A GRAVIMETRIC GEOID COMPUTATION AND COMPARISON WITH GPS RESULTS IN NORTHERN ANDALUSIA (SPAIN)

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Summary: *Two new GPS surveys have been carried out to check the accuracy of an existing gravimetric geoid in a test area located in northern Andalusia (Spain). The fast collocation method and the remove-restore procedure have been used for the computation of the quasigeoid model. The Spanish height system is based on orthometric heights, so the gravimetrically determined quasigeoid has been transformed to a geoid model and then compared to geoid undulations provided by GPS and levelling at benchmarks belonging to the Spanish first-order levelling network. The discrepancies between the gravimetric solution and GPS/levelling undulations amount to ±2 cm for one survey and ±5 cm for another after fitting a plane to the geoid model.*

Keywords: GPS, levelling, quasigeoid, fast collocation, RTM.

1. INTRODUCTION

Most of the Iberian Peninsula ($36° \leq \varphi \leq 44°$, $-10° \leq \lambda \leq -3°$) is occupied by the Meseta, a large plateau that is almost completely surrounded by mountain ranges. A region of Andalusia has been chosen to test the accuracy of a previously computed gravimetric geoid. This geoid solution has been computed from a validated gravity data set covering the area $37.3° \leq \varphi \leq 38.5°$, $-4.3° \leq \lambda \leq -2.3°$ and a Digital Terrain Model (DTM) of the land area $37.1° \leq \varphi \leq 38.7°$, $-4.5° \leq \lambda \leq -2.1°$, having a grid spacing of 100 m. The method used to compute the quasigeoid is the remove-restore and fast collocation techniques. The residual terrain model (RTM) effects have been computed from the available DTM. The OSU91A geopotential model has been used as a reference field in order to remove and restore part of the long wavelength components of gravity and geoid, respectively. The gravimetric quasigeoid on a 3'× 3' grid in the area with limits $37.3° \leq \varphi \leq 38.5°$, $-4.3° \leq \lambda \leq -2.3°$ has been transformed to a geoid model to be compared with GPS/levelling undulations at benchmarks.

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In the first analysis of our quasigeoid, the European quasigeoid EGG97 (*Denker, 1997*) was downloaded from the International Geoid Service (IGeS) and used in the test area to contrast the quasigeoid solution.

In the analysis of the accuracy of the geoid, the differences between the gravimetric geoid and GPS/levelling data have been computed, before and after modelling the systematic biases and tilts, (*Denker and Wenzel, 1987*). In the latter case, a similarity transformation has been used and the discrepancies amounted to ±2 cm. However, this transformation also models any errors present in the GPS and levelling data, thus sometimes giving over-optimistic error estimates. In order to test the accuracy obtained, a second GPS survey at different benchmarks has been performed independently of the first. The results of the comparisons between the geoid and GPS/levelling data, before and after fitting a plane, are presented in Section 3.

2. GPS SURVEYS

Two GPS surveys have been carried out to study the accuracy of the gravimetric geoid in a test area. Both of them have been carried out independently of time and executing agency. Therefore, the data provide a good working example of what may be achieved with GPS when used in a production environment.

GPS Network

The first GPS network was observed in October 1996. The measurement of the network baselines was carried out by collecting at each point for two hours with two dual frequency (L1 and L2 carrier phases) GPS receivers operating simultaneously in each observing session. The baseline lengths range from 3 to 30 km in length. This network contains six benchmarks of the Spanish first-order levelling network (NGO782, NGO771, NGO766, NGN657, NGN678 and NGN681), three benchmarks belong to the Spanish second-order levelling networlc (SSK138, SSK305 and SSK298) and three points to the Spanish first order geodetic network (Muela, Piedra Hincada and Atalaya de Mengíbar). Figure 1 shows the GPS network. A total of 32 baselines were observed and processed with Bernese 4.0 software (*Rothacher et al., 1996*) and precise ephemerides.

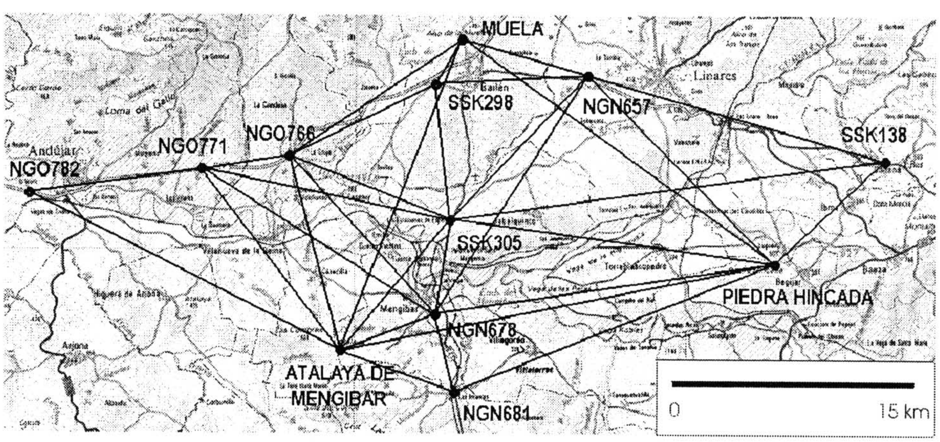

Fig. 1: GPS network established in 1996 in the test area.

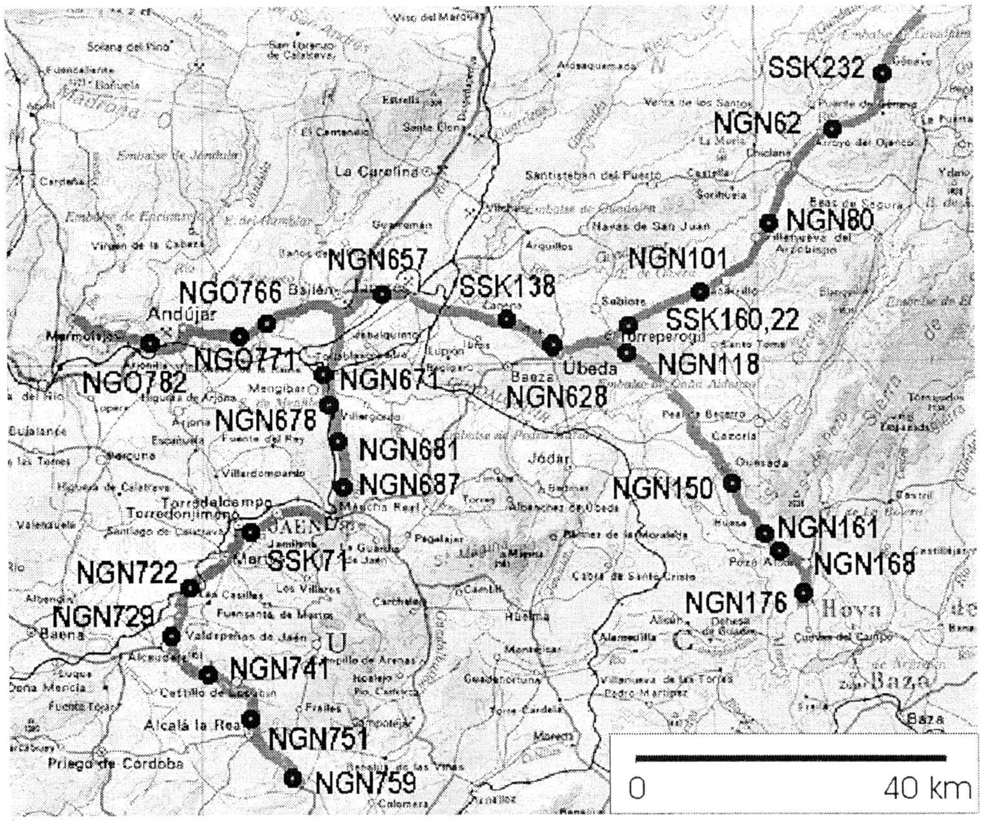

Fig. 2: GPS traverses observed in 1998 in the test area.

GPS Traverses

A second GPS survey was carried out in March 1998. These traverses contain 26 GPS control points, of which 22 are benchmarks of the Spanish first-order levelling network, and the others belong to the second-order levelling network. These latter appear in Figure 2 with the letters SSK71, SSK138, SSK232 and SSK160. For geodetic establishment, the GPS constellation was tracked for 3-4 hour sessions over baseline lengths averaging less than 21 km; 38 baselines were observed during 25 observation sessions. The equipment used throughout the survey comprised three Leica SR399 and one Leica 9500 GPS dual frequency carrier phase GPS receivers. Bernese 4.0 (*Rothacher et al., 1996*) with precise ephemerides was used to process the GPS observations, and NETGPS software (*Crespi, 1996*) was used to adjust the GPS traverses.

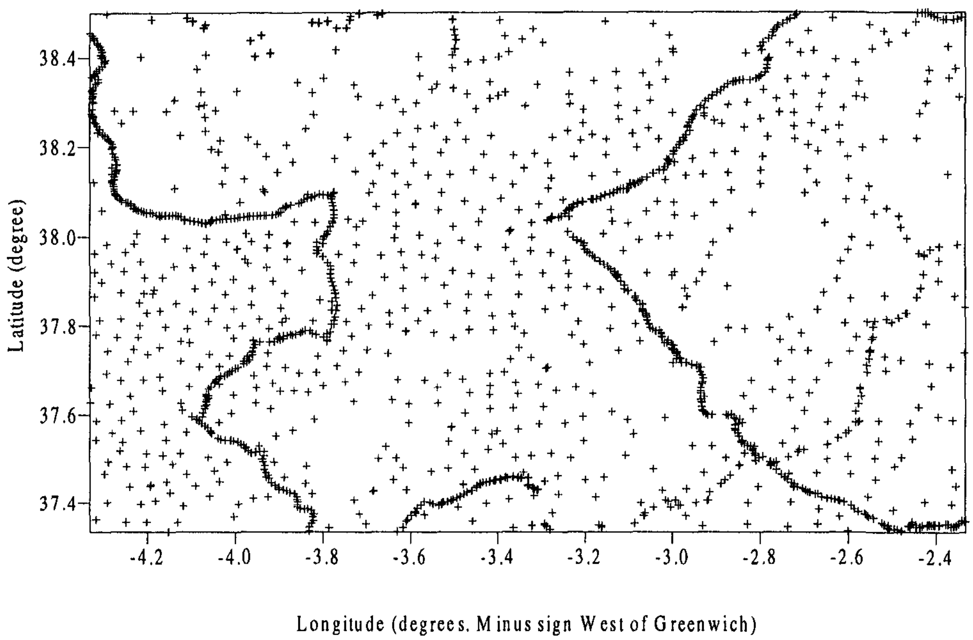

Fig. 3: Distribution of gravity obsevations in the test area.

3. QUASIGEOID COMPUTATION IN THE TEST AREA

Data used

In 1983 the Instituto Geográfico Nacional supplied Spanish gravity data with a total of 27691 free-air gravity anomalies covering the Iberian Peninsula and Baleares Islands. Gravity anomalies in the Geodetic Reference System 1980 were computed from the International Gravity Formula (1967), according to the following transformation (*National Geodetic Survey, 1986*):

$$\Delta g_{1980} = \Delta g_{1967} - \left(0.8316 + 0.0782 \sin^2 \varphi - 0.0007 \sin^4 \varphi\right) \tag{1}$$

From this data base, a set of 1874 free-air anomalies covering our test area, (Δg_{fa}), was collected. Figure 3 shows the distribution of gravity anomalies in the test area.

The high-degree global geopotential model OSU91A (*Rapp et al., 1991*) was used for this work. This model is complete up to degree and order 360. Gravity anomalies can be computed, in a spherical approximation, from the geopotential coefficient set by (*Rapp, 1997*):

$$\Delta g_n = \frac{GM}{R^2} \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n (n-1) \sum_{m=0}^{n} [\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} m\lambda]\, \overline{P}_{nm}(\cos\theta) \tag{2}$$

where θ, λ are the geocentric colatitude and longitude of the point at which Δg is to be determined; $\overline{C}_{nm}$, $\overline{S}_{nm}$ are the fully normalized spherical geopotential coefficients of the anomalous potential; $\overline{P}_{nm}$ are the fully normalized associated Legendre polynomials; $n_{\max}$ is the maximum degree of the geopotential model, and GM/R^2 is the mean gravity.

The RTM (*Forsberg, 1994*) effect was computed using the DTM provided by Servicio Geográfico del Ejército. This data set contains 3001 × 2001 heights on a regular UTM grid of 100m × 100m spacing.

Quasigeoid Solution

The fast collocation and the remove-restore procedures were used to compute the quasigeoid estimate (*Barzaghi et al., 1996*). In order to obtain a smoothed gravity field as input for the collocation procedure, gravity data must be reduced for the geopotential model and residual terrain effect. In the following the computation process is summarized:

- The spherical harmonic coefficient set 05U91A was used to remove the long wave length component of the gravity data:

$$\Delta g_{fa} - \Delta g_{\mathrm{mod}} \tag{3}$$

- The RTM effect (g_{rtm}) was computed using the formulas for the gravitational effects of a homogeneous rectangular prism with the available DTM and with respect to a 10' × 10' reference grid. Only residual topography out to a distance of 38 km was taken into account. To select this value, contributions of the residual terrain model for some points to different distances from the calculation point were computed and the value beyond, whose results were quite similar, was chosen, showing that the effect was negligible beyond this distance (*Gil et al., 1993*). To compute the effect of topography, the TC (*Tschering et al., 1992*) program has been used. After that, the RTM effect was substracted to get the residual gravity anomalies (Δg_r):

$$\Delta g_r = \Delta g_{fa} - \Delta g_{\mathrm{mod}} - g_{rtm} \tag{4}$$

  Finally, a set of 1874 residual gravity anomalies was available.
- In order to detect outliers in this data set, the differences between the observed and predicted values were compared (*Tscherning, 1991*).

  The gravity anomaly at point P, $\overline{\Delta g_p}$, is predicted from a set of values Δg_i, $i = 1, 2, ..., n$ in the neighbourhood as regularly as possible in all directions by,

$$\overline{\Delta g_p} = \sum_{i=1}^{n} a_i \Delta g_i \tag{5}$$

  where

$$a_i = C_{ip}^T \left(C_{ij} + D_{ij}\right)^{-1} \tag{6}$$

with,

C_{ip}: the covariance between observation i and predicted value p

C_{ij}: the covariance of the observations

D_{ij} : the covariance of the observation errors (associated with points i and j).

The error estimate of the predicted gravity anomaly is then computed as,

$$\sigma_p^2(\Delta g_p - \overline{\Delta g_p}) = C_0 - C_{ip}^T (C_{ij} + D_{ij})^{-1} C_{jp} \tag{7}$$

where C_0 is the value of covariance function $C(\psi)$ for argument $\psi = 0$.
A measurement is rejected or considered suspect if

$$|\Delta g_p - \overline{\Delta g_p}| > k\left(\sigma_p^2(\Delta g_p - \overline{\Delta g_p}) + \sigma_p^2(\Delta g_p)\right)^{1/2} \tag{8}$$

where a value of $k = 3$ was taken, and $\sigma_p^2(\Delta g_p)$ is the error variance of observation Δg_p.
By using this technique 1.1% of the outliers were detected and removed from the data prior to quasigeoid computation.

- The gridding of Δg_r to produce a 3'× 3' grid of residual anomalies was carried out with the GEOGRID program of the GRAVSOFT package (*Tscherning et al., 1992*).
- The empirical and model covariance functions of the gravity anomalies are required to estimate the residual quasigeoid, ζ_r, via fast collocation. The covariance function model used in this work was (*Knudsen, 1987*):

$$C(P,Q) = a \sum_{n=2}^{N_{\max}} c_n \left(\frac{R^2}{rr'}\right)^{n+2} P_n(\cos(\psi)) +$$

$$+ \sum_{n=N_{\max}+1}^{\infty} \frac{A(n-1)}{(n-2)\,(n+24)} \left(\frac{R_B^2}{rr'}\right)^{n+2} P_n(\cos(\psi)) \tag{9}$$

where ψ is the spherical distance between points P and Q with radial distances r and r', $P_n(\cos(\psi))$ are the Legendre polynomials, R is the mean radius of the Earth, R_B the radius of the Bjerhammar sphere, c_n, are the error anomaly degree variances associated with the model coefficients.
The free parameters are: a, the factor to scale the error degree variances, A, the scale factor of the degree variances, and the summation limit $N_{\max}$, whose value reflects the degree to which the spherical harmonic expansion is considered reliable for the area. In this case $a = 0.525316$, $N_{\max} = 360$, $A = 68056001$.

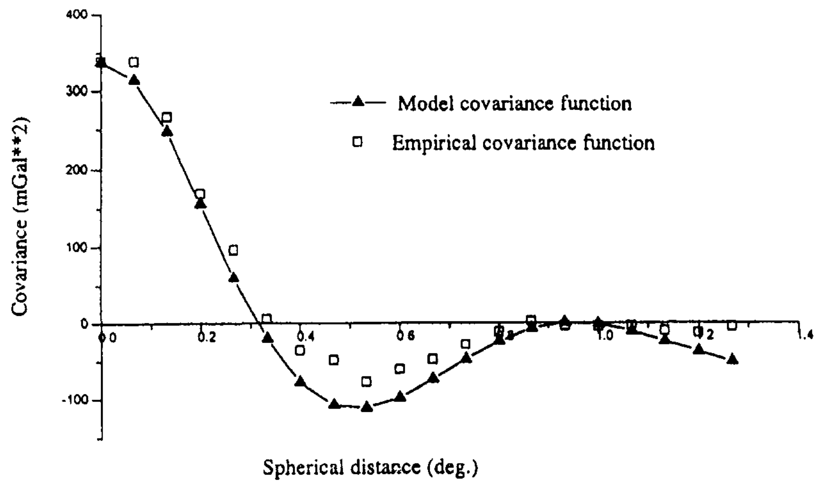

Fig. 4: The empirical and synthetic covariance functions of residual gravity.

The covariance functions used in the prediction of the height anomaly via fast collocation are plotted in Figure 4. COVFIT (*Tscherning et al., 1992*) and FASTCOLB (*Bottoni and Barzaghi, 1993*) programs have been used for the analytical approximation of the empirical covariance functions and for the computation of height anomalies.

- Restoration of the model and of the RTM efFect on the prediction grid to get the quasigeoid model (Figure 5):

$$\zeta = \zeta_r + \zeta_{rtc} + \zeta_m \tag{10}$$

- Conversion of the quasigeoid model to the geoid model (Figure 6) using the expres sion, (*Heiskanen and Moritz, 1967*):

$$(\zeta - N) \approx 0.1\, H^{av} H \tag{11}$$

where the l. h. s. of the equation is given in meters, H is the orthometric height of the station in km and H^{av} is the average height of the area considered in km

The numerical results of the estimation procedure are summarized in Table 1.

The EGG97 quasigeoid (*Denker, 1997*) was downloaded from the International Geoid Service (IGeS). It is a quasigeoid covering the area $25.083° \leq \varphi \leq 76.916°$, $-34.875° \leq \lambda \leq 67.375°$ with a grid mesh of 10'× 15'. These values are referred to the GRS80 ellipsoid. The EGG quasigeoid solution for our test area was extrapolated from the area with limits $37.3° \leq \varphi \leq 38.5°$, $-4.3° \leq \lambda \leq -2.3°$. A set of 72 points was obtained. The corresponding mean value and standard deviation are 51.23 m and 1.49 m, respectively, with values ranging from 47.76 m to 53.7 m. The statistics show good agreement between both quasigeoid solutions.

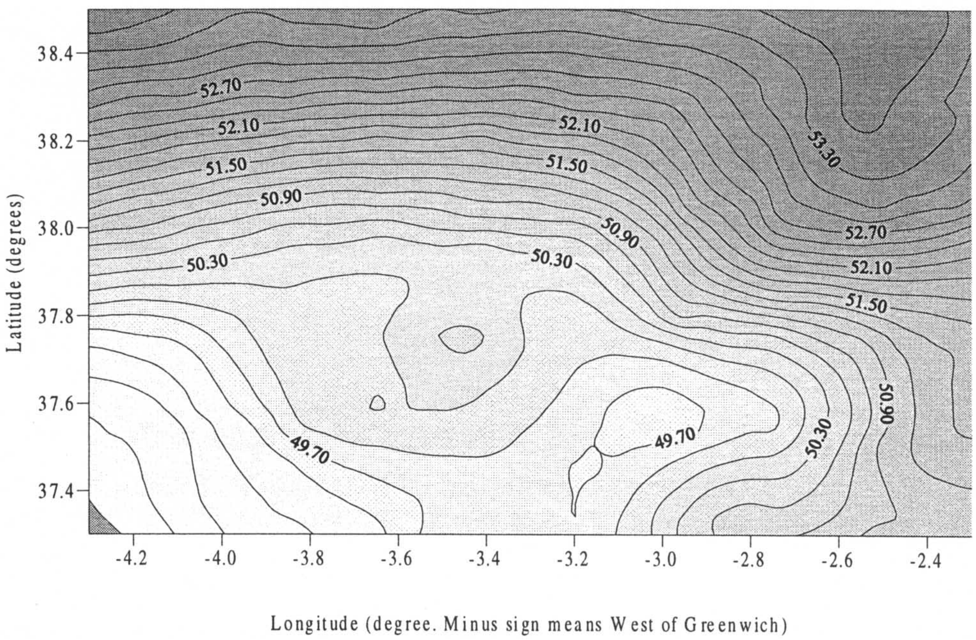

Fig. 5: Gravimetric quasigeoid solution in the test area. (Contour interval is 20 cm).

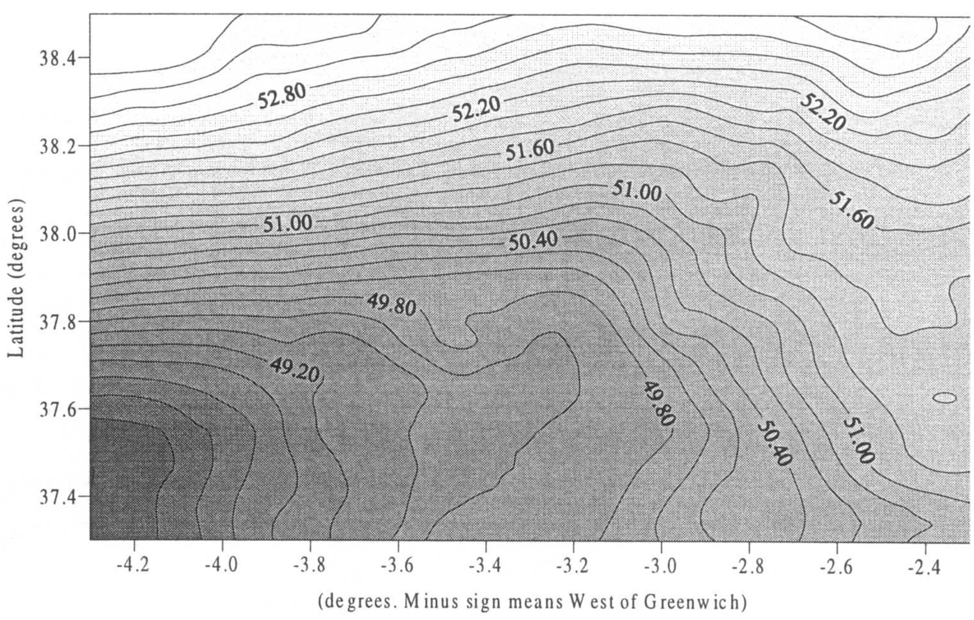

Fig. 6: Gravimetric geoid solution in the test area. (Contour interval is 20 cm).

Table 1: Statistical summary of remove-restore procedure.

	Δg_{fa} (mGal)	$\Delta g_{fa} - \Delta g_m$ (mGal)	Δg_r (mGal)	ζ_r (m)	ζ (m)	N (m)
$n°$ of points	1874	1874	1853	1025	1025	1025
mean	-2.16	-11.29	-1.38	0.28	51.18	50.90
st. dev.	30.19	23.07	18.48	0.37	1.41	1.37
max.	135.32	145.54	64.45	1.27	53.78	53.74
min.	-77.78	-81.56	-66.83	-0.43	48.52	48.18

Δg_{fa}: free-air gravity anomaly
Δg_m: model gravity anomaly
Δg_r: residual gravity anomaly
ξ_r: residual quasigeoid
ξ: quasigeoid
N: geoid

4. COMPARISON WITH GPS/LEVELLING DATA

Geoid undulations (N) have been interpolated to 12 and 26 stations belonging to the first and the second GPS surveys, respectively. The $N_{GPS/levelling}$ values were calculated at these stations by using the expression

$$N = h - H \tag{12}$$

where h is the ellipsoidal height provided by GPS and H is the orthometric height provided by the levelling and gravity surveys. Only the standard deviation of the differences between N and $N_{GPS/levelling}$ is used to give an indication of the precision of the gravimetric solution because any gravimetric determination of the geoid is deficient in the zero and first-degree terms, (*Featherstone et al., 1996*). The values of the standard deviation of the differences in Table 2 show an improvement is achieved when only benchmark points (nine points) are included in the comparison. In the second survey the differences between the statistics for 26 and 22 points are negligible. The statistics of the differences between GPS/levelling results and the geoid show a standard deviation of about 7 cm with GPS data from the first GPS survey and about 16 cm if the GPS data belonging to the second GPS survey are used.

Table 2: Statistics of the differences between GPS/levelling results and the geoid (in meters).

Before Fitting				
$n°$ of points	12	9	26	22
mean	0.43	0.40	0.30	0.31
st. dev.	0.12	0.07	0.16	0.17
max.	0.72	0.51	0.54	0.54
min.	0.27	0.28	0.02	0.02

Figure 7 shows that there are some systematic differences between the GPS/levelling and geoid values. The short wavelength differences are either due to errors in the GPS/levelling data or due to localized errors in the gravity or terrain data, or in both of them. The long wavelength discrepancy is due to one, or all, of the following factors, (*Featherstone et al, 1996*) long wavelength errors propagating from the OSU91A global geopotential model, long wavelength errors propagating from the gravity and/or terrain data. In our test area the errors from the gravity data could be present due to inhomogeneous distribution of the gravity anomalies (Figure 3).

To minimize the long wavelength error, the systematic datum difference between the gravimetric geoid and the GPS/levelling values was removed by a four-parameter transformation. The use of the datum shift eliminates the possible tilt of the gravimetric geoid as well, yielding a fit of only ±5 cm versus ±2 cm for the first GPS network. This fact proves a good fit between the geoid of the test area and GPS/levelling data has been

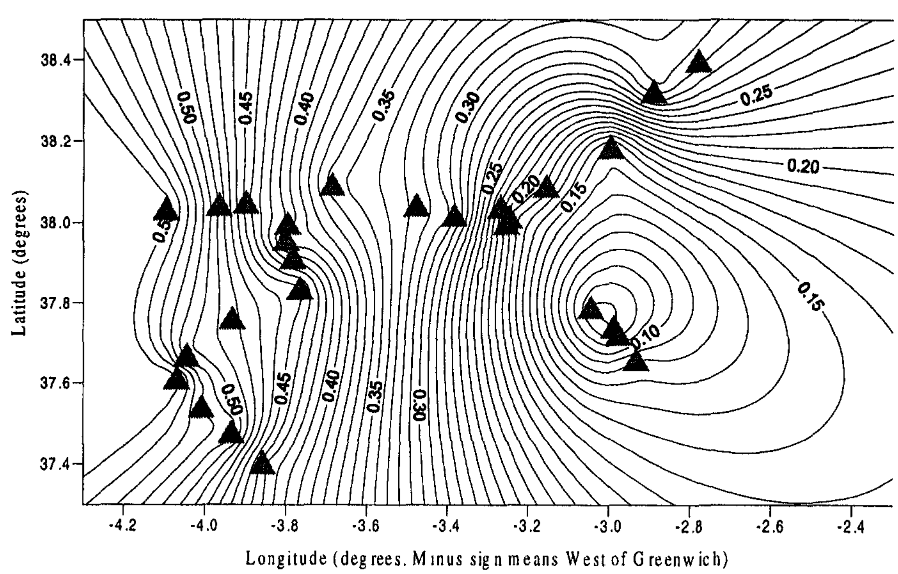

Fig. 7: Systematic differences between GPS/levelling and geoid values before fit. (Contour interval is 1 cm). Δ is the symbol used to represent the GPS benchmarks belonging to GPS traverses.

achieved, (Table 3). Nevertheless, a standard deviation of ±2 cm could be thought an over-optimistic result due to remove not only bias and tilt when a similarity transformation has been applied. Another important fact is the poor spacial density of the benchmarks in the first GPS survey.

Table 3: Comparison of GPS/levelling and the geoid in the first-order benchmarks (in meters).

After Fitting		
$n°$ of points	9	22
mean	0	0
st. dev.	0.02	0.05
max.	0.02	0.18
min.	-0.02	-0.09

5. CONCLUSIONS

This paper presents a study of the accuracy of a gravimetric geoid computed in a test area in northern Andalusia (Spain) by comparison with GPS/levelling data. The GPS measurements which cover the test area were carried out in two different GPS surveys. The standard deviations of the discrepancies between N and $N_{GPS/levelling}$ amount to ±17 and ±5 cm before and after fitting a plane, respectively. Therefore, a good fit between the gravimetric geoid solution and GPS/levelling values has been achieved.

The results of this work show the fast collocation technique and the remove-restore procedure with residual terrain modelling have been used successfully for the determination of a quasigeoid in a test area. Therefore, the same method will be applied to compute a quasigeoid over the whole of Andalusia in a near future.

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