + 2x^2 - y + 2xy, -2 + 3x - 2x^2 - 3y - 3xy + y^2)$

and

$g(u,v,w) = (2u - v^2, -2uv + 2w - 2uw + 2w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-3,3)$.

1) 719.164

2) 1992.

3) 1653.6

4) 2905.5

5) 2324.61

Exercise 2

Given the system

$2x^3 - 3xu_2 - 3yu_3^2 - u_3^3 + 2xu_1u_4 == -55$
$-yu_1 - 2x^2u_3 + 3u_4^2 + 2xu_4^2 == -54$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4 arround the point $p=(x,y,u_1,u_2,u_3,u_4)=(5,4,3,0,5,4)$. Compute if possible $\frac{\partial x}{\partial u_1}(3,0,5,4)$.

1) $\frac{\partial x}{\partial u_1}(3,0,5,4) = -\frac{69}{937}$

2) $\frac{\partial x}{\partial u_1}(3,0,5,4) = -\frac{66}{937}$

3) $\frac{\partial x}{\partial u_1}(3,0,5,4) = -\frac{67}{937}$

4) $\frac{\partial x}{\partial u_1}(3,0,5,4) = -\frac{68}{937}$

5) $\frac{\partial x}{\partial u_1}(3,0,5,4) = -\frac{70}{937}$

Exercise 3

Given the function

$f(x,y,z) = 25 - 6x + x^2 - 6y + y^2 - 4z + z^2$ defined over the domain D≡
$\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-4.83914, -0.32878, -0.571454}

2) We have a maximum at {3, 3, 2}

3) We have a maximum at {-5.13914, -0.52878, -0.371454}

4) We have a maximum at {-4.43914, -0.62878, -0.0714537}

5) We have a maximum at {-5.33914, -0.72878, -0.871454}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x+y}{x - 3x\left(1 - x + x^2 + 2x^3\right) + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 4x^3 - 4y^3$ defined over the domain D≡
$6x^2 + 6y^2 \leqslant 12$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.5****

3) The value of the minimum is ****.3****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 22

Exercise 1

Given the functions

$f(x,y) = (2 - x + 2x^2 + y - xy - y^2, -1 - x - 2x^2 + 2y - 3xy, 2 + 2x + 3x^2 + 2y + 2xy - 2y^2)$

and

$g(u,v,w) = (1 + 2u + 2u^2 + v - uv, -3v + 3w - 3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,-3)$.

1) -215359.

2) -324376.

3) -310773.

4) -227442.

5) -185472.

Exercise 2

Given the system

$-3x^2 u_1 - 2y u_2 u_3 - 3xy u_4 + 2x u_4^2 == -350$
$-3x^2 u_1 + x u_2 + 2u_3 u_4 + 2y u_3 u_4 == -45$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4 arround the point $p=(x,y,u_1,u_2,u_3,u_4) = (5,5,3,0,3,5)$. Compute if possible $\frac{\partial y}{\partial u_4}(3,0,3,5)$.

1) $\frac{\partial y}{\partial u_4}(3,0,3,5) = -\frac{31}{170}$

2) $\frac{\partial y}{\partial u_4}(3,0,3,5) = -\frac{63}{340}$

3) $\frac{\partial y}{\partial u_4}(3,0,3,5) = -\frac{3}{17}$

4) $\frac{\partial y}{\partial u_4}(3,0,3,5) = -\frac{59}{340}$

5) $\frac{\partial y}{\partial u_4}(3,0,3,5) = -\frac{61}{340}$

Exercise 3

Given the function

$f(x,y,z) = -20 + 6x - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.79306, 0.769727, 1.80555}

2) We have a maximum at {1.07085, 0.228063, 1.08333}

3) We have a maximum at {0.890291, 0.589172, 2.52776}

4) We have a maximum at {3, 1, 2}

5) We have a maximum at {0.890291, 1.49195, 1.26388}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2y + xy}{2x - 3x^2 + y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 3x^3 + 3y^3$ defined over the domain D≡ $9x^2 + 9y^2 \leqslant 72$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.3****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 23

Exercise 1

Given the functions

$f(x_1, x_2, x_3, x_4) = (3 x_2 + 3 x_2^2 + x_2 x_3 + 2 x_4, 1 - 3 x_1 x_2 - x_2^2 + 2 x_2 x_3 - x_4^2, 3 x_1^2 + x_2 - 3 x_1 x_2 + 3 x_2 x_3 - 2 x_3^2 - 2 x_4 - 3 x_1 x_4 + 2 x_2 x_4 - 3 x_3 x_4)$

and

$g(u, v, w) = (2 u v + 2 w, -3 - 2 v w, 3 u - 2 v + 2 u v + 2 v w, -v - 3 u w + 2 v w + 2 w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (3, 3, 3, -3)$.

1) -0.681044

2) -0.657997

3) 0.

4) -0.630152

5) -0.889567

Exercise 2

Given the system

$-u^2 x_1 - 2 x_1 x_2 - 2 v x_1 x_4 == -85$
$u x_3 x_4 == -9$
$3 v^3 + 3 x_1^2 + x_3^2 == -108$
$2 u v - 3 v x_1 x_2 - 2 v x_3 - 3 u x_4^2 == -229$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (5, -4, -3, -3, -1, -4)$. Compute if possible $\frac{\partial x_1}{\partial u}(-1, -4)$.

1) $\frac{\partial x_1}{\partial u}(-1, -4) = -\frac{32}{61}$

2) $\frac{\partial x_1}{\partial u}(-1, -4) = -\frac{639}{1220}$

3) $\frac{\partial x_1}{\partial u}(-1, -4) = -\frac{641}{1220}$

4) $\frac{\partial x_1}{\partial u}(-1, -4) = -\frac{319}{610}$

5) $\frac{\partial x_1}{\partial u}(-1, -4) = -\frac{637}{1220}$

Exercise 3

Given the function

$f(x,y,z) = -10 + 4x - x^2 + 6y - y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-3.83916, -1.72815, -0.485329}

2) We have a minimum at {-3.43916, -1.82815, -0.685329}

3) We have a minimum at {-3.43916, -2.12815, -0.585329}

4) We have a minimum at {-3.33916, -1.62815, -0.185329}

5) We have a minimum at {2, 3, 1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^3 + 2y^3}{x^2 + y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = x^3 - 5y^3$ defined over the domain D≡ $6x^2 + 30y^2 \leqslant 576$, compute its absolute maxima and minima.

1) The value of the minimum is ****.7****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.9****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.6****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 24

Exercise 1

Given the functions

$f(x,y,z) = (2x^2 + 3y^2, -y^2, 1 - x^2 + 2yz)$

and

$g(u,v,w) = (-u + 3uv - w - uw, -2w^2, u + v^2 + 2w + uw - vw)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-2,-2,-3)$.

1) -1.04141×10^6

2) -1.97168×10^6

3) -1.38321×10^6

4) -1.2825×10^6

5) $-665325.$

Exercise 2

Given the system

$u x + 3 v x z + 3 y z - y^2 z == 188$
$-3u - x y z == -30$
$-u x^2 + u x z == 8$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point $p = (x,y,z,u,v) = (-4,3,-3,-2,5)$. Compute if possible $\frac{\partial z}{\partial v}(-2,5)$.

1) $\frac{\partial z}{\partial v}(-2,5) = \frac{45}{104}$

2) $\frac{\partial z}{\partial v}(-2,5) = \frac{6}{13}$

3) $\frac{\partial z}{\partial v}(-2,5) = \frac{49}{104}$

4) $\frac{\partial z}{\partial v}(-2,5) = \frac{47}{104}$

5) $\frac{\partial z}{\partial v}(-2,5) = \frac{23}{52}$

Exercise 3

Given the function

$f(x,y,z)=1-2x+x^2-4y+y^2-4z+z^2$ defined over the domain $D\equiv \frac{x^2}{25}+\frac{y^2}{9}+\frac{z^2}{9}\leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {1, 2, 2}

2) We have a minimum at {0.2, 2.6, 1.4}

3) We have a minimum at {2., 1.8, 2.6}

4) We have a minimum at {0.8, 1.6, 2.2}

5) We have a minimum at {0.6, 2.4, 1.4}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{2x^2y^3}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=4x^3-3y^3$ defined over the domain $D\equiv 6x^2+27y^2\leq 978$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.0****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.7****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 25

Exercise 1

Given the functions

$f(x,y) = (-1 - 2x + 2y + xy - 2y^2, -1 + 2x + 2x^2 - y - 3xy + 3y^2, -1 + x + 3x^2 - y - 3xy - 2y^2)$

and

$g(u,v,w) = (-2 + 3u^2, -2u - v)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(0,-1)$.

1) -270.

2) -360.441

3) -220.966

4) -130.192

5) -121.77

Exercise 2

Given the system

$-v + 3w^2 x - 3wy == 131$
$3u^2 v - v^2 + 3vw + 2u^2 x - 3w^2 x + y^2 == -225$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point $p=(x,y,u,v,w)=(2,-1,0,4,-5)$. Compute if possible $\frac{\partial x}{\partial v}(0,4,-5)$.

1) $\frac{\partial x}{\partial v}(0,4,-5) = -\frac{344}{975}$

2) $\frac{\partial x}{\partial v}(0,4,-5) = -\frac{346}{975}$

3) $\frac{\partial x}{\partial v}(0,4,-5) = -\frac{343}{975}$

4) $\frac{\partial x}{\partial v}(0,4,-5) = -\frac{347}{975}$

5) $\frac{\partial x}{\partial v}(0,4,-5) = -\frac{23}{65}$

Exercise 3

Given the function

$f(x,y,z) = 11 - 2x + x^2 - 2y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.8, 1.5, 0.4}

2) We have a minimum at {1.1, 1.5, -0.3}

3) We have a minimum at {1.2, 1.3, 0.5}

4) We have a minimum at {1.5, 0.6, -0.1}

5) We have a minimum at {1, 1, 0}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^2 y + y^2}{3x^2 - 2y^3}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 4x^3 + 3y^3$ defined over the domain D≡ $6x^2 + 9y^2 \leqslant 42$, compute its absolute maxima and minima.

1) The value of the minimum is ****.0****

2) The value of the minimum is ****.6****

3) The value of the minimum is ****.5****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 26

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (-2 x_3 - x_1 x_3 + 2 x_2 x_3, 3 x_1 + x_2^2 + 2 x_1 x_4, 3 x_1^2 - 2 x_1 x_2 - x_1 x_3 + 2 x_2 x_3 + 2 x_3 x_4, 2 x_3^2 + 2 x_4^2)$

and

$g(u_1,u_2,u_3,u_4) = (u_1 u_2 - 3 u_1 u_3 - 3 u_2 u_3, 3 u_1^2 - 2 u_1 u_2 + 3 u_3 u_4, -3 u_1 u_2 - 2 u_2^2 + 3 u_3 - 3 u_2 u_3, 3 + 2 u_1 u_3 + 2 u_3^2 + u_4 + u_3 u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,0,1,0)$.

1) $-102487.$

2) $-193924.$

3) $-140079.$

4) $-165888.$

5) -93592.9

Exercise 2

Given the system

$$w x_1 x_2 + 2 x_1 x_2 x_4 == 55$$
$$-3 v x_1 x_4 == -75$$
$$3 u x_3 + u^2 x_4 == 9$$
$$-2 v x_2 x_3 == -200$$

determine if it is possible to solve for variables x_1,x_2,x_3,x_4 in terms of variables u,v,w arround the point $p=(x_1,x_2,x_3,x_4,u,v,w)=(1,5,4,5,-3,5,1)$. Compute if possible $\frac{\partial x_2}{\partial v}(-3,5,1)$.

1) $\frac{\partial x_2}{\partial v}(-3,5,1) = \frac{51}{59}$

2) $\frac{\partial x_2}{\partial v}(-3,5,1) = \frac{54}{59}$

3) $\frac{\partial x_2}{\partial v}(-3,5,1) = \frac{53}{59}$

4) $\frac{\partial x_2}{\partial v}(-3,5,1) = \frac{55}{59}$

5) $\frac{\partial x_2}{\partial v}(-3,5,1) = \frac{52}{59}$

Exercise 3

Given the function

$f(x,y,z)=-16-x^2+4y-y^2+6z-z^2$ defined over the domain D≡ $\frac{x^2}{16}+\frac{y^2}{16}+\frac{z^2}{16}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0.3, 3.2, 1.8}

2) We have a maximum at {0, 2, 3}

3) We have a maximum at {-1.2, 0.8, 4.2}

4) We have a maximum at {0.3, 2.6, 2.7}

5) We have a maximum at {-0.3, 2.6, 1.8}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^4+3x^2y-3y^4}{3x^4-2y^3}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=3x^3+4y^3$ defined over the domain D≡ $18x^2+18y^2\leqslant 450$, compute its absolute maxima and minima.

1) The value of the minimum is ****.0****

2) The value of the minimum is ****.2****

3) The value of the minimum is ****.4****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 27

Exercise 1

Given the functions

$f(x,y) = (-1 - x^2 - 2y + xy - 3y^2, -2 + 2x - 2x^2 - 3y + 2xy)$

and

$g(u,v) = (-2 + u + u^2 - 2v - uv + v^2, -3 - 3u - 3u^2 + 3v - 2uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1,0)$.

1) 3130.8

2) 4666.64

3) 2472.

4) 3071.52

5) 1042.57

Exercise 2

Given the system

$$-3u^3 - 2ux^2 - 3y - 3uxy + y^2 - 2uy^2 + xy^2 == -151$$
$$-3u^3 + x + 3u^2x + 2x^2 - 3ux^2 + x^3 - uy - 3xy + x^2y - 3xy^2 == -15$$

determine if it is possible to solve for variables x,y in terms of variable u arround the point $p = (x,y,u) = (-4,-4,1)$. Compute if possible $\frac{\partial y}{\partial u}(1)$.

1) $\frac{\partial y}{\partial u}(1) = \frac{10}{17}$

2) $\frac{\partial y}{\partial u}(1) = \frac{848}{1445}$

3) $\frac{\partial y}{\partial u}(1) = \frac{847}{1445}$

4) $\frac{\partial y}{\partial u}(1) = \frac{849}{1445}$

5) $\frac{\partial y}{\partial u}(1) = \frac{851}{1445}$

Exercise 3

Given the function

$f(x,y,z) = -4 - x^2 + 2y - y^2 - z^2$ defined over the domain D≡
$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.2, -0.3, -2.24955}

2) We have a minimum at {0., -0.8, -2.74955}

3) We have a minimum at {-0.2, -0.7, -2.24955}

4) We have a minimum at {-0.5, -0.4, -2.54955}

5) We have a minimum at {0, 1, 0}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3 + 2y^3}{3x + 6x^2 + x^3 - y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -x^3 - 4y^3$ defined over the domain D≡
$3x^2 + 12y^2 \leq 60$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 28

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (x_1^2 - x_2 - x_1 x_2 + 2 x_4^2, 2 x_1 - 3 x_1^2 - 2 x_2 + 3 x_2^2 + 3 x_1 x_3 - 2 x_4)$

and

$g(u,v) = (2 + u - v + 2 v^2, 2 + 2 u + 2 u^2 + v + 2 u v - 3 v^2, 1 - u + v + 3 u v + 3 v^2, 3 + u - 2 u^2 + v + 2 v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(1,-3,0,-1)$.

1) 0.430681

2) 0.551464

3) 0.

4) −0.587124

5) −0.156864

Exercise 2

Given the system

$-u x_2^2 + u x_2 x_4 - 3 x_4^2 == -23$
$-x_1 x_2 x_3 == 60$
$2 u^3 + 2 u x_2^2 + u x_1 x_3 == 19$
$2 u x_1 x_4 + 2 x_4^2 == 12$

determine if it is possible to solve for variables x_1,x_2,x_3,x_4 in terms of variable u arround the point $p=(x_1,x_2,x_3,x_4,u)=(-5,4,3,-1,1)$. Compute if possible $\frac{\partial x_4}{\partial u}(1)$.

1) $\frac{\partial x_4}{\partial u}(1) = \frac{376}{395}$

2) $\frac{\partial x_4}{\partial u}(1) = \frac{76}{79}$

3) $\frac{\partial x_4}{\partial u}(1) = \frac{377}{395}$

4) $\frac{\partial x_4}{\partial u}(1) = \frac{379}{395}$

5) $\frac{\partial x_4}{\partial u}(1) = \frac{378}{395}$

Exercise 3

Given the function

$f(x,y,z) = 8 - 2x + x^2 - 4y + y^2 - 2z + z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{25} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.831375, -4.88033, -0.12802}

2) We have a maximum at {-0.331375, -5.28033, -0.62802}

3) We have a maximum at {-0.631375, -5.18033, -0.62802}

4) We have a maximum at {1, 2, 1}

5) We have a maximum at {-1.13138, -4.58033, 0.17198}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{-9x - 9x^2 + 17x^3 + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 3x^3 + y^3$ defined over the domain D≡ $27x^2 + 6y^2 \leqslant 1068$, compute its absolute maxima and minima.

1) The value of the minimum is ****.5****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.1****

4) The value of the minimum is ****.8****

5) The value of the minimum is ****.7****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 29

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (-x_2 - 3x_2^2 - x_3^2 - 2x_1x_4 + 3x_2x_4 + 2x_4^2, x_1 - 2x_2 + x_1x_2 + 2x_4^2, 2 + x_3 + 2x_1x_4, -1 + 2x_1 + 2x_1^2 - 3x_3^2 - 3x_4^2)$

and

$g(u_1,u_2,u_3,u_4) = (-u_2u_3 + 3u_4, -1 - 3u_1u_2 + 3u_2u_3 + u_1u_4 + 2u_2u_4 - u_3u_4, -1 + 3u_1^2 + u_3 - 2u_1u_3 + u_3u_4, 2 - u_1^2 + 2u_2 + 3u_3 - u_1u_3 + 3u_3u_4 + 3u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,1,1,2)$.

1) 2.72147×10^6

2) 1.84314×10^6

3) 1.58218×10^6

4) 1.40195×10^6

5) 1.60944×10^6

Exercise 2

Given the system

$2uvw - uvx_1 + 3ux_1x_4 == 25$
$-x_1x_2 - 2wx_1x_3 + 3wx_4 == -175$
$-ux_2x_3 == -3$
$-3vx_1x_2 - 3vx_2x_3 == 24$

determine if it is possible to solve for variables x_1,x_2,x_3,x_4 in terms of variables u,v,w arround the point $p=(x_1,x_2,x_3,x_4,u,v,w)=(5,-1,3,-2,-1,1,5)$. Compute if possible $\frac{\partial x_4}{\partial u}(-1,1,5)$.

1) $\frac{\partial x_4}{\partial u}(-1,1,5) = -\frac{1063}{265}$

2) $\frac{\partial x_4}{\partial u}(-1,1,5) = -\frac{1064}{265}$

3) $\frac{\partial x_4}{\partial u}(-1,1,5) = -\frac{1066}{265}$

4) $\frac{\partial x_4}{\partial u}(-1,1,5) = -\frac{1067}{265}$

5) $\frac{\partial x_4}{\partial u}(-1,1,5) = -\frac{213}{53}$

Exercise 3

Given the function

$f(x,y,z) = -4 + 2x - x^2 + 4y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1, 2, 0}

2) We have a maximum at {1.6, 1.4, 0.8}

3) We have a maximum at {1.4, 2.4, 1.}

4) We have a maximum at {1.2, 2.2, 0.2}

5) We have a maximum at {1.6, 1., -1.}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x + y}{-5x + 12x^2 + 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -4x^3 + 4y^3$ defined over the domain D≡ $6x^2 + 30y^2 \leqslant 756$, compute its absolute maxima and minima.

1) The value of the minimum is ****.9****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.3****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 30

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (2 x_4 - 2 x_1 x_4 + x_3 x_4 + x_4^2, 1 + 2 x_1^2 + 3 x_2 x_3 + 3 x_1 x_4 - 2 x_2 x_4, -2 x_3 + 2 x_1 x_3 - x_1 x_4, 2 x_1 + 2 x_1 x_2 + 2 x_1 x_3 - 2 x_4 + 3 x_2 x_4)$

and

$g(u_1,u_2,u_3,u_4) = (-1 - u_2 u_3 - 2 u_1 u_4 + 3 u_2 u_4 - 2 u_4^2, -3 - 3 u_2 u_3 + 3 u_3^2 + u_1 u_4 - 2 u_3 u_4, -u_1 - u_2 u_3 - 3 u_4, 2 u_2 + u_2 u_3 - 3 u_1 u_4 - u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(2,-3,3,-1)$.

1) -1.91952×10^{8}

2) -2.21492×10^{8}

3) -1.97878×10^{8}

4) -2.82332×10^{7}

5) -7.91461×10^{7}

Exercise 2

Given the system

$-3 v^2 w - 3 x_2 - x_3^3 - 2 u w x_4 == -308$
$2 u x_2 - 2 x_3 x_4 == 4$
$2 x_2^2 x_3 == -2$
$-2 v^2 x_1 + x_1 x_3 == -99$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u,v,w arround the point $p=(x_1,x_2,x_3,x_4,u,v,w)=(3,-1,-1,-3,-5,4,4)$. Compute if possible $\frac{\partial x_1}{\partial u}(-5,4,4)$.

1) $\frac{\partial x_1}{\partial u}(-5,4,4)=\frac{12}{443}$

2) $\frac{\partial x_1}{\partial u}(-5,4,4)=\frac{131}{4873}$

3) $\frac{\partial x_1}{\partial u}(-5,4,4)=\frac{130}{4873}$

4) $\frac{\partial x_1}{\partial u}(-5,4,4)=\frac{128}{4873}$

5) $\frac{\partial x_1}{\partial u}(-5,4,4)=\frac{129}{4873}$

Exercise 3

Given the function

$f(x,y,z)=8-4x+x^2-6y+y^2-2z+z^2$ defined over the domain D≡ $\frac{x^2}{16}+\frac{y^2}{16}+\frac{z^2}{4}\leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {2.55278, 4.41828, 1.22566}

2) We have a minimum at {2.84734, 2.65097, 0.0474563}

3) We have a minimum at {3.43644, 3.24007, 2.10932}

4) We have a minimum at {1.96368, 2.94552, 0.931112}

5) We have a minimum at {2, 3, 1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)}\frac{3x^2y+3y^2+2y^3}{3x^2+2xy^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-4x^3+3y^3$ defined over the domain D≡ $30x^2+18y^2\leq 1038$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.2****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.7****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 31

Exercise 1

Given the functions

$f(x,y) = (-2 - 3x^2 + 2xy + 3y^2, -2 - x + 3x^2 - 2xy + 2y^2, 3 + x - 3x^2 - 2y + 2xy - 2y^2, -1 - 3x - 3x^2 + y + xy + y^2)$

and

$g(u_1,u_2,u_3,u_4) = (3u_2 + 2u_1u_2 + u_3 - u_2u_4 + u_3u_4, u_1^2 + 2u_2 - 3u_1u_3)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(0,0).

1) 195.04

2) 293.194

3) 160.

4) 67.1485

5) 236.023

Exercise 2

Given the system

$2ux - vy^2 == 85$

$-3uw^2 + uvx - uy - 2x^2y - 2y^2 == -180$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point p=(x,y,u,v,w)=(5,5,-4,-5,0). Compute if possible $\frac{\partial y}{\partial v}(-4,-5,0)$.

1) $\frac{\partial y}{\partial v}(-4,-5,0) = \frac{118}{283}$

2) $\frac{\partial y}{\partial v}(-4,-5,0) = \frac{117}{283}$

3) $\frac{\partial y}{\partial v}(-4,-5,0) = \frac{116}{283}$

4) $\frac{\partial y}{\partial v}(-4,-5,0) = \frac{119}{283}$

5) $\frac{\partial y}{\partial v}(-4,-5,0) = \frac{115}{283}$

Exercise 3

Given the function

$f(x,y,z) = -16 + 4x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.195533, -0.820808, -4.23897}

2) We have a minimum at {2, 1, 1}

3) We have a minimum at {-0.204467, -1.52081, -4.63897}

4) We have a minimum at {-0.304467, -1.12081, -4.73897}

5) We have a minimum at {-0.504467, -1.42081, -5.03897}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x + y}{8x + 18x^2 + 9x^3 - 9x^4 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = x^3 - 4y^3$ defined over the domain D≡ $6x^2 + 12y^2 \leqslant 144$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.8****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.5****

5) The value of the minimum is ****.9****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 32

Exercise 1

Given the functions

$f(x,y,z) = (3xy + 2xz + yz, 3x - 2y - 3yz, 2y - xz + 2yz, -2 - 2x + 3y)$

and

$g(u_1,u_2,u_3,u_4) = (u_1^2, u_2 - 2u_3 + u_1 u_3 + 3u_4^2, 2u_1 - 3u_2 + 3u_1 u_4 + u_2 u_4 - u_4^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(1,2,0).

1) 15840.

2) 5273.41

3) 21129.1

4) 4975.22

5) 8960.68

Exercise 2

Given the system

$-xy == -5$
$-2uxz - 2y^2 z == 52$
$-3x^2 + uwy - wy^2 == -83$

determine if it is possible to solve for variables x,y,z in terms of variables u,v,w arround the point p=(x,y,z,u,v,w)=(5,1,-1,5,5,-2). Compute if possible $\frac{\partial y}{\partial u}(5,5,-2)$.

1) $\frac{\partial y}{\partial u}(5,5,-2) = \frac{5}{72}$

2) $\frac{\partial y}{\partial u}(5,5,-2) = \frac{1}{24}$

3) $\frac{\partial y}{\partial u}(5,5,-2) = \frac{1}{36}$

4) $\frac{\partial y}{\partial u}(5,5,-2) = \frac{1}{18}$

5) $\frac{\partial y}{\partial u}(5,5,-2) = \frac{1}{72}$

Exercise 3

Given the function

$f(x,y,z) = -16 + 2x - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.0173655, -3.09584, -1.79722}

2) We have a minimum at {-0.417365, -3.29584, -2.19722}

3) We have a minimum at {-0.617365, -3.09584, -1.79722}

4) We have a minimum at {1, 3, 2}

5) We have a minimum at {-0.617365, -2.99584, -2.59722}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x - 3y}{8x + 18x^2 + 18x^3 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -4x^3 - 5y^3$ defined over the domain D≡ $18x^2 + 45y^2 \leq 1782$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.1****

5) The value of the minimum is ****.8****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 33

Exercise 1

Given the functions

$f(x,y)=(-1-x-2x^2+y-3xy-3y^2, 3+3x-x^2+3y+xy, -1-x+3y-2xy-2y^2, 2x-3x^2+3xy+3y^2)$

and

$g(u_1,u_2,u_3,u_4)=(-u_1^2-3u_2u_4+3u_4^2, 3u_1+u_3-3u_2u_4+2u_3u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(0,0).

1) -110.232

2) -24.0253

3) -68.

4) -109.046

5) -52.514

Exercise 2

Given the system

$-3-2u^2+2u^3-3x+3u^2x+2ux^2-x^3-2y+3uy-3u^2y-xy-2x^2y-y^3==-188$
$3u^3-2u^2x+2x^3+2y-xy+3uxy-3uy^2-xy^2-2y^3==-330$

determine if it is possible to solve for variables x,y in terms of variable u arround the point p=(x,y,u)=(2,5,1). Compute if possible $\frac{\partial x}{\partial u}(1)$.

1) $\frac{\partial x}{\partial u}(1)=\frac{5188}{10115}$

2) $\frac{\partial x}{\partial u}(1)=\frac{5189}{10115}$

3) $\frac{\partial x}{\partial u}(1)=\frac{1038}{2023}$

4) $\frac{\partial x}{\partial u}(1)=\frac{5186}{10115}$

5) $\frac{\partial x}{\partial u}(1)=\frac{741}{1445}$

Exercise 3

Given the function

$f(x,y,z) = -7 + 4x - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.32256, 1.70043, 2.88118}

2) We have a maximum at {2, 3, 2}

3) We have a maximum at {2.17277, 1.98383, 1.74756}

4) We have a maximum at {1.03915, 1.98383, 0.897346}

5) We have a maximum at {1.88937, 2.83405, 1.46416}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{x^4 y}{(x^2+y^2)^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 3x^3 + 5y^3$ defined over the domain D≡ $18x^2 + 30y^2 \leqslant 768$, compute its absolute maxima and minima.

1) The value of the maximum is ****.0****

2) The value of the maximum is ****.8****

3) The value of the maximum is ****.2****

4) The value of the maximum is ****.4****

5) The value of the maximum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 34

Exercise 1

Given the functions

$f(x,y) = (2x - 2x^2 - 2y - 2xy - 3y^2, -2 - x^2 + 3y + 2xy + 2y^2, 3x - 2x^2 + y - 2y^2)$

and

$g(u,v,w) = (-3u^2, -3uv + 2v^2 - 3w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(0,3)$.

1) 2.00108×10^6

2) 3.17479×10^6

3) 2.30234×10^6

4) 2.91724×10^6

5) 3.93443×10^6

Exercise 2

Given the system

$-vy + 2uxy + 3y^3 == -183$
$-x^3 + 2xy == 45$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point $p=(x,y,u,v,w)=(-3,-3,-5,-4,0)$. Compute if possible $\frac{\partial y}{\partial u}(-5,-4,0)$.

1) $\frac{\partial y}{\partial u}(-5,-4,0) = -\frac{39}{241}$

2) $\frac{\partial y}{\partial u}(-5,-4,0) = -\frac{197}{1205}$

3) $\frac{\partial y}{\partial u}(-5,-4,0) = -\frac{196}{1205}$

4) $\frac{\partial y}{\partial u}(-5,-4,0) = -\frac{194}{1205}$

5) $\frac{\partial y}{\partial u}(-5,-4,0) = -\frac{198}{1205}$

Exercise 3

Given the function

$f(x,y,z)=5-4x+x^2-2y+y^2+z^2$ defined over the domain D≡ $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.751203, -2.23898, -0.4}

2) We have a maximum at {-1.2512, -2.33898, 0.4}

3) We have a maximum at {-1.3512, -2.33898, 0.5}

4) We have a maximum at {-0.951203, -2.63898, 0.}

5) We have a maximum at {2, 1, 0}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2x^2+4x^2y}{-2y^2+xy^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=2x^3-2y^3$ defined over the domain D≡ $3x^2+6y^2\leqslant 27$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.5****

5) The value of the maximum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 35

Exercise 1

Given the functions

$f(x,y,z) = (-x + 2x^2 + 2y + 3z, 2y, -1 - x^2 + z, -2x + 2y^2)$

and

$g(u_1,u_2,u_3,u_4) = (u_1^2 + 2u_1u_4, u_1 + 3u_2^2 + u_3^2 - u_3u_4, u_1u_2 - u_3 - u_2u_3 + 2u_3^2 - 3u_2u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(1,-2,0).

1) -45480.

2) -60580.5

3) -23436.9

4) -25808.1

5) -19279.3

Exercise 2

Given the system

$3u^2y + 3uz == -126$
$2uvz + 2xz^2 == -4$
$-1 - 3wxz == 89$

determine if it is possible to solve for variables x,y,z in terms of variables u,v,w arround the point p=(x,y,z,u,v,w)=(-5,-4,-2,3,-3,-3). Compute if possible $\frac{\partial y}{\partial w}(3,-3,-3)$.

1) $\frac{\partial y}{\partial w}(3,-3,-3) = -\frac{16}{9}$

2) $\frac{\partial y}{\partial w}(3,-3,-3) = -\frac{19}{9}$

3) $\frac{\partial y}{\partial w}(3,-3,-3) = -\frac{17}{9}$

4) $\frac{\partial y}{\partial w}(3,-3,-3) = -2$

5) $\frac{\partial y}{\partial w}(3,-3,-3) = -\frac{20}{9}$

Exercise 3

Given the function

$f(x,y,z) = -2 + 4x - x^2 + 2y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-1.02172, -3.21351, -0.275378}

2) We have a minimum at {-1.52172, -3.31351, -0.475378}

3) We have a minimum at {-1.12172, -2.91351, -0.375378}

4) We have a minimum at {2, 1, 2}

5) We have a minimum at {-1.62172, -2.81351, -0.175378}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^2 + y + 2y^2}{x - 3xy}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 2x^3 - 4y^3$ defined over the domain D≡ $12x^2 + 12y^2 \leqslant 240$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.9****

5) The value of the maximum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 36

Exercise 1

Given the functions

$f(x,y) = (-1 + 2x^2 - 3y - 2xy - y^2, 3 - 3x + 2x^2 - 2xy + 3y^2, 2 - 2x - 3x^2 - 2y - xy + 2y^2, -x + x^2 - 3y + 3xy - 3y^2)$

and

$g(u_1,u_2,u_3,u_4) = (1 - u_1 + 3u_2 - 2u_3^2 + 2u_1u_4 - 2u_2u_4, 1 - 2u_1 - 3u_2u_3 + 3u_3u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-3,-3)$.

1) 1.22598×10^6

2) 936 845.

3) 655 964.

4) 776 370.

5) 1.28047×10^6

Exercise 2

Given the system

$2yu_2^2 - xu_3u_5 == 52$

$y^2 + 3xy^2 - 2u_1^2u_5 == 24$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4,u_5 arround the point $p = (x,y,u_1,u_2,u_3,u_4,u_5) = (-3,-1,2,-2,-5,-2,-4)$. Compute if possible $\frac{\partial x}{\partial u_5}(2,-2,-5,-2,-4)$.

1) $\frac{\partial x}{\partial u_5}(2,-2,-5,-2,-4) = -\frac{22}{43}$

2) $\frac{\partial x}{\partial u_5}(2,-2,-5,-2,-4) = -\frac{21}{43}$

3) $\frac{\partial x}{\partial u_5}(2,-2,-5,-2,-4) = -\frac{20}{43}$

4) $\frac{\partial x}{\partial u_5}(2,-2,-5,-2,-4) = -\frac{18}{43}$

5) $\frac{\partial x}{\partial u_5}(2,-2,-5,-2,-4) = -\frac{19}{43}$

Exercise 3

Given the function

$f(x,y,z) = -2x + x^2 + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-4.9, 0.5, 0.5}

2) We have a maximum at {-5.1, 0.2, 0.5}

3) We have a maximum at {-5., 0., 0.}

4) We have a maximum at {1, 0, 0}

5) We have a maximum at {-4.5, -0.2, 0.1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^2 - 2y^2}{9x - 17x^2 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -x^3 + 5y^3$ defined over the domain D≡ $6x^2 + 45y^2 \leqslant 1716$, compute its absolute maxima and minima.

1) The value of the maximum is ****.6****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.8****

4) The value of the maximum is ****.7****

5) The value of the maximum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 37

Exercise 1

Given the functions

$f(x,y,z) = (2y, -y^2 + 2z^2)$

and

$g(u,v) = (-2 + 3u - 3u^2 - 2v - 2uv + 3v^2, -1 - 3u + 2u^2 - 2v + uv, -1 - 3u^2 + 3v + uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-1,0,1)$.

1) 0.

2) 0.840936

3) 0.803386

4) 0.679925

5) −0.245466

Exercise 2

Given the system

$$3u + 3u^2 + 3y - xy^2 + uz - uyz - 3xyz == 13$$
$$2u^2x + 2uy + uz + 3x^2z + 2uz^2 == 198$$
$$2 - 3ux - 3x^2 + 2z - uxz + z^2 == -28$$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point $p=(x,y,z,u)=(2,1,4,3)$. Compute if possible $\frac{\partial z}{\partial u}(3)$.

1) $\frac{\partial z}{\partial u}(3) = -\frac{15588}{28397}$

2) $\frac{\partial z}{\partial u}(3) = -\frac{15587}{28397}$

3) $\frac{\partial z}{\partial u}(3) = -\frac{15585}{28397}$

4) $\frac{\partial z}{\partial u}(3) = -\frac{15586}{28397}$

5) $\frac{\partial z}{\partial u}(3) = -\frac{15584}{28397}$

Exercise 3

Given the function

$f(x,y,z)=14+x^2+y^2-6z+z^2$ defined over the domain $D\equiv \frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.8, -0.8, 2.2}

2) We have a minimum at {0., 0., 2.}

3) We have a minimum at {0, 0, 3}

4) We have a minimum at {-0.4, 0.4, 2.6}

5) We have a minimum at {-1., -0.4, 1.}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x^2+y^2}{-9x-17x^2+9x^3+18x^4+3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=x^3+y^3$ defined over the domain $D\equiv 3x^2+6y^2\leqslant 108$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.0****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.5****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 38

Exercise 1

Given the functions

$f(x,y,z) = (-2x^2 + y - 2z, 3x - 3yz)$

and

$g(u,v) = (-1 - 3u^2 - 2v - 3uv + v^2, 1 + 2uv, -2 + 2v - uv - 2v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(3,-1,1)$.

1) -0.257056

2) 0.

3) -0.256027

4) 0.895796

5) 0.309781

Exercise 2

Given the system

$$u x y - 2 x z + x^2 z + z^3 == 59$$
$$-3 u x^2 - u y + 2 x y == 160$$
$$-3 u x^2 + 3 x^3 + 2 z - 3 u x z - y z - u y z == 563$$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point $p=(x,y,z,u)=(5,-5,-1,-3)$. Compute if possible $\frac{\partial z}{\partial u}(-3)$.

1) $\frac{\partial z}{\partial u}(-3) = \frac{339857}{26717}$

2) $\frac{\partial z}{\partial u}(-3) = \frac{339858}{26717}$

3) $\frac{\partial z}{\partial u}(-3) = \frac{339856}{26717}$

4) $\frac{\partial z}{\partial u}(-3) = \frac{339859}{26717}$

5) $\frac{\partial z}{\partial u}(-3) = \frac{339855}{26717}$

Exercise 3

Given the function

$f(x,y,z)=-7+4x-x^2+4y-y^2+2z-z^2$ defined over the domain D≡ $\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {2.58133, 0.604834, -0.127805}

2) We have a maximum at {1.72089, 1.46528, 0.732639}

3) We have a maximum at {1.37671, 1.80946, -0.127805}

4) We have a maximum at {2.40924, 0.604834, 0.388461}

5) We have a maximum at {2, 2, 1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-3x^2+xy^2}{3xy+3x^2y+2y^3}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-3x^3-2y^3$ defined over the domain D≡ $27x^2+6y^2\leqslant 996$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.9****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.5****

5) The value of the maximum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 39

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4)=(-2+3x_3^2+x_4-2x_1x_4,2x_1x_2-3x_1x_4+2x_2x_4,1-2x_1^2-x_1x_2+3x_3+x_4,2x_1x_3)$

and

$g(u_1,u_2,u_3,u_4)=(u_1+2u_1u_2-3u_2^2+3u_3-2u_1u_3-u_3u_4+3u_4^2,2u_2^2-3u_3+3u_1u_3$
$,3u_1^2-3u_1u_3-2u_2u_3-u_3^2-u_1u_4,-3u_1-u_1^2+3u_1u_2+3u_1u_3-2u_2u_3-2u_3^2+u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-1,1,0,2)$.

1) 1.49194×10^6

2) 5.24492×10^6

3) 3.34354×10^6

4) 1.6039×10^6

5) 2.1373×10^6

Exercise 2

Given the system

$x_1^2 x_3 == 125$
$v^2+u^2 w-2wx_1x_4 == 5$
$-3wx_2x_4 == -24$
$-2v+2w^2x_2+2wx_4+3vwx_4 == 62$

determine if it is possible to solve for variables x_1,x_2,x_3 $,x_4$ in terms of variables u,v,w arround the point $p=(x_1,x_2,x_3,x_4,u,v,w)=(-5,2,5,-1,3,-1,-4)$. Compute if possible $\frac{\partial x_4}{\partial u}(3,-1,-4)$.

1) $\frac{\partial x_4}{\partial u}(3,-1,-4)=0$

2) $\frac{\partial x_4}{\partial u}(3,-1,-4)=3$

3) $\frac{\partial x_4}{\partial u}(3,-1,-4)=1$

4) $\frac{\partial x_4}{\partial u}(3,-1,-4)=2$

5) $\frac{\partial x_4}{\partial u}(3,-1,-4)=4$

Exercise 3

Given the function

$f(x,y,z) = -9 + 4x - x^2 + 4y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-3.43553, -3.23553, -0.3}

2) We have a minimum at {-3.63553, -3.93553, 0.1}

3) We have a minimum at {-3.53553, -3.53553, 0.}

4) We have a minimum at {-3.93553, -3.03553, -0.2}

5) We have a minimum at {2, 2, 0}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-x + y}{-x + 4x^2 + 2x^3 + y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = x^3 + 2y^3$ defined over the domain D≡ $3x^2 + 15y^2 \leqslant 387$, compute its absolute maxima and minima.

1) The value of the maximum is ****.5****

2) The value of the maximum is ****.4****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.3****

5) The value of the maximum is ****.1****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 40

Exercise 1

Given the functions

$f(x,y) = (3 + x + y - 2xy, -3 + 3x + 3x^2 + 3y - 3y^2, 2 + x - x^2 - 3xy + 3y^2)$

and

$g(u,v,w) = (-1 + v^2 + 2uw, u^2 + v + uv + 3v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (-1,-3)$.

1) -1.44006×10^6

2) $-162928.$

3) -1.90373×10^6

4) -1.1926×10^6

5) $-321570.$

Exercise 2

Given the system

$-1 + wx - w^2 y == -91$
$-3v^2 - 2v^3 + uvx + y + 2x^2 y == 21$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point $p = (x,y,u,v,w) = (2,4,-5,1,5)$. Compute if possible $\frac{\partial x}{\partial w}(-5,1,5)$.

1) $\frac{\partial x}{\partial w}(-5,1,5) = \frac{23}{40}$

2) $\frac{\partial x}{\partial w}(-5,1,5) = \frac{21}{40}$

3) $\frac{\partial x}{\partial w}(-5,1,5) = \frac{11}{20}$

4) $\frac{\partial x}{\partial w}(-5,1,5) = \frac{19}{40}$

5) $\frac{\partial x}{\partial w}(-5,1,5) = \frac{1}{2}$

Exercise 3

Given the function

$f(x,y,z) = -6 + 6x - x^2 + 2y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.559116, -5.26647, -0.2}

2) We have a minimum at {3, 1, 0}

3) We have a minimum at {-0.359116, -4.36647, -0.4}

4) We have a minimum at {-0.159116, -5.16647, 0.1}

5) We have a minimum at {-0.459116, -4.86647, 0.}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^2 y + 2 x y^2 + 3 y^3}{x^2 + y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = x^3 - 3y^3$ defined over the domain D≡
$6x^2 + 27y^2 \leqslant 1068$, compute its absolute maxima and minima.

1) The value of the minimum is ****.1****

2) The value of the minimum is ****.9****

3) The value of the minimum is ****.8****

4) The value of the minimum is ****.3****

5) The value of the minimum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 41

Exercise 1

Given the functions

$f(x,y,z) = (2y^2, 2yz + 2z^2, 1 + 3y^2 - 3z - 3z^2)$

and

$g(u,v,w) = (3 + 2v^2 - 2w - 3vw, -3 + 2v + 3w^2, -u^2 - 3v^2 - w + 3uw + 3w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2,-3,-3)$.

1) -0.827687

2) 0.

3) -0.819128

4) 0.313637

5) 0.510141

Exercise 2

Given the system

$-1 - 2v - 2vx^2 + 2x^2z == -99$
$-3u^2 - 2u^2x - 3v^2x - 3y + 2xz == 117$
$-2v^2x + 3vx^2 - 2vy^2 - 3y^3 + 2vxz == 135$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point $p = (x,y,z,u,v) = (-4,-3,-2,4,1)$. Compute if possible $\frac{\partial x}{\partial u}(4,1)$.

1) $\frac{\partial x}{\partial u}(4,1) = \frac{922}{603}$

2) $\frac{\partial x}{\partial u}(4,1) = \frac{307}{201}$

3) $\frac{\partial x}{\partial u}(4,1) = \frac{308}{201}$

4) $\frac{\partial x}{\partial u}(4,1) = \frac{920}{603}$

5) $\frac{\partial x}{\partial u}(4,1) = \frac{923}{603}$

Exercise 3

Given the function

$f(x,y,z)=20-2x+x^2-4y+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{9}+\frac{y^2}{9}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-1.00178, -1.50357, -2.50535}

2) We have a maximum at {-1.20178, -1.30357, -2.10535}

3) We have a maximum at {-0.801784, -1.60357, -2.40535}

4) We have a maximum at {-0.601784, -1.40357, -2.70535}

5) We have a maximum at {1, 2, 3}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{-2x^2+y^2}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-4x^3+5y^3$ defined over the domain D≡ $12x^2+30y^2\leqslant 528$, compute its absolute maxima and minima.

1) The value of the maximum is ****.6****

2) The value of the maximum is ****.4****

3) The value of the maximum is ****.3****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.1****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 42

Exercise 1

Given the functions

$f(x,y) = (3 - 3x + 3x^2 - 3y - xy + y^2, -1 + 2x - x^2 - 3y - 2xy + 2y^2, 2 - 2x + x^2 - y + 2xy, 3 + x - 3x^2 - 3y - 3xy)$

and

$g(u_1,u_2,u_3,u_4) = (-3u_1^2 + 2u_2 - 2u_1u_2 - 3u_3 - 3u_3^2 - 3u_2u_4, -2u_1^2 - 3u_1u_3 + u_4 - 3u_1u_4)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(-3,3).

1) 2.79599×10^6

2) 1.94279×10^6

3) 1.39891×10^6

4) 4.6134×10^6

5) 3.20364×10^6

Exercise 2

Given the system

$3xy^2 == -375$
$-3y^2u_5 - 2xu_5^2 == -125$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4,u_5 arround the point $p=(x,y,u_1,u_2,u_3,u_4,u_5) = (-5,5,-3,-5,-1,4,5)$. Compute if possible $\frac{\partial x}{\partial u_3}(-3,-5,-1,4,5)$.

1) $\frac{\partial x}{\partial u_3}(-3,-5,-1,4,5)=0$

2) $\frac{\partial x}{\partial u_3}(-3,-5,-1,4,5)=2$

3) $\frac{\partial x}{\partial u_3}(-3,-5,-1,4,5)=1$

4) $\frac{\partial x}{\partial u_3}(-3,-5,-1,4,5)=4$

5) $\frac{\partial x}{\partial u_3}(-3,-5,-1,4,5)=3$

Exercise 3

Given the function

$f(x,y,z)=7-4x+x^2-6y+y^2+z^2$ defined over the domain D≡ $\frac{x^2}{25}+\frac{y^2}{25}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {3.5, 2.7, 1.2}

2) We have a minimum at {2, 3, 0}

3) We have a minimum at {1.1, 1.8, -0.6}

4) We have a minimum at {2.3, 2.4, 0.9}

5) We have a minimum at {2.3, 2.1, -0.9}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{4xy^2}{x^2+y^2}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=3x^3+5y^3$ defined over the domain D≡ $18x^2+15y^2\leqslant 348$, compute its absolute maxima and minima.

1) The value of the maximum is ****.9****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.6****

5) The value of the maximum is ****.2****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 43

Exercise 1

Given the functions

$f(x,y) = (2 + x - 3x^2 + y - xy - 2y^2, 2 - 3x - 2x^2 - 2y - xy + 3y^2)$

and

$g(u,v) = (-1 + u + 3u^2 - 2v - 3uv - 2v^2, 3 - 2u + 2u^2 - 3v - 3uv)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(2,0).

1) -10 263.5

2) -7651.92

3) -18 065.7

4) -1949.67

5) -12 243.

Exercise 2

Given the system

$-2u^2 v - 3x^2 + 3uxy - vxy - uy^2 == 277$
$-3u - 2v^2 - 2vxy - 3xy^2 + 3y^3 == -612$

determine if it is possible to solve for variables x,y in terms of variables u,v

arround the point p=(x,y,u,v)=(5,-4,-4,-4). Compute if possible $\frac{\partial x}{\partial v}(-4,-4)$.

1) $\frac{\partial x}{\partial v}(-4,-4) = \frac{2}{23}$

2) $\frac{\partial x}{\partial v}(-4,-4) = \frac{16}{161}$

3) $\frac{\partial x}{\partial v}(-4,-4) = \frac{12}{161}$

4) $\frac{\partial x}{\partial v}(-4,-4) = \frac{13}{161}$

5) $\frac{\partial x}{\partial v}(-4,-4) = \frac{15}{161}$

Exercise 3

Given the function

$f(x,y,z) = 15 - 6x + x^2 + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {-1., 0., 3.4641}

2) We have a maximum at {-1.34641, 1.73205, 4.50333}

3) We have a maximum at {-2.73205, 1.03923, 2.07846}

4) We have a maximum at {3, 0, 0}

5) We have a maximum at {-2.38564, -1.38564, 5.19615}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 + y^3}{4x - 4x^2 - 3x^3 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 4x^3 + 4y^3$ defined over the domain $D \equiv 30x^2 + 30y^2 \leq 1500$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.1****

3) The value of the maximum is ****.0****

4) The value of the maximum is ****.2****

5) The value of the maximum is ****.5****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 44

Exercise 1

Given the functions

$f(x,y,z) = (-x + 2z + 3z^2, xz - 3yz)$

and

$g(u,v) = (3 + 3u + 2u^2 - v - 2uv + 2v^2, -1 + 3u + 3u^2 - v - 3uv - v^2, -u + 3u^2 + v - 3uv + v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(0,1,2).

1) 0.329454

2) 0.46016

3) 0.215826

4) 0.

5) 0.752798

Exercise 2

Given the system

$$u x z + x y z + 3 z^3 == -12$$
$$-3 x^3 - 2 y - 3 u^2 y + 2 x^2 y + 3 y^3 - 2 u x z == 910$$
$$2 x + u^2 y - 3 y^2 - u z + 2 u x z + 3 u y z + 2 y^2 z + 2 z^3 == -21$$

determine if it is possible to solve for variables x,y,z in terms of variable u arround the point p=(x,y,z,u)=(-5,5,1,-2). Compute if possible $\frac{\partial y}{\partial u}(-2)$.

1) $\frac{\partial y}{\partial u}(-2) = -\frac{24944}{16655}$

2) $\frac{\partial y}{\partial u}(-2) = -\frac{24941}{16655}$

3) $\frac{\partial y}{\partial u}(-2) = -\frac{4988}{3331}$

4) $\frac{\partial y}{\partial u}(-2) = -\frac{24943}{16655}$

5) $\frac{\partial y}{\partial u}(-2) = -\frac{24942}{16655}$

Exercise 3

Given the function

$f(x,y,z)=23-6x+x^2-2y+y^2-6z+z^2$ defined over the domain D≡ $\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {−2.6079, −0.524606, −2.2079}

2) We have a maximum at {3, 1, 3}

3) We have a maximum at {−1.8079, −0.124606, −2.4079}

4) We have a maximum at {−1.6079, −0.324606, −1.9079}

5) We have a maximum at {−2.1079, −0.224606, −2.1079}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{2(x-y)}{-x+2x^2+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=5x^3-4y^3$ defined over the domain D≡ $45x^2+18y^2\leqslant 1782$, compute its absolute maxima and minima.

1) The value of the minimum is ****.2****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.7****

4) The value of the minimum is ****.0****

5) The value of the minimum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 45

Exercise 1

Given the functions

$f(x,y,z) = (-3x + 3x^2 - xy, -3xy + xz + 3z^2, 3y^2 + 3z + xz + 2yz, -1 + 3x^2)$

and

$g(u_1,u_2,u_3,u_4) = (3u_2 - 2u_1u_2 - u_2^2 + u_3 + u_4 - 3u_2u_4 + 2u_4^2, u_1 - 2u_2^2 - u_2u_3 + 2u_2u_4 + 3u_4^2, -2u_1 - 2u_1u_2 - 3u_3^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(2,-3,-2).

1) 1.03326×10^{10}

2) 1.73612×10^{10}

3) 6.25635×10^{9}

4) 6.59795×10^{9}

5) 1.88534×10^{10}

Exercise 2

Given the system

$2x^2 z == -90$
$-vx - wy - 3vwy == 15$
$3ux + 2vxy == -63$

determine if it is possible to solve for variables x,y,z in terms of variables u,v,w arround the point p=(x,y,z,u,v,w)=(-3,-3,-5,5,-1,-3). Compute if possible $\frac{\partial z}{\partial u}(5,-1,-3)$.

1) $\frac{\partial z}{\partial u}(5,-1,-3) = -\frac{13}{11}$

2) $\frac{\partial z}{\partial u}(5,-1,-3) = -1$

3) $\frac{\partial z}{\partial u}(5,-1,-3) = -\frac{15}{11}$

4) $\frac{\partial z}{\partial u}(5,-1,-3) = -\frac{12}{11}$

5) $\frac{\partial z}{\partial u}(5,-1,-3) = -\frac{14}{11}$

Exercise 3

Given the function

$f(x,y,z) = -9 + 6x - x^2 + 2y - y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {2.56502, 0.116243, -0.443604}

2) We have a maximum at {1.42501, 1.82626, 0.411403}

3) We have a maximum at {2.85002, 0.971251, 0.981408}

4) We have a maximum at {3.99003, 1.25625, 0.696406}

5) We have a maximum at {3, 1, 1}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{x^2 - 3y^2}{6x - 5x^2 - 3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = -5x^3 + y^3$ defined over the domain D≡ $15x^2 + 9y^2 \leq 384$, compute its absolute maxima and minima.

1) The value of the minimum is ****.6****

2) The value of the minimum is ****.4****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.8****

5) The value of the minimum is ****.3****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 46

Exercise 1

Given the functions

$f(x,y) = (x - 3x^2 + 2y + xy - 2y^2, 2 - x - 2x^2 + y + 3xy - 2y^2)$

and

$g(u,v) = (3 - 2u + 2u^2 + uv + 2v^2, 2 - 3u + 3uv - v^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(2,2).

1) -2337.97

2) -2946.

3) -1411.9

4) -1956.6

5) -3455.46

Exercise 2

Given the system

$u^2 - 2x - 3ux^2 + 3x^3 + 2uy + 2uxy + x^2y - xy^2 == -111$
$-u^2 + 3x - ux - 2x^2 - ux^2 - 2x^3 + 3uy + 3xy - 3uxy - 2x^2y + y^2 == 24$

determine if it is possible to solve for variables x,y in terms of variable u arround the point p=(x,y,u)=(3,-5,1). Compute if possible $\frac{\partial x}{\partial u}(1)$.

1) $\frac{\partial x}{\partial u}(1) = -\frac{287}{190}$

2) $\frac{\partial x}{\partial u}(1) = -\frac{144}{95}$

3) $\frac{\partial x}{\partial u}(1) = -\frac{29}{19}$

4) $\frac{\partial x}{\partial u}(1) = -\frac{289}{190}$

5) $\frac{\partial x}{\partial u}(1) = -\frac{291}{190}$

Exercise 3

Given the function

$f(x,y,z)=7-6x+x^2-2y+y^2+z^2$ defined over the domain $D\equiv \frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {3, 1, 0}

2) We have a maximum at {-1.79737, -0.132456, 0.2}

3) We have a maximum at {-1.59737, -0.832456, -0.4}

4) We have a maximum at {-1.69737, -0.132456, -0.4}

5) We have a maximum at {-1.89737, -0.632456, 0.}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} -\frac{3x^2}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=5x^3+2y^3$ defined over the domain $D\equiv 30x^2+9y^2\leqslant 561$, compute its absolute maxima and minima.

1) The value of the maximum is ****.2****

2) The value of the maximum is ****.6****

3) The value of the maximum is ****.9****

4) The value of the maximum is ****.0****

5) The value of the maximum is ****.5****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 47

Exercise 1

Given the functions

$f(x,y,z) = (1 + 2 x y - 3 x z - z^2, x^2 + y + 2 y z + 3 z^2, 2 x z - y z)$

and

$g(u,v,w) = (2 u v, 2 u + 3 u v - 3 w + 3 v w - 3 w^2, -2 v)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p = (2,2,-2)$.

1) 253 933.

2) 645 967.

3) 510 300.

4) 416 160.

5) 507 910.

Exercise 2

Given the system

$$x - 2 x^3 - 3 x y^2 + v x z + v y z - 2 x y z - z^2 + 3 z^3 = -32$$
$$-2 v x - 3 v y - u v y + 2 v^2 y + x y^2 - x z^2 = 24$$
$$-3 x^2 y = -12$$

determine if it is possible to solve for variables x,y,z in terms of variables u,v arround the point $p = (x,y,z,u,v) = (2,1,-1,-5,4)$. Compute if possible $\frac{\partial x}{\partial u}(-5,4)$.

1) $\frac{\partial x}{\partial u}(-5,4) = -\frac{17}{231}$

2) $\frac{\partial x}{\partial u}(-5,4) = -\frac{6}{77}$

3) $\frac{\partial x}{\partial u}(-5,4) = -\frac{16}{231}$

4) $\frac{\partial x}{\partial u}(-5,4) = -\frac{5}{77}$

5) $\frac{\partial x}{\partial u}(-5,4) = -\frac{19}{231}$

Exercise 3

Given the function

$f(x,y,z)=3+4x-x^2+2y-y^2-z^2$ defined over the domain $D\equiv \frac{x^2}{25}+\frac{y^2}{9}+\frac{z^2}{9}\leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {2, 1, 0}

2) We have a minimum at {-5.36678, -0.245266, -0.1}

3) We have a minimum at {-4.56678, -0.645266, 0.3}

4) We have a minimum at {-4.76678, -0.0452656, -0.2}

5) We have a minimum at {-4.96678, -0.345266, 0.}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3(x^2+y^2)}{9x+17x^2-3y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=-5x^3-4y^3$ defined over the domain $D\equiv 15x^2+30y^2\leqslant 810$, compute its absolute maxima and minima.

1) The value of the minimum is ****.3****

2) The value of the minimum is ****.1****

3) The value of the minimum is ****.2****

4) The value of the minimum is ****.6****

5) The value of the minimum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 48

Exercise 1

Given the functions

$f(x_1,x_2,x_3,x_4) = (x_1 - x_2 + x_4 + 3 x_2 x_4, -3 x_1^2 + 2 x_2 x_3 + x_3^2 - 3 x_3 x_4 - 2 x_4^2, 3 x_1 x_2 - x_2^2 - x_3^2 + x_1 x_4 + x_2 x_4)$

and

$g(u,v,w) = (-3 u^2 + 3 v w - 3 w^2, 1 + 2 v, -2 u + 3 v - 3 u v + 2 u w - v w + w^2, w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-1,0,-2,2)$.

1) 0.749632

2) 0.651128

3) 0.

4) 0.821696

5) -0.405115

Exercise 2

Given the system

$u^3 - 3 x_1 x_2^2 + 3 x_2 x_4 == -61$
$-u x_2 x_4 == 12$
$-3 v x_1 x_2 == 24$
$3 x_3 - 2 x_1 x_3 - v x_4^2 == -36$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p=(x_1,x_2,x_3,x_4,u,v)=(2,-1,0,-3,-4,4)$. Compute if possible $\frac{\partial x_2}{\partial u}(-4,4)$.

1) $\frac{\partial x_2}{\partial u}(-4,4) = -\frac{67}{8}$

2) $\frac{\partial x_2}{\partial u}(-4,4) = -\frac{33}{4}$

3) $\frac{\partial x_2}{\partial u}(-4,4) = -8$

4) $\frac{\partial x_2}{\partial u}(-4,4) = -\frac{63}{8}$

5) $\frac{\partial x_2}{\partial u}(-4,4) = -\frac{65}{8}$

Exercise 3

Given the function

$f(x,y,z)=10+x^2-2y+y^2+z^2$ defined over the domain $D\equiv$ $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{4}\leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0, 1, 0}

2) We have a maximum at {3.61403, -1.28571, 0.}

3) We have a maximum at {4.69824, -0.924311, 1.80702}

4) We have a maximum at {2.52982, -3.09273, -1.08421}

5) We have a maximum at {2.16842, -1.64712, 1.08421}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)}\frac{2x^3+y^3}{-2x-4x^2+x^3+4x^4+y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y)=x^3+5y^3$ defined over the domain $D\equiv$ $6x^2+30y^2\leqslant 576$, compute its absolute maxima and minima.

1) The value of the maximum is ****.8****

2) The value of the maximum is ****.5****

3) The value of the maximum is ****.6****

4) The value of the maximum is ****.3****

5) The value of the maximum is ****.4****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 49

Exercise 1

Given the functions

$f(x,y) = (-3x - 2x^2 + xy + 2y^2, 3x^2 + xy - 2y^2, 2x - 3x^2 - 2y)$

and

$g(u,v,w) = (uv - 3v^2 - 2w + 2w^2, w - 3uw - 3vw + 2w^2)$,

compute the determinant of the Jacobian matrix of the composition gof at the point $p=(-1,0)$.

1) 16462.

2) 27367.1

3) 9007.58

4) 27220.5

5) 3601.45

Exercise 2

Given the system

$-x^2 y == 48$
$-3xyu_5 + 3u_1u_2u_5 == -255$

determine if it is possible to solve for variables x,y in terms of variables u_1,u_2,u_3,u_4,u_5 arround the point $p=(x,y,u_1,u_2,u_3,u_4,u_5) = (-4,-3,-1,5,-4,1,5)$. Compute if possible $\frac{\partial x}{\partial u_4}(-1,5,-4,1,5)$.

1) $\frac{\partial x}{\partial u_4}(-1,5,-4,1,5)=4$

2) $\frac{\partial x}{\partial u_4}(-1,5,-4,1,5)=2$

3) $\frac{\partial x}{\partial u_4}(-1,5,-4,1,5)=1$

4) $\frac{\partial x}{\partial u_4}(-1,5,-4,1,5)=0$

5) $\frac{\partial x}{\partial u_4}(-1,5,-4,1,5)=3$

Exercise 3

Given the function

$f(x,y,z) = -2 - 2x + x^2 - 2y + y^2 - 4z + z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{4} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {1.8, 1.6, 2.4}

2) We have a minimum at {1, 1, 2}

3) We have a minimum at {1.4, 2., 3.}

4) We have a minimum at {1.4, 0.2, 2.6}

5) We have a minimum at {0.8, 0., 1.2}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 3y^3}{6x + 6x^2 - 11x^3 - 2y}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 5x^3 + 5y^3$ defined over the domain D≡ $45x^2 + 45y^2 \leqslant 3240$, compute its absolute maxima and minima.

1) The value of the maximum is ****.4****

2) The value of the maximum is ****.7****

3) The value of the maximum is ****.5****

4) The value of the maximum is ****.1****

5) The value of the maximum is ****.0****

Further Mathematics - 2023/2024
Exam - 1 - Multivariate Functions for serial number: 50

Exercise 1

Given the functions

$f(x,y) = (3 - 3x + 2x^2 + y - xy + y^2, 2 - x - 2x^2 - y - xy + 3y^2, 1 - x + 3x^2 + 2y - 3xy + 2y^2)$

and

$g(u,v,w) = (2u^2, u - w)$,

compute the determinant of the Jacobian matrix of the composition gof at the point p=(-2,-2).

1) 543.713

2) 420.

3) 786.03

4) 595.241

5) 348.784

Exercise 2

Given the system

$3uw^2 + xy == -3$
$uvw - 2x^2y + 2uy^2 - vy^2 == 14$

determine if it is possible to solve for variables x,y in terms of variables u,v,w arround the point p=(x,y,u,v,w)=(-1,-3,-2,-4,1). Compute if possible $\frac{\partial y}{\partial w}(-2,-4,1)$.

1) $\frac{\partial y}{\partial w}(-2,-4,1) = -26$

2) $\frac{\partial y}{\partial w}(-2,-4,1) = -27$

3) $\frac{\partial y}{\partial w}(-2,-4,1) = -25$

4) $\frac{\partial y}{\partial w}(-2,-4,1) = -24$

5) $\frac{\partial y}{\partial w}(-2,-4,1) = -28$

Exercise 3

Given the function

$f(x,y,z) = 16 - 6x + x^2 - 2y + y^2 + z^2$ defined over the domain $D \equiv \frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.459116, -4.86647, 0.}

2) We have a maximum at {-0.0591159, -4.76647, 0.4}

3) We have a maximum at {3, 1, 0}

4) We have a maximum at {-0.259116, -5.06647, -0.1}

5) We have a maximum at {-0.259116, -4.56647, 0.4}

Exercise 4

Study the limit, $\lim_{(x,y)\to(0,0)} \frac{6y^2}{\sqrt{x^2+y^2}}$.

1) The limit exists.

2) For any line passing through the point we obtain the same limit
but there is a parabolic curve along which we obtain different limit.

3) We obtain different limit for different lines passing through the point.

Exercise 5

Given the function

$f(x,y) = 3x^3 + y^3$ defined over the domain $D \equiv 27x^2 + 3y^2 \leqslant 984$, compute its absolute maxima and minima.

1) The value of the minimum is ****.0****

2) The value of the minimum is ****.9****

3) The value of the minimum is ****.7****

4) The value of the minimum is ****.2****

5) The value of the minimum is ****.3****